Orthogonal Range Searching

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Computaional Geometry
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max planck institut
informatik

Overview

- Intro and problem definition

Overview

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- Kd trees

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- Intro and problem definition

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- Range trees

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- Kd trees
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- Fractional cascading

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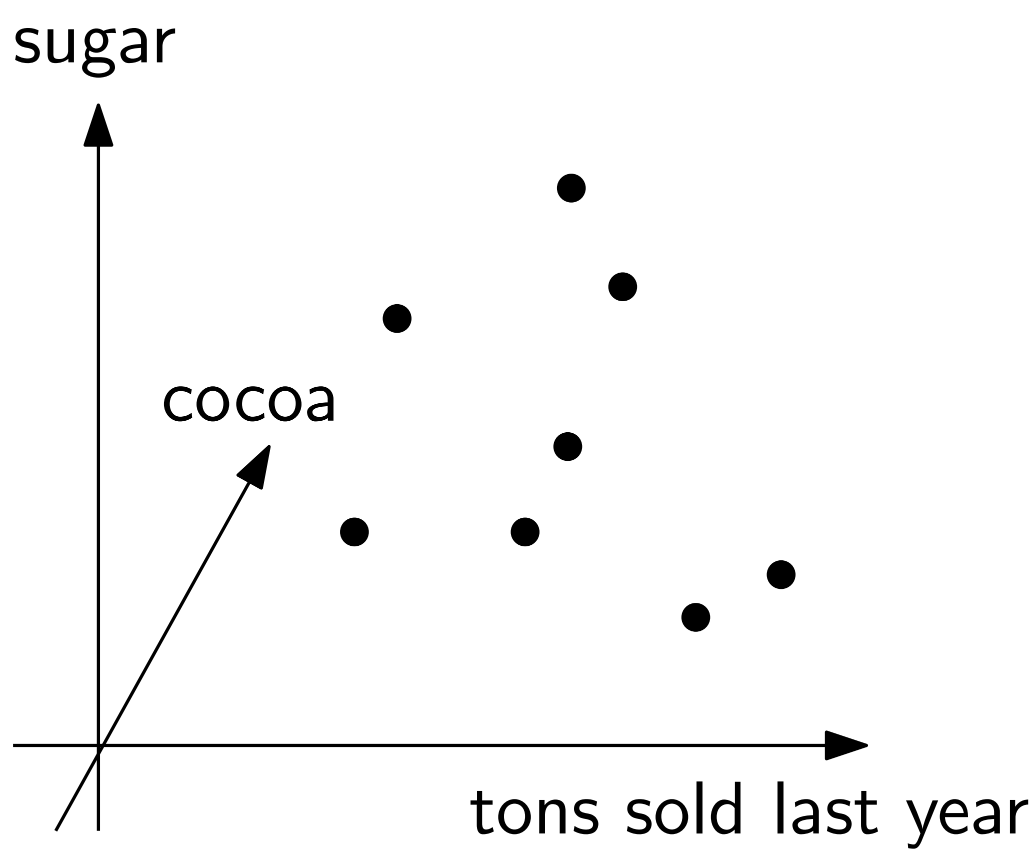
- Kd trees
- Range trees
- Fractional cascading
- Priority search trees

The obvious applciation

Database of cakes.



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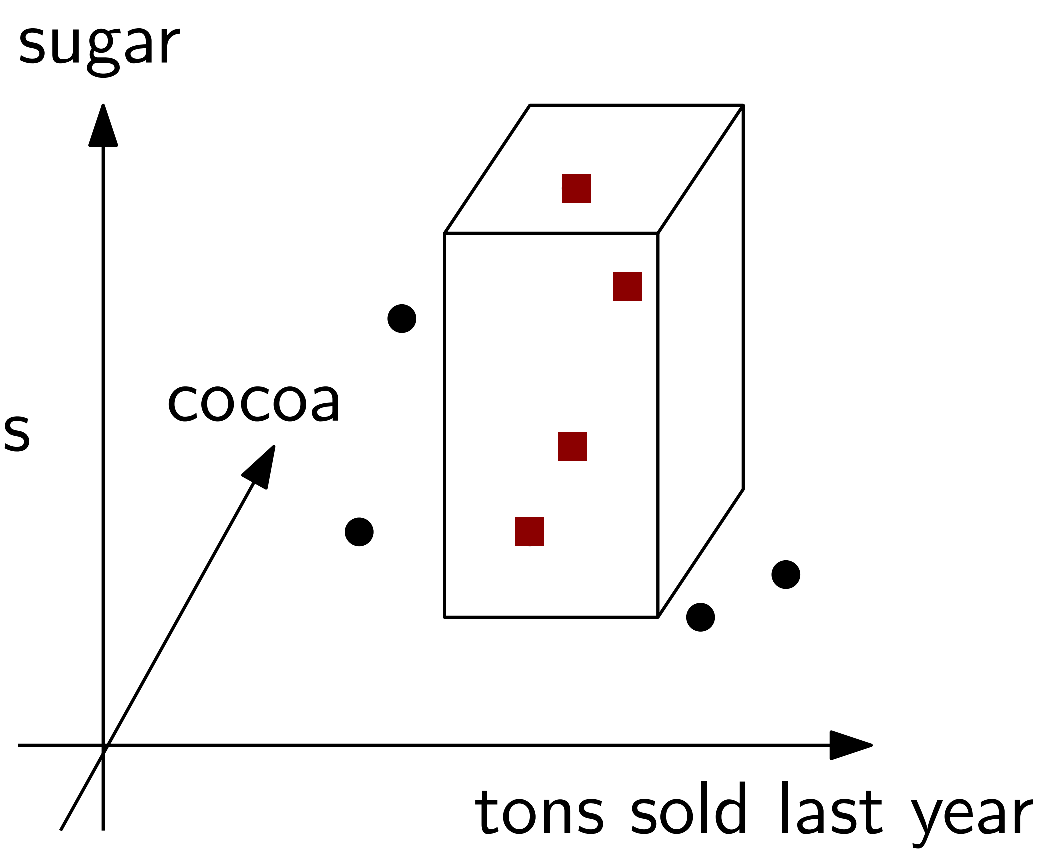
Which cakes have

- sugar content [0.12,0.17]
- cocoa content [0.05,0.1]
- and sold btw. 3 and 4 tons last year?



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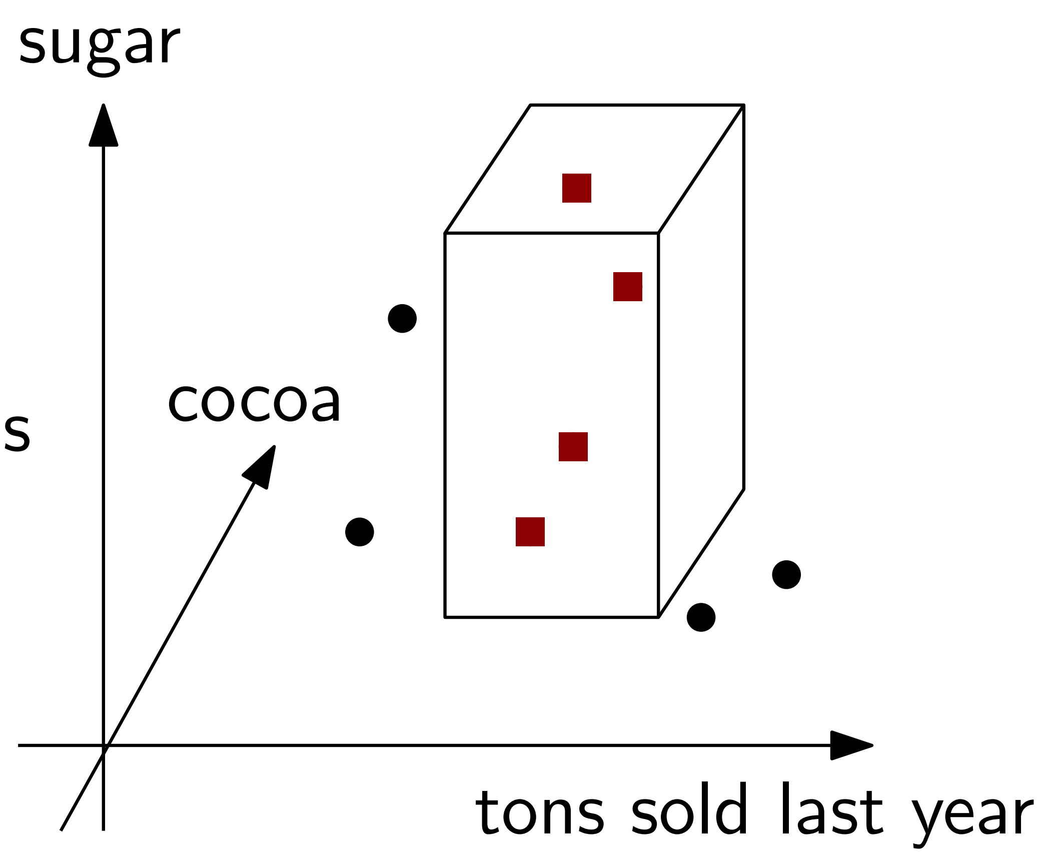
Orthogonal range query



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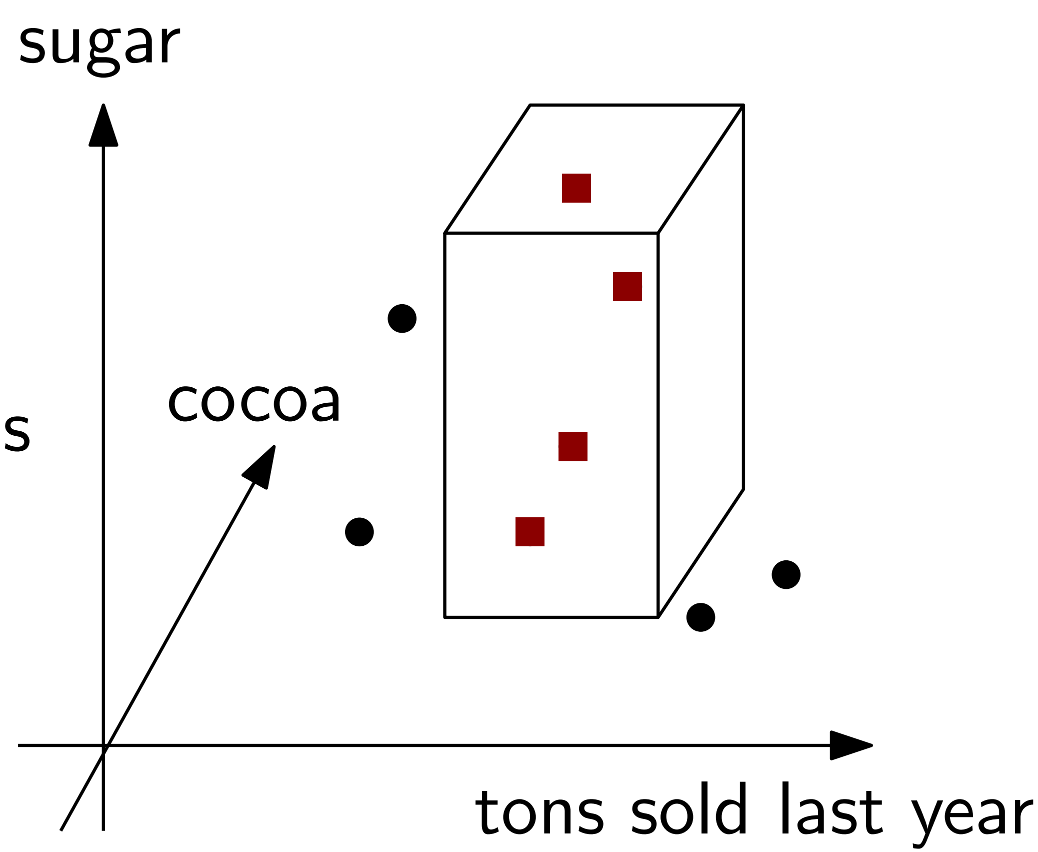

Orthogonal range query

Task: support such queries efficiently

Problem definition

Given n points in $\mathbb{Z}^d$ or $\mathbb{Q}^d$,

1. Preprocess them in $\widetilde{O}(n)$ time and space to
2. support orthogonal range queries in $\mathrm{poly}(\log n) + O(k)$
3. on a Word RAM.

