Homework Sheet 2: Low Diameter Decomposition

Due Date: 03-05-2019

Sequential Algorithms

1. Given a graph G with a partial $\Delta + 1$ coloring of it, i.e. only some of its vertices are colored (properly, i.e. no edge has both endpoints of same color). Show that it is always possible to extend this partial coloring to a proper $\Delta + 1$ coloring of G.

2. a. Provide two classes of graphs which they do not admit Δ coloring.

   b. (Bonus Exercise) Show that deciding whether a graph can be colored by Δ colors is polynomial time solvable.

3. Suppose Minimum Dominating Set has no $o(\log n)$-approximation unless $P = NP$. Then show that unless $P = NP$, there is no polynomial time $o(\log n)$-approximation of minimum dominating set problem in graphs of diameter ≤ 4. Is it possible to show the same for diameter 3? What about diameter 2?

4. Suppose G is a graph, and X is a subset of vertices. Then, $S \subset V(G)$ is a Minimum X-Dominating Set, if it is a minimum sized set such that for any $v \in X$, either $x \in S$ or $x \in N(u)$ for some $u \in S$. Suppose X_1 and X_2 are two vertex subsets, and let S_i be a minimum-X_i-dominating set for $i \in \{1, 2\}$. Then show that if $N(X_1) \cap N(X_2) = \emptyset$, then $S_1 \cup S_2$ is a minimum $X_1 \cup X_2$-dominating set. Here $N(X) = \{v \mid v \in X \text{ or } \exists u \in X, v \in N(u)\}$.

Distributed Algorithms

5. An **edge coloring** of a graph G is to color its edges so that no two edges that intersect on the same endpoint have the same color. Let suppose a graph G with maximum degree Δ and its (c, D)-network decomposition are given. Provide an algorithm to calculate $O(\Delta^2)$ edge coloring of G in $O(c \cdot D)$ communication rounds. Can you improve the number of colors without exploring the entire graph? (In the sequential settings by Vizing's theorem we know that every graph has $\Delta + 1$ edge coloring).

6. A Maximal Independent Set (MIS) problem asks for finding a set of vertices $M \subseteq V(G)$ such that no two of them are neighbors of each other and in addition to that every vertex $u \in V(G) - M$ is a neighbor of a node in M. Suppose a (c, D)-network decomposition of a graph G is given. Let Δ be a maximum degree of G. Find a MIS of G in $O(c \cdot D + \Delta)$.

7. Suppose we have a Weak-(c, D)-Decomposition of a graph G. Then show that there is a LOCAL distributed algorithm for computing a $\Delta + 1$ coloring of G in $O(cD)$ rounds.

8. Let P be a path of length ℓ. Show that P can be properly colored using 2-colors. Show that any distributed algorithm for coloring P with 2 colors needs more than $\ell/10$ rounds.