Homework Sheet 9: Graph Connectivity: Sequential Algorithms

Due Date: AA July 2019

1. Recall the notion of a (global) mincut in a graph. Let G be a graph such that mincut of this graph has at least k edges. Consider any subset of edges $X = \{e_1, e_2, \ldots, e_t\}$ and let G' be the graph obtained by contracting all edges of X in G. Show that the minimum degree of G' is at least k.

2. (i) Let G be a undirected graph. Let D_G be the digraph obtained from G by converting each edge $(u, v) \in E(G)$ into two directed edges (u, v) and (v, u). Show that G is k-edge-connected if and only if D_G is k-edge-connected.

   (ii) Let D be a directed graph. Let G_D be the graph obtained from G by adding an edge (u, v) whenever one of the two directed edges $(u, v), (v, u)$ exists in D. Show that D is k-edge-connected if and only if G_D is k-edge-connected.

3. (i) Let D be a directed graph. Let $r \in V(D)$ be a vertex. Let $T_1^O, T_2^O, \ldots, T_k^O$ be a collection of edge disjoint spanning out-branchings in D. Then show that, for any vertex $v \in V(D) - r$, there are at least k edge disjoint paths in D from r to v.

   (ii) Similarly, let $T_1^I, T_2^I, \ldots, T_k^I$ be a collection of edge disjoint spanning in-branchings in D. Then show that, for any vertex $v \in V(D) - r$, there are at least k edge disjoint paths in D from v to r.

   (iii) From the above, conclude that if D is a digraph and r is a vertex such that,

   – there is a collection of k-edge disjoint spanning out-branchings in D,

   – and there is a collection of k-edge disjoint spanning in-branchings in D,

   then D is k-edge-connected.