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Static: preprocess and answer queries

$\updownarrow$

Dynamic: update insertions and deletions in $\mathrm{poly}(\log(n))$

The 1-dimensional problem

Query: $[x, x']$

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Option 1. Use sorted array:

Binary search for x

Report next until exceeds x'

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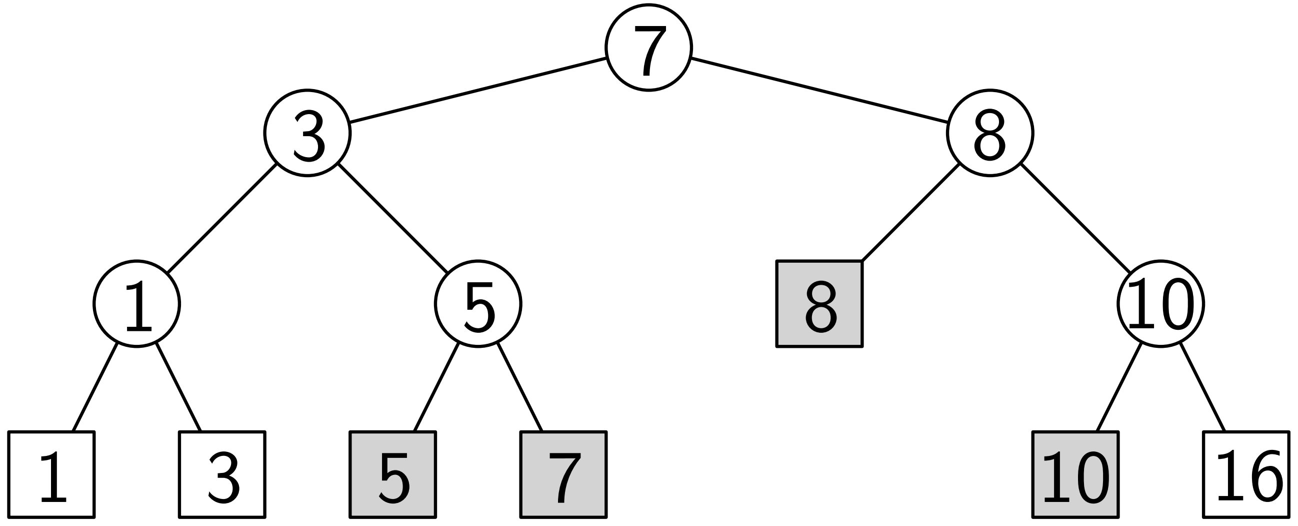

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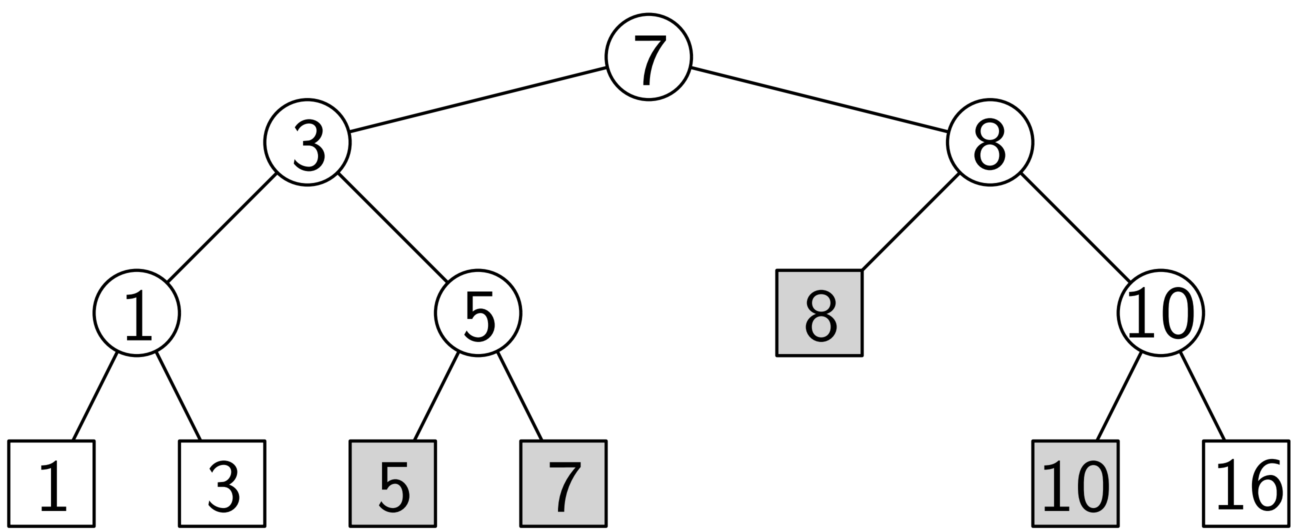


P in the leaves

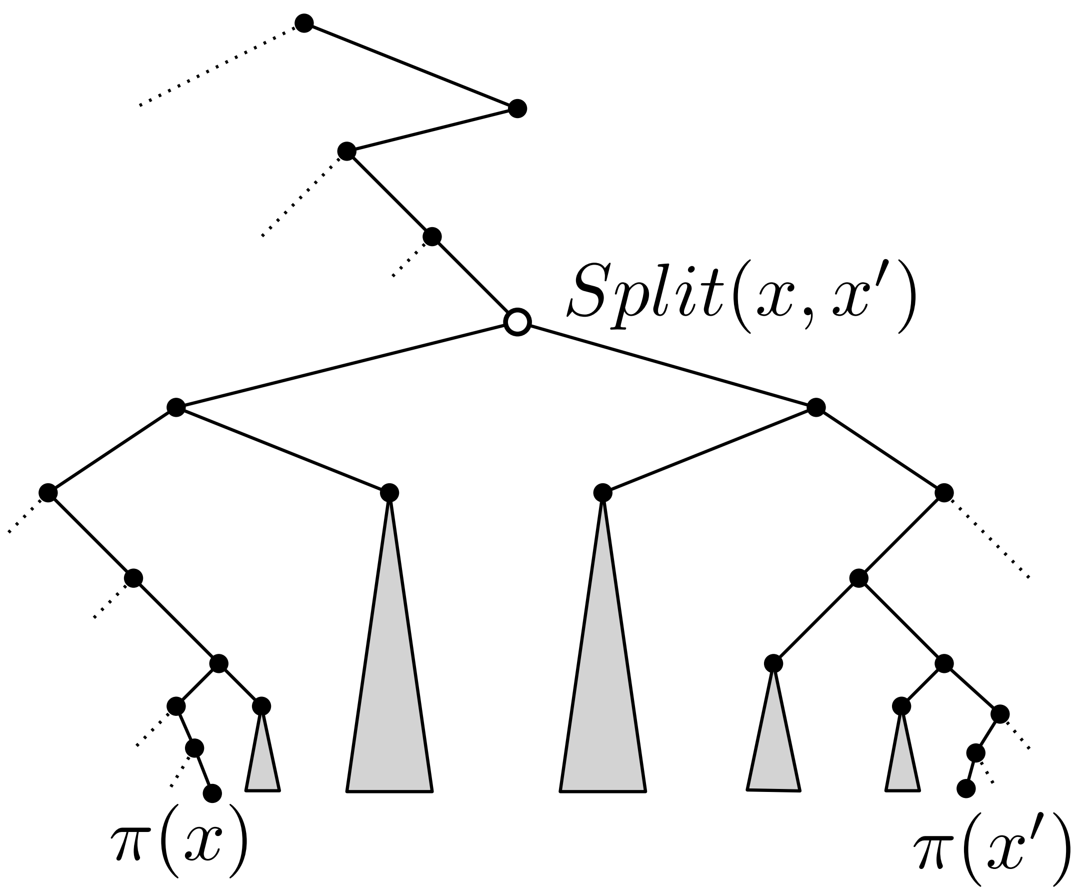
Query: $[4, 11]$

inner vertex $=$ largest value in left child's subtree

Dynamic solution in $\mathbb{R}^1$

Answering a query

- Binary search for $Split(x, x')$
- Search for x, reporting right child subtrees
- Search for x', reporting left child subtrees



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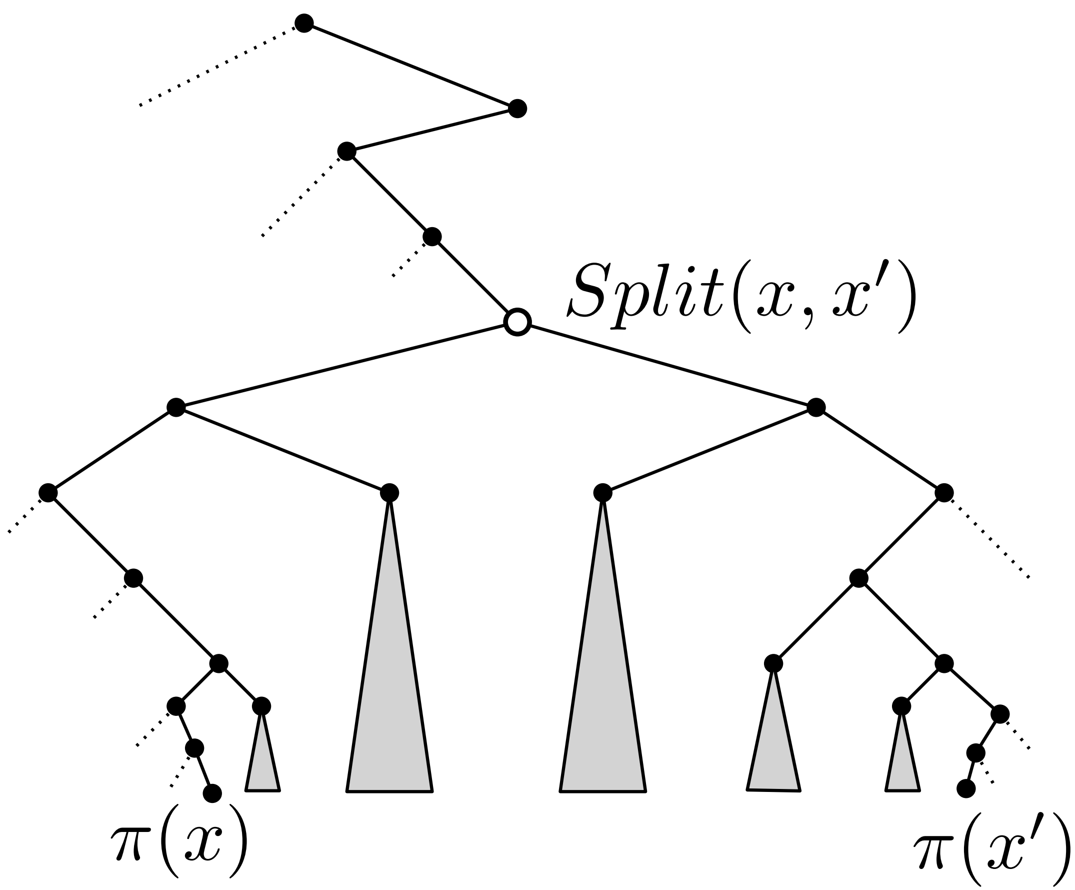

