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Homework Sheet 4: Distributed Graph Coloring

Due Date: 21. May 2019

This lecture was about distributed graph coloring, we have provided a handwritten lecture notes, however, we would like to encourage you to have a look at chapter 3 and 5 of the monograph [1] by Elkin and Barenboim for further reading.

You may find the solution to some of the exercises in the aforementioned monograph.

1. **Arboricity and Degeneracy:**

    1. Let G be a graph of degeneracy d and let $H \subseteq G$. Prove that $|E(H)|/|V(H)| \leq d$.
    2. Use Cole-Vishkins algorithm and the fact that every graph G has arboricity at most Δ, provide a $\Delta + 1$-coloring of G in $O(3^{\Delta})$ rounds.

2. **Acyclic Orientation:** Recall how we obtained acyclic orientation in one round: if $e = \{u, v\}$ then we direct $e : u \to v$ if $ID(u) > ID(v)$ otherwise we direct it the other way around. Now answer the following questions:

    1. Why the above construction provides an acyclic orientation of the graph?
    2. Let k be the length of the longest directed path obtained by the above orientation in a graph G. Provide a $k + 1$ coloring of G in $O(k)$ rounds.
    3. Prove that if a graph G and k-coloring of G are given, then in $O(1)$ round we can obtain an acyclic orientation of G where the length of the longest path is at most $k - 1$.

4. Use above and provide an acyclic orientation of a graph G such that the length of the longest path is at most Δ in $O(t)$, where t is the time we spend to provide a $(O(\log n), O(\log n))$ network decomposition of G. (Recall that in a such a decomposition, every cluster has diameter of size $O(\log n)$, clusters of the same color/type are of distance 2 from each other and we can color clusters with $\log n$ colors.)

References

[1] L. Barenboim and M. Elkin "Distributed Graph Coloring," 2013. `https://www.cs.bgu.ac.il/ elkinm/BarenboimElkin-monograph.pdf`