Exercise 9: Stabilization

Task 1: Recovering Reasonable Constraints

In this exercise, we want to get a better handle on the set of constraints from the lecture:

\[ S(1) \geq 2 \left( \delta + \left( 1 - \frac{1}{\vartheta} \right) T \right) \]

\[ \frac{R^-}{\vartheta} \geq \sigma_h + \vartheta S(1) + d \]

\[ \frac{B_2}{\vartheta} > \sigma_h + R^+ + T + 2S(1) \]

\[ B_1 > \sigma_h + R^+ \]

\[ B_3 > R^+ + (M - 1)(T + S(1)) + (\vartheta + 1)S(M) + \sigma_h \]

\[ B_2 \leq \frac{R^-}{\vartheta} + (M - 1) \left( \frac{T}{\vartheta} - S(1) \right) + S(M) \]

\[ \frac{R^+}{\vartheta} \geq (\vartheta + 1)S(M) + \sigma_h \]

\[ 2(S(1) - S(M)) \geq \sigma_h \]

a) Choose \( R^- \) tight according to (2) and \( R^+ / \vartheta \) equal to the r.h.s. of (7) plus \( d \), respectively. Show that for these choices (1) and (8) hold. (Hint: Use these equalities to express \( S(1) \) in terms of \( S(M) \) and other terms not involving \( R^+ \) or \( R^- \)).

b) Fix these choices and consider the remaining inequalities. Which of the terms on the r.h.s. of the inequalities are in \( O(\sigma_h + d) \), if \( M \) is not treated as a constant? (Hint: Recall that \( \vartheta \in O(1) \).)

c) In addition, suppose now that for \( \alpha > 1 \), we have that \( B_3 = \alpha B_2 = \alpha^2 B_1 \) and can choose \( B_1 > 0 \) sufficiently large. For which values of \( \alpha \) can you choose \( M \) so that the system of inequalities is satisfied?

Task 2: If it Were so Simple...

a) Modify the Srikanth-Toueg algorithm so that its pulses can be triggered by an external NEXT signal in the vein of Definition 9.7. (Hint: Add a second, smaller timeout to the transition from READY to PROPOSE.)

b) What choices of \( B_1 \), \( B_2 \), and \( B_3 \) can this solution support?