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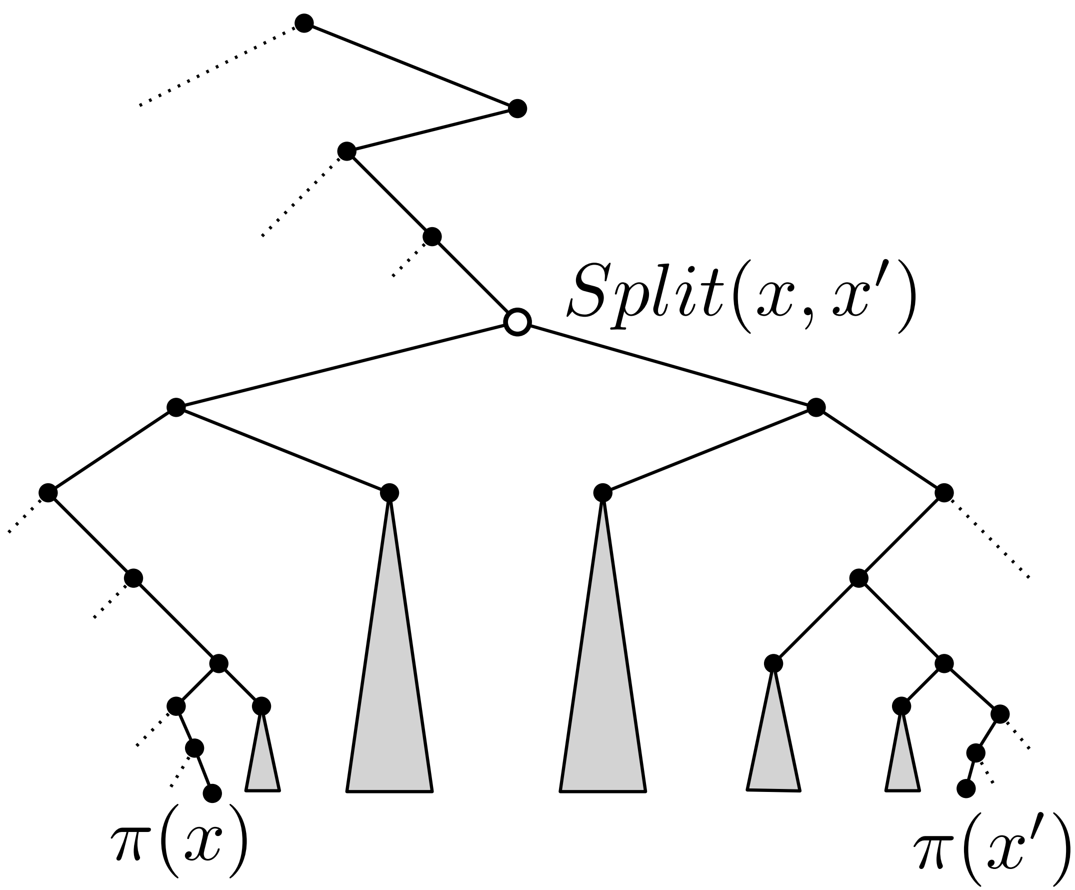

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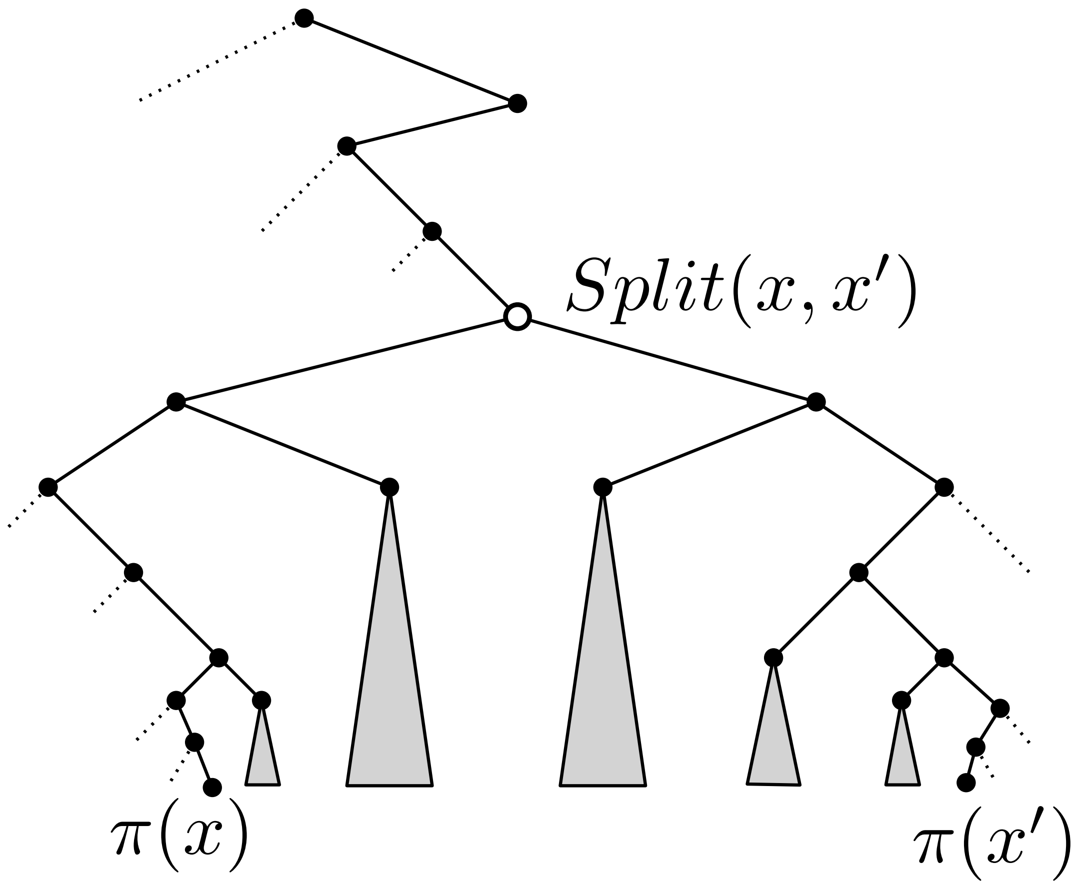

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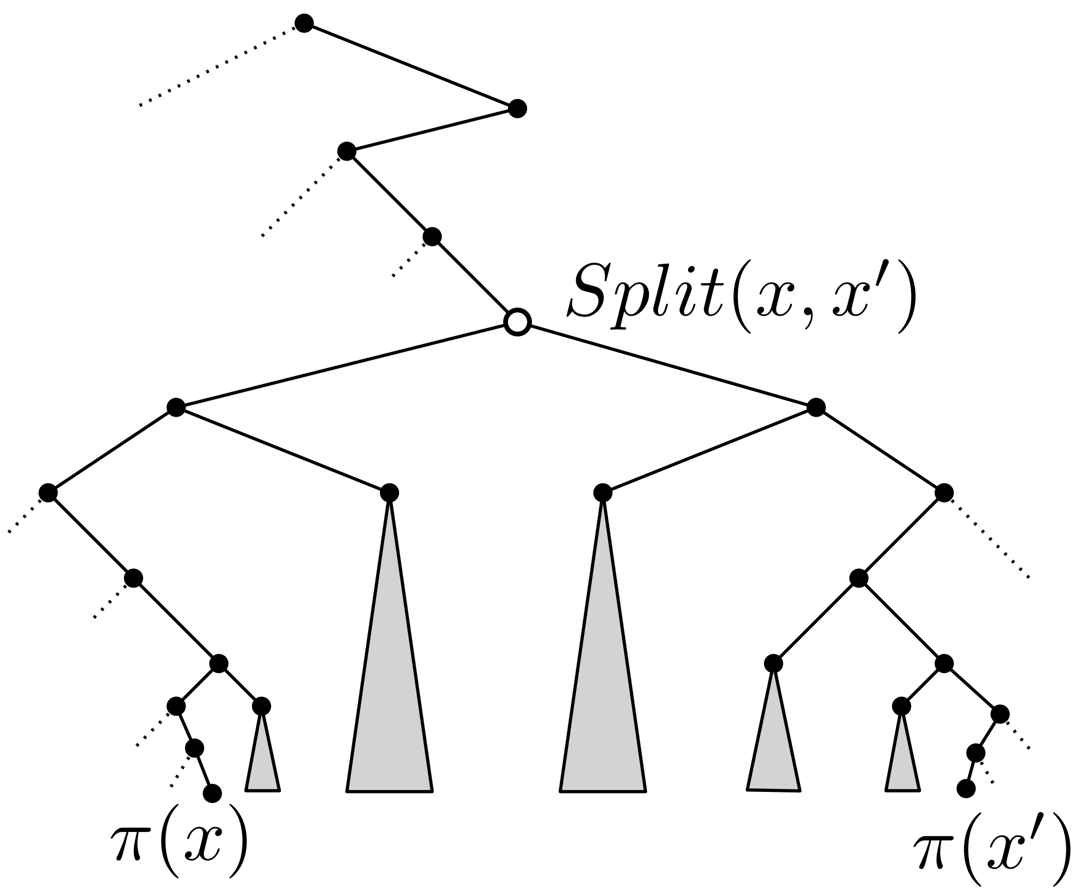

Space $= O(n)$

Preprocess $= O(n \log n)$

Update $= O(\log n)$

Query $= O(\log n + k)$

Kd trees
Bentley 1975

Kd trees in $\mathbb{R}^2$

Assume distinct x- and y-coords

Idea:

