Homework Sheet 1: Minimum Dominating Set (MDS)

Due Date: 30-04-2019

Sequential Algorithms

1. Recall the greedy sequential algorithm for Dominating Set, and let S be the solution output by it. Construct an example where $\frac{|S|}{|S^*|} \geq \log(\Delta)$, where S^* is an optimal solution and Δ is the maximum degree.
2. Extend that the greedy sequential algorithm to the weighted case. Here, the input is a graph $G(V, E)$ and a weight function $w : V \to \mathbb{N}$, and we want to compute a *minimum weight* dominating set of G. Show that this algorithm is a $(\ln(\Delta+1))+1$-factor approximation to the optimal solution.

Distributed Algorithms

3. Consider the Dominating Set problem on trees. Design a distributed algorithm that outputs a constant factor approximation, and terminates in constantly many rounds.
4. (Bonous Exercise) Let $G = (V, E)$ be a graph, $G^2 = (V, E(G^2))$ is defined as a graph with vertex set $V(G^2) = V(G)$ and the edge set

$$E(G^2) = \{\{u, v\} \mid \text{distance(u,v) in } G \text{ is at most } 2\}.$$

The graph G^2 is called power of the graph G. Provide a local algorithm that runs in constant number of rounds and outputs a constant factor approximation for the MDS in power graph of trees (in T^2).