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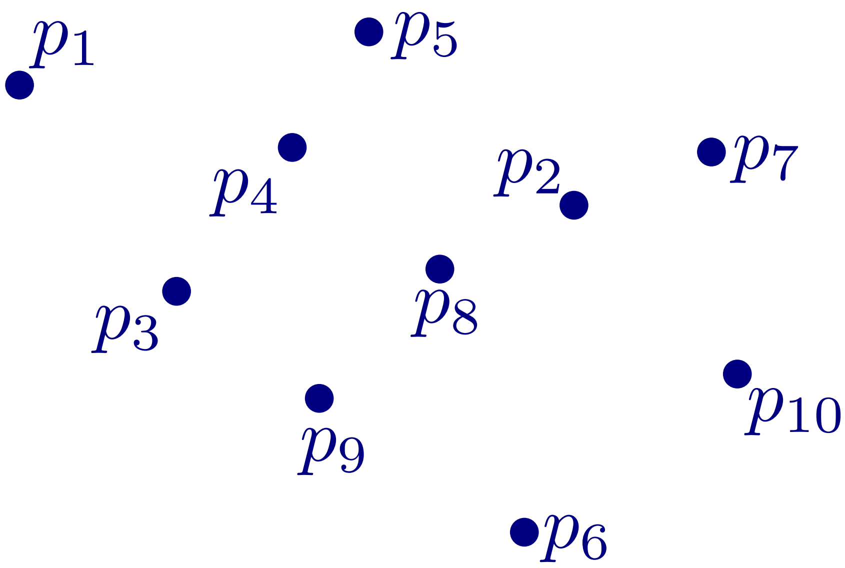

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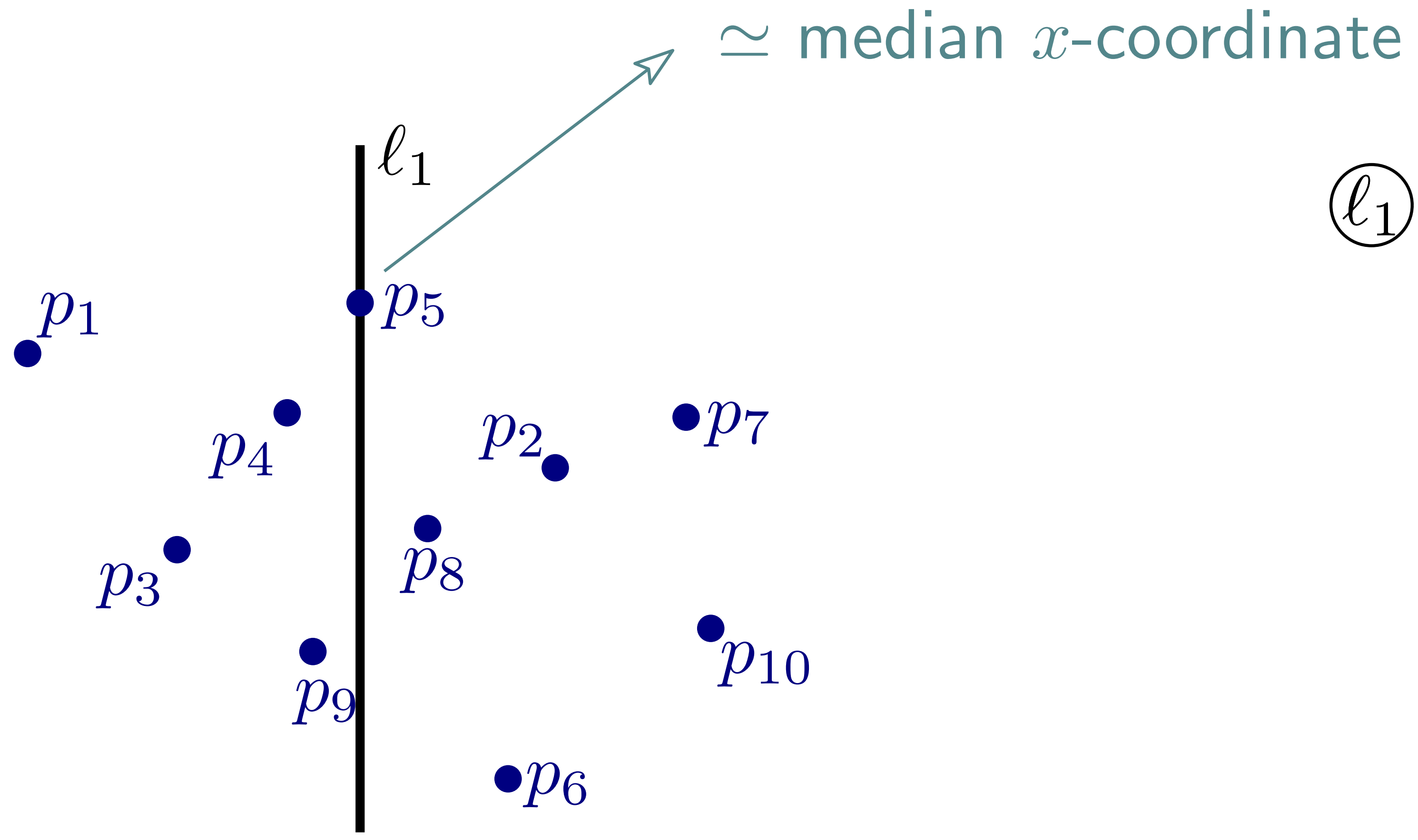

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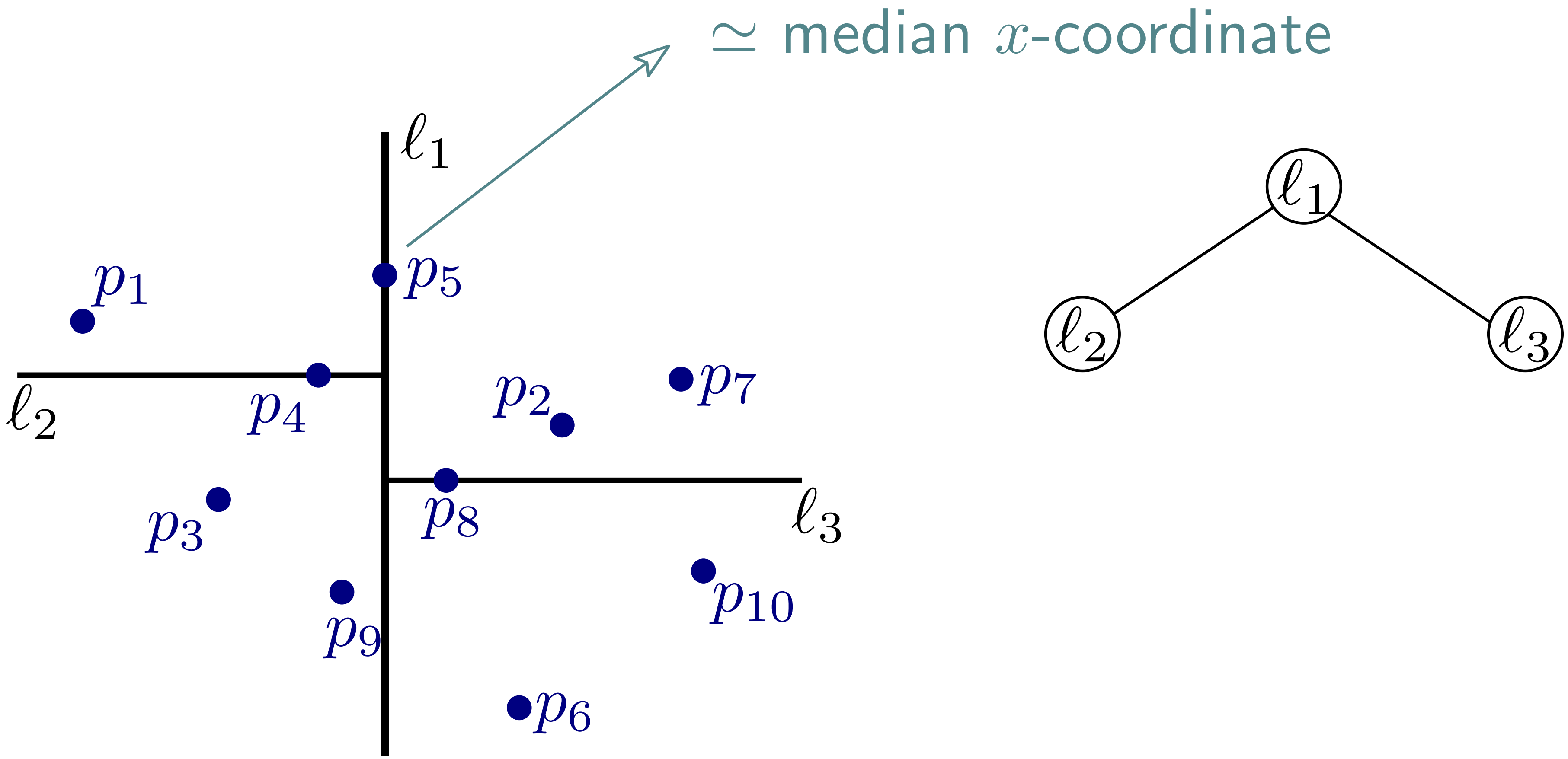

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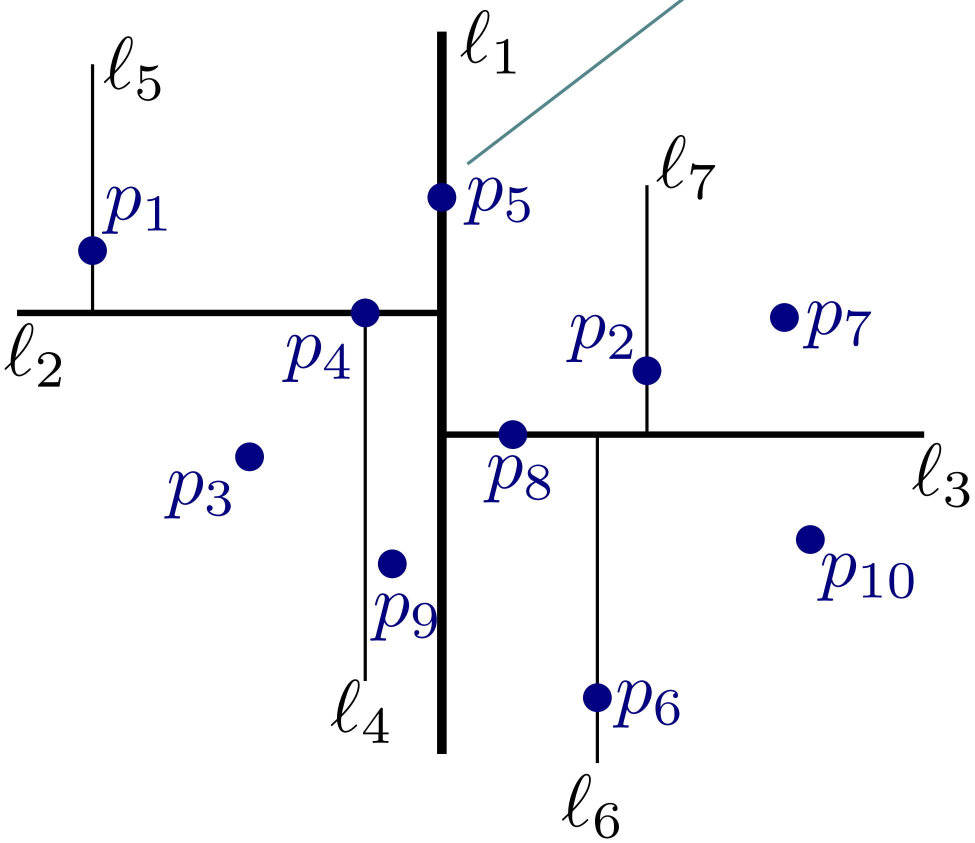
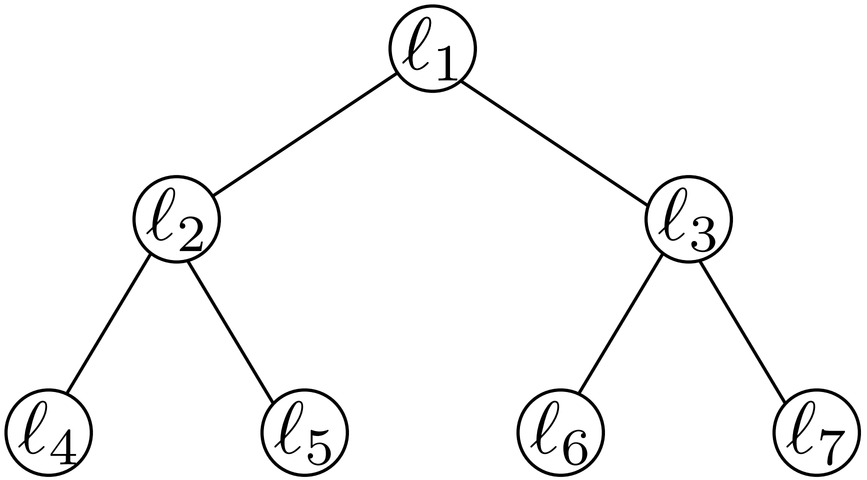
$\simeq$ median x-coordinate





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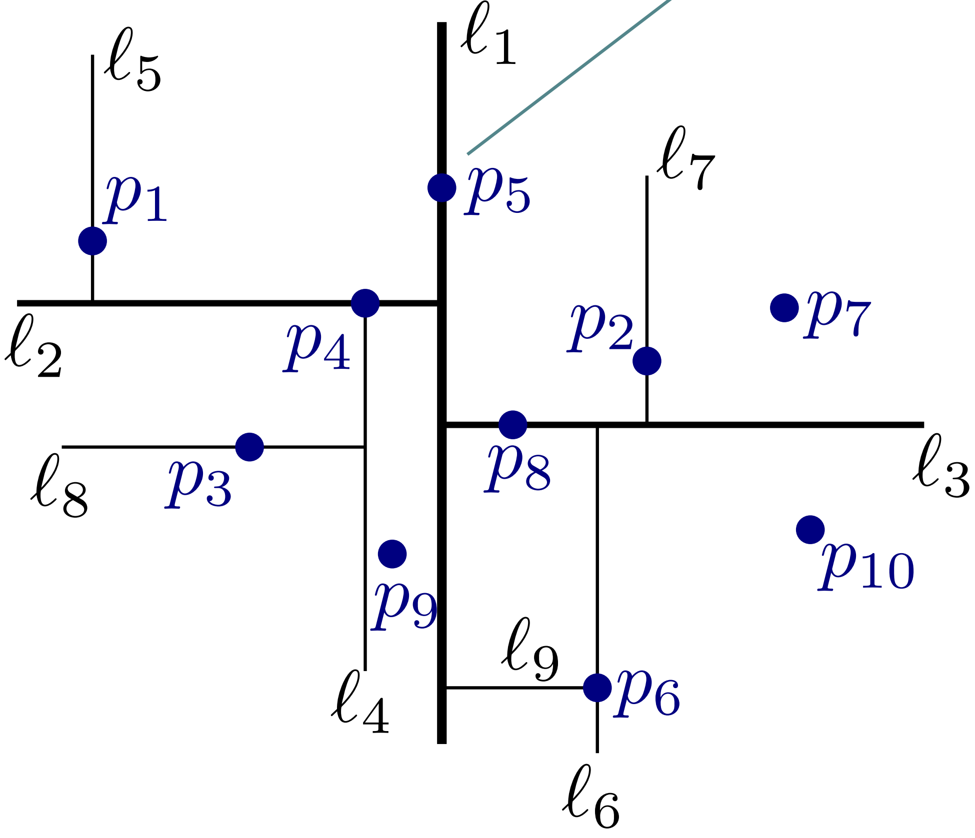

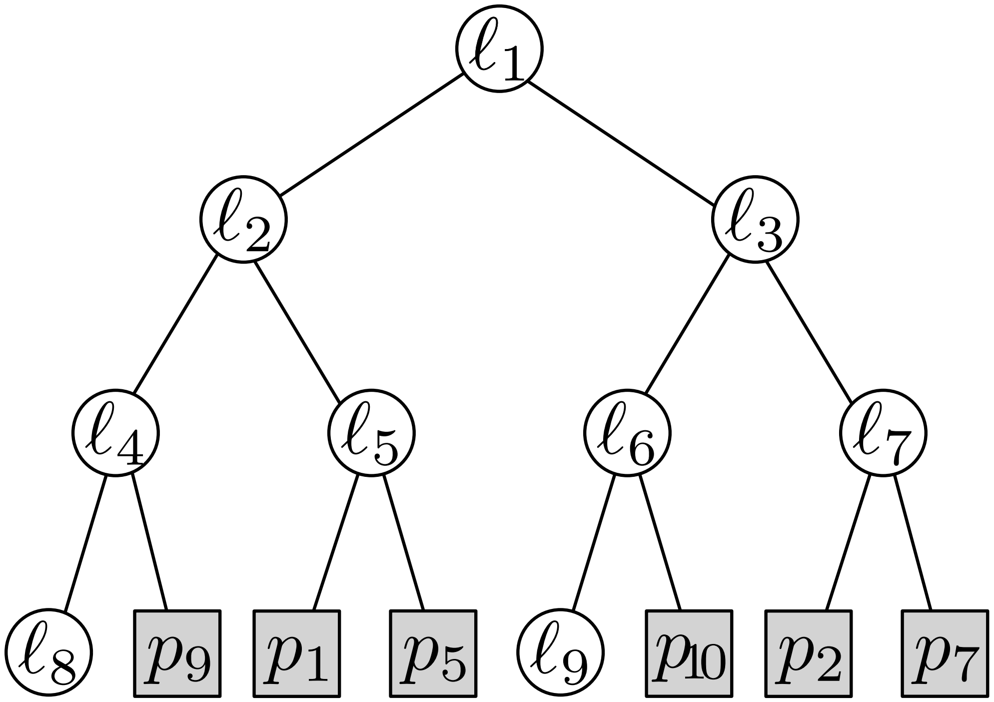

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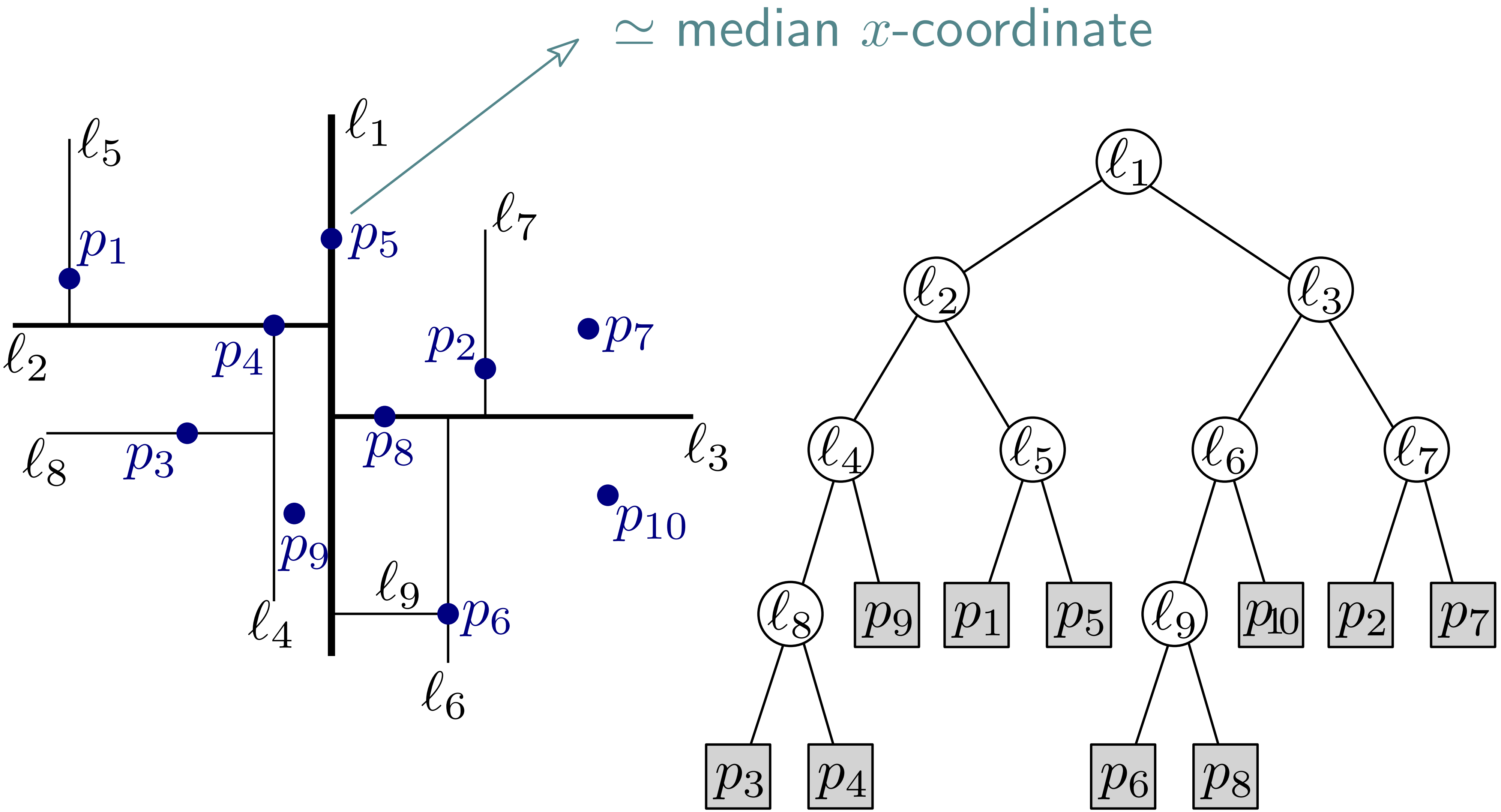

Kd tree anatomy

Space: $O(n)$
Tree has $O(\log n)$ depth.

Preprocessing: use linear time median:

$$T(n) = 2T(n/2) + O(n) \ \Rightarrow O(n \log n)$$

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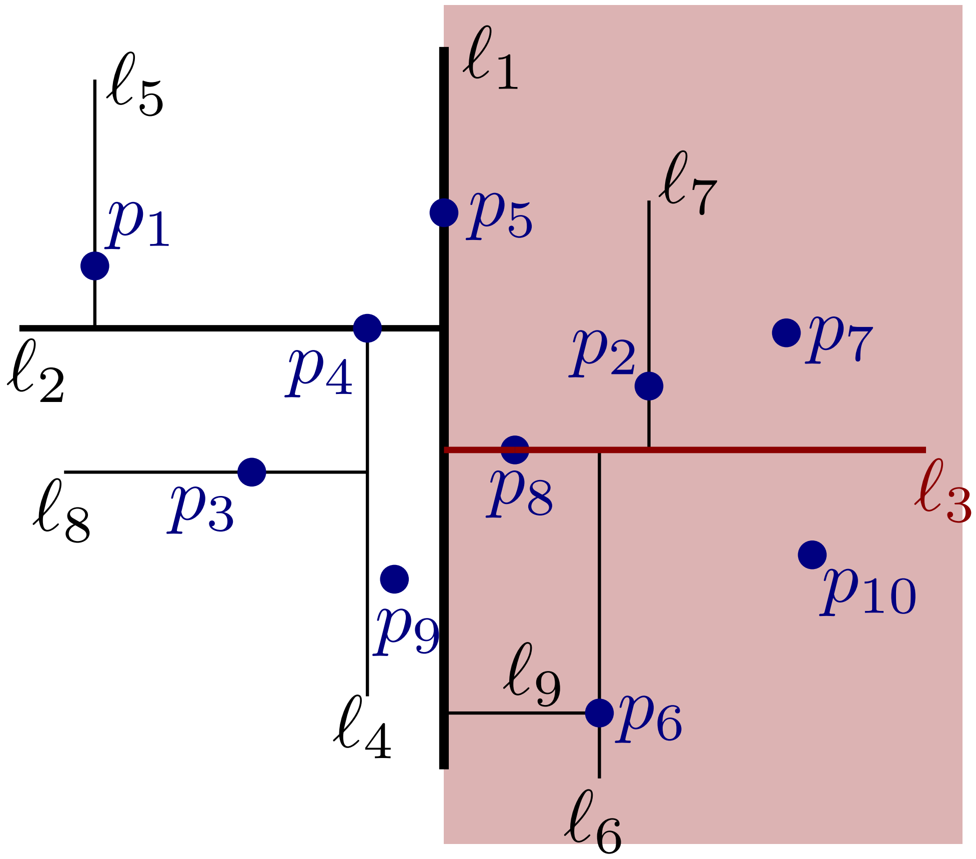

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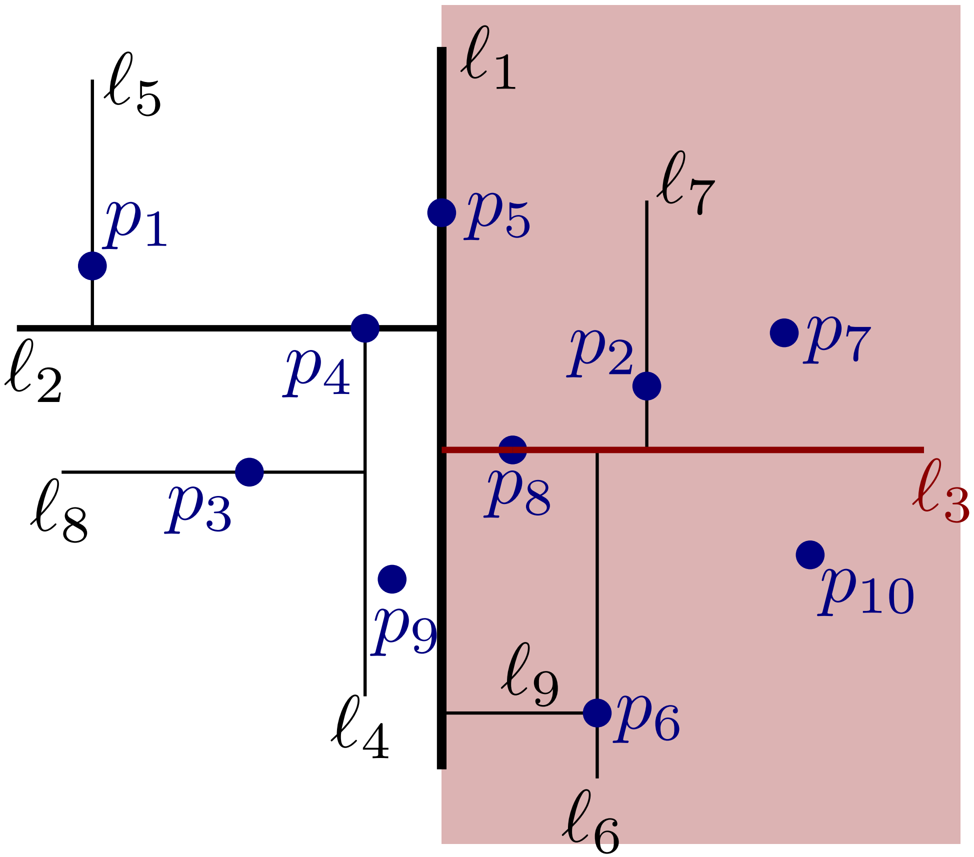

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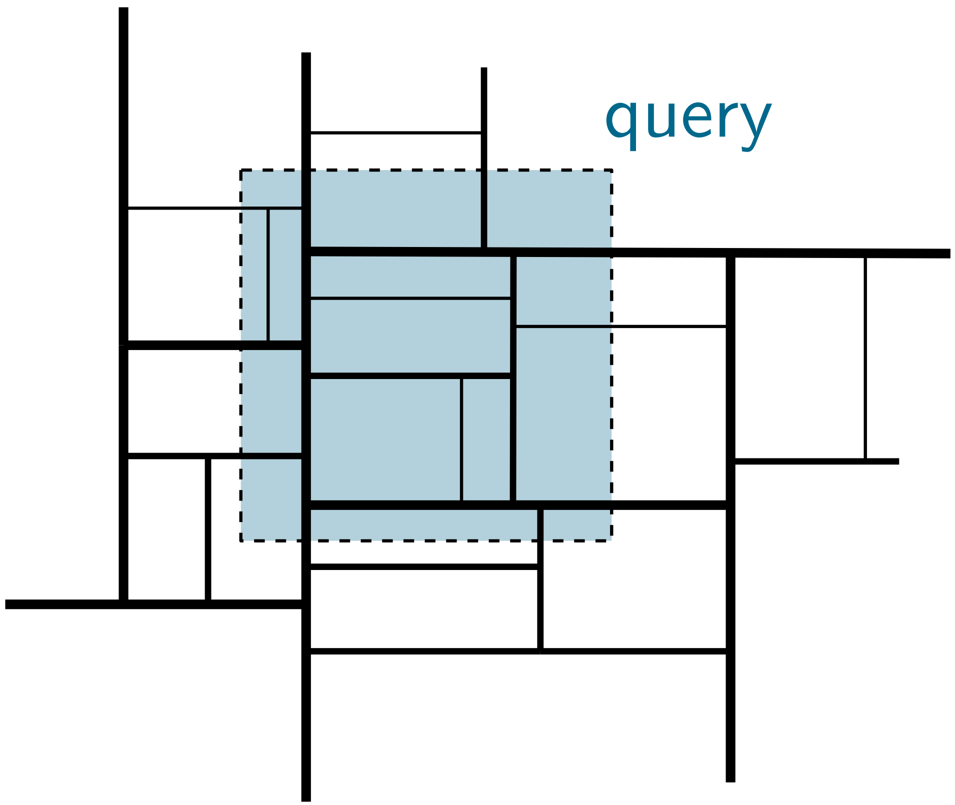
$$reg(\ell_1) = \mathbb{R}^2$$

Child regions of ℓ:
separated by ℓ

Querying a kd tree



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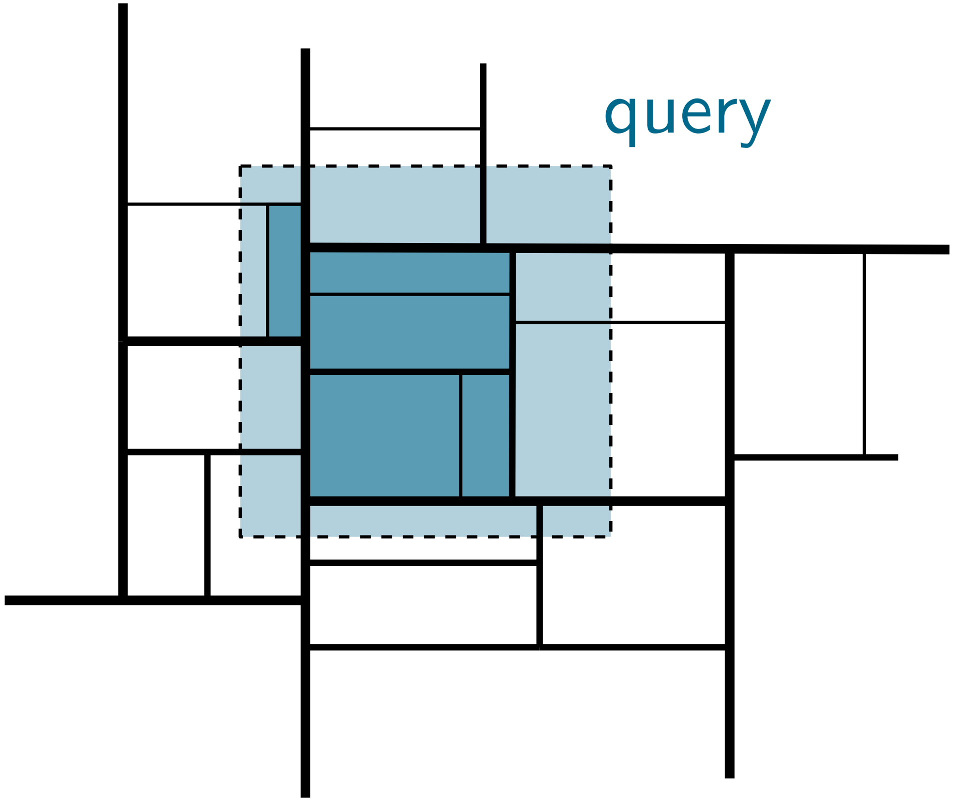

Report subtree if $reg(\ell) \subseteq Q$

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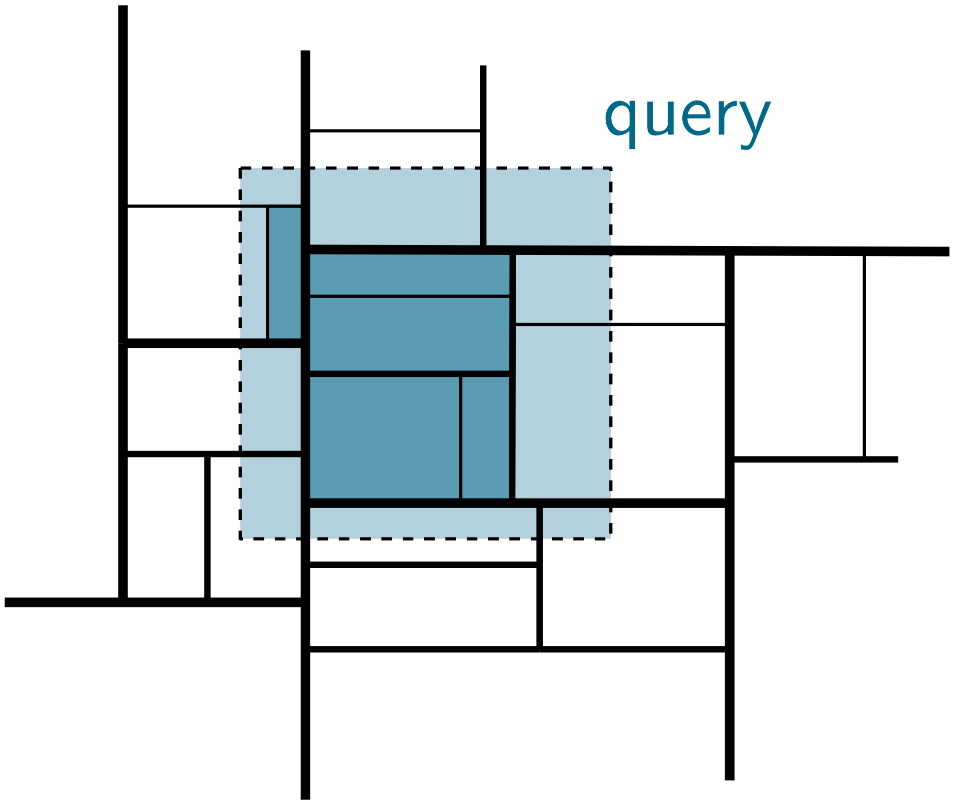

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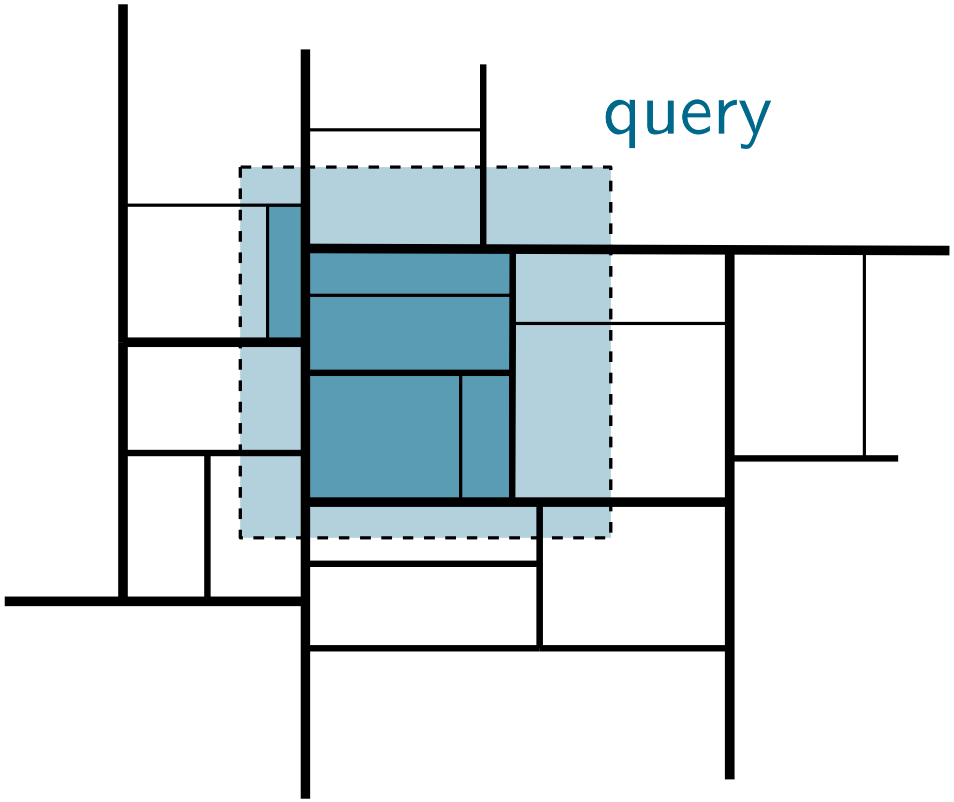

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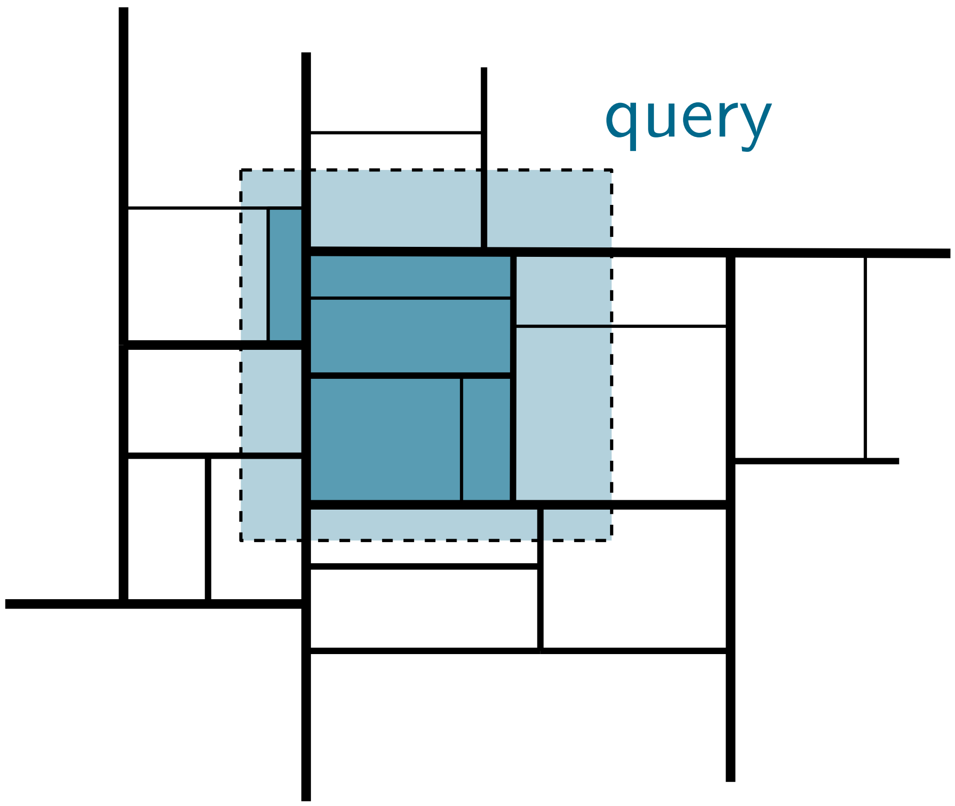

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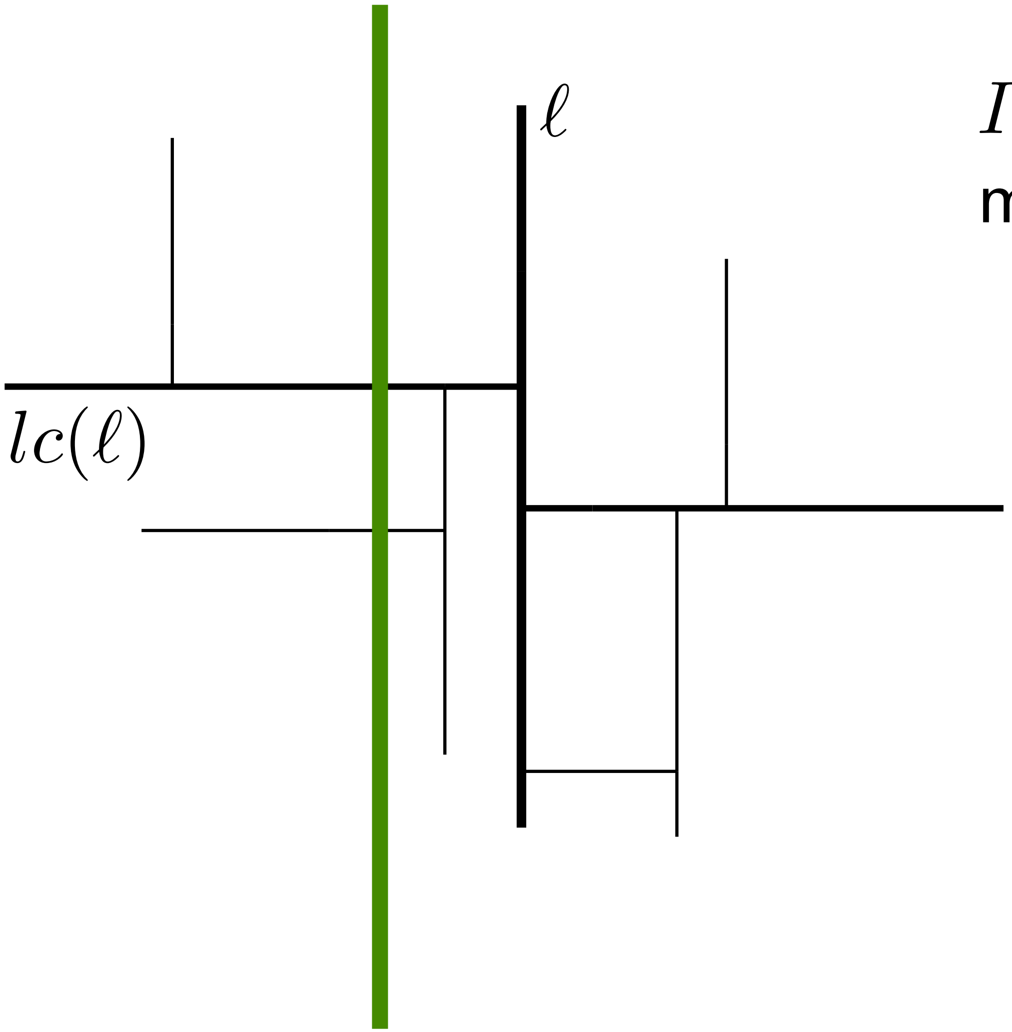
How many regions can intersect ∂Q?

Regions interseted by the boundary

Claim. A vertical line intersects at most $O(\sqrt{n})$ node regions.



$I(n)$: vert. line intersects this many regions in kd tree of size n

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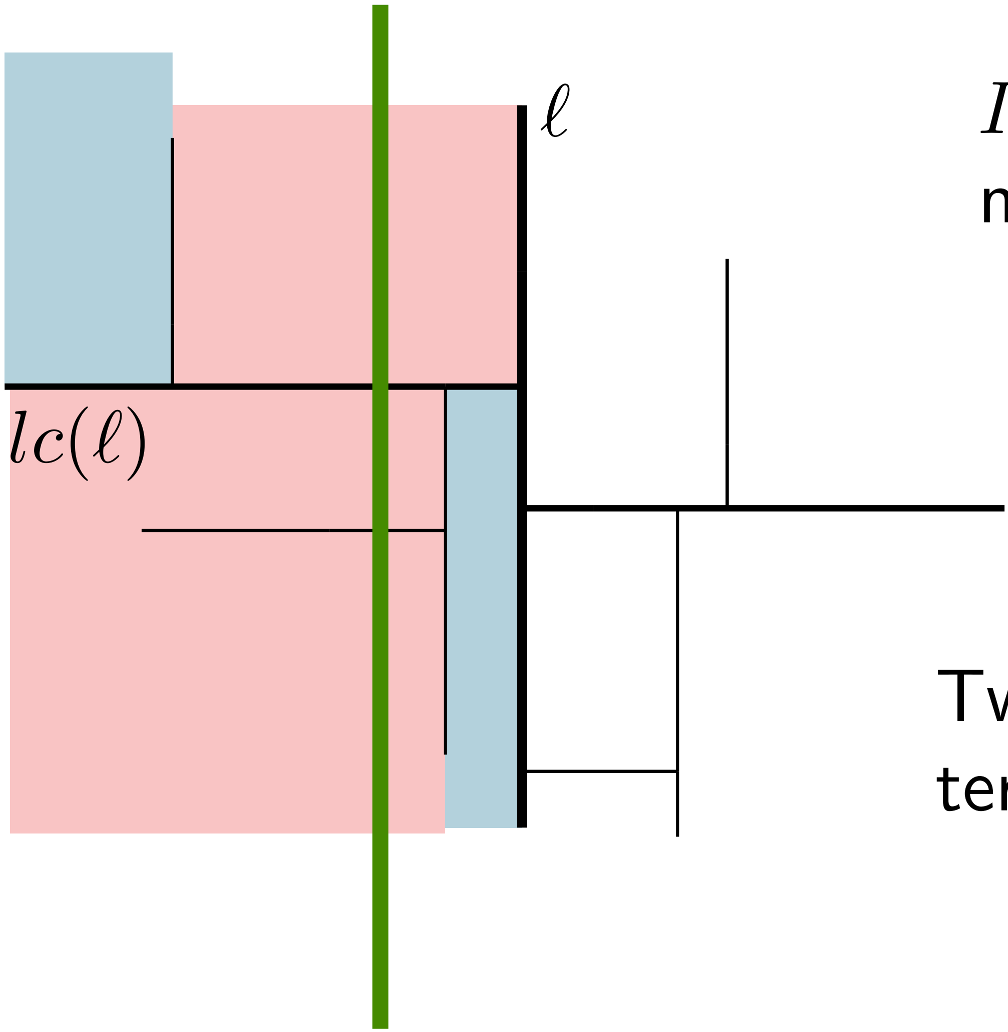

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Two grandchild regions are not intersected

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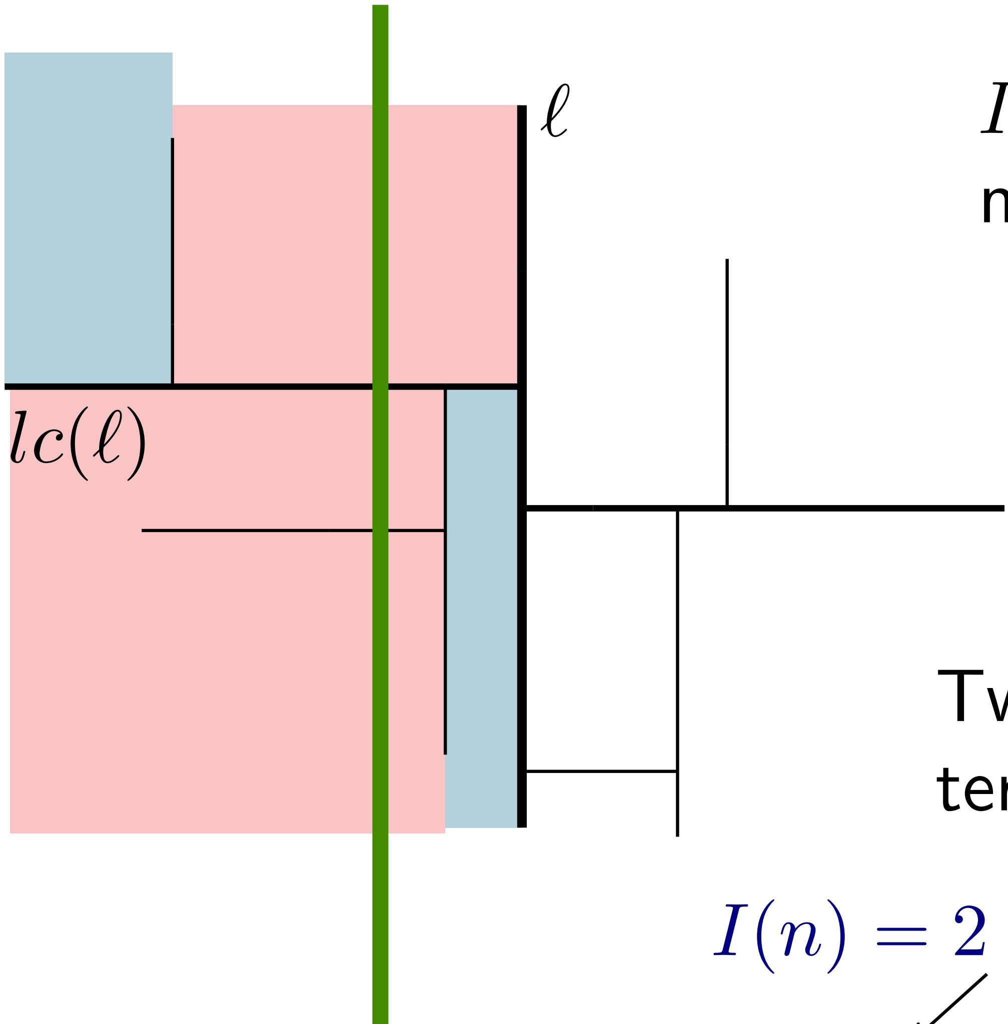

$I(n)$: vert. line intersects this many regions in kd tree of size n

Two grandchild regions are not intersected

$$I(n) = 2 + 2I(n/4)$$

$reg(\ell)$ and $reg(\text{leftchild}(\ell))$

Intersections in red subtrees

Kd tree query time

$$I(n) = 2 + 2I(n/4) = 2 + 2 \cdot (2 + 2 \cdot (\dots)) = 2 + 4 + \dots + 2^{\log_4 n}$$

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Q has ≤ 2 vertical and ≤ 2 horizontal sides
$\Rightarrow \partial Q$ intersects $O(\sqrt{n})$ regions

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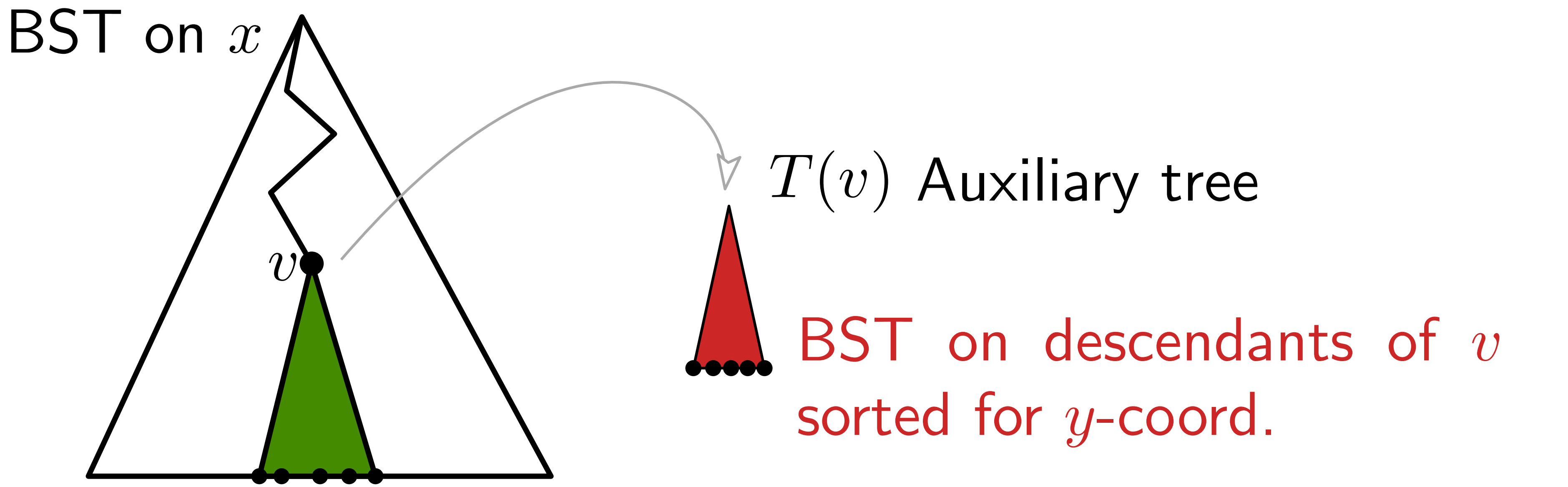
Orthogonal range queries of a 2-dim kd tree take $O(\sqrt{n} + k)$ time.

Range trees

Range tree in $\mathbb{R}^2$

Binary search tree on x-xoordinates

Each node has a new BST for y-coords of descendant leaves



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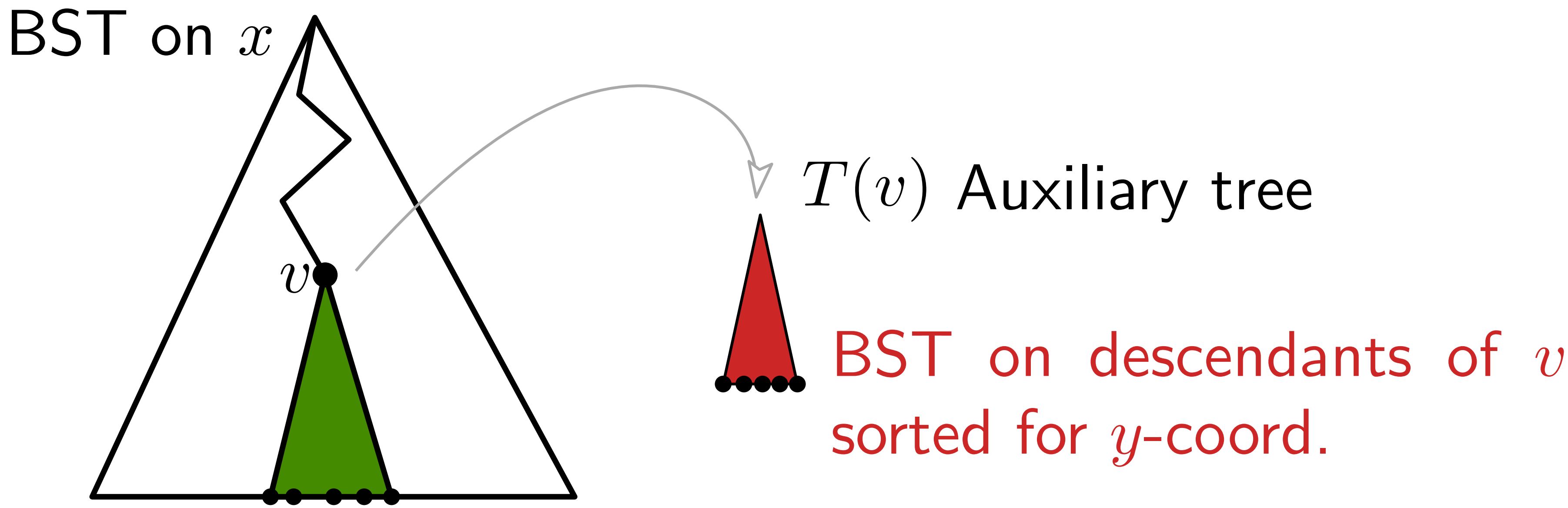

Construction:

- Presort on y-coord to array A

Construct(v, A):

- Construct $T(v)$ using A
- Find median x, split A to A_{left}, A_{right}
- Construct($lc(v), A_{left}$), Construct($rc(v), A_{right}$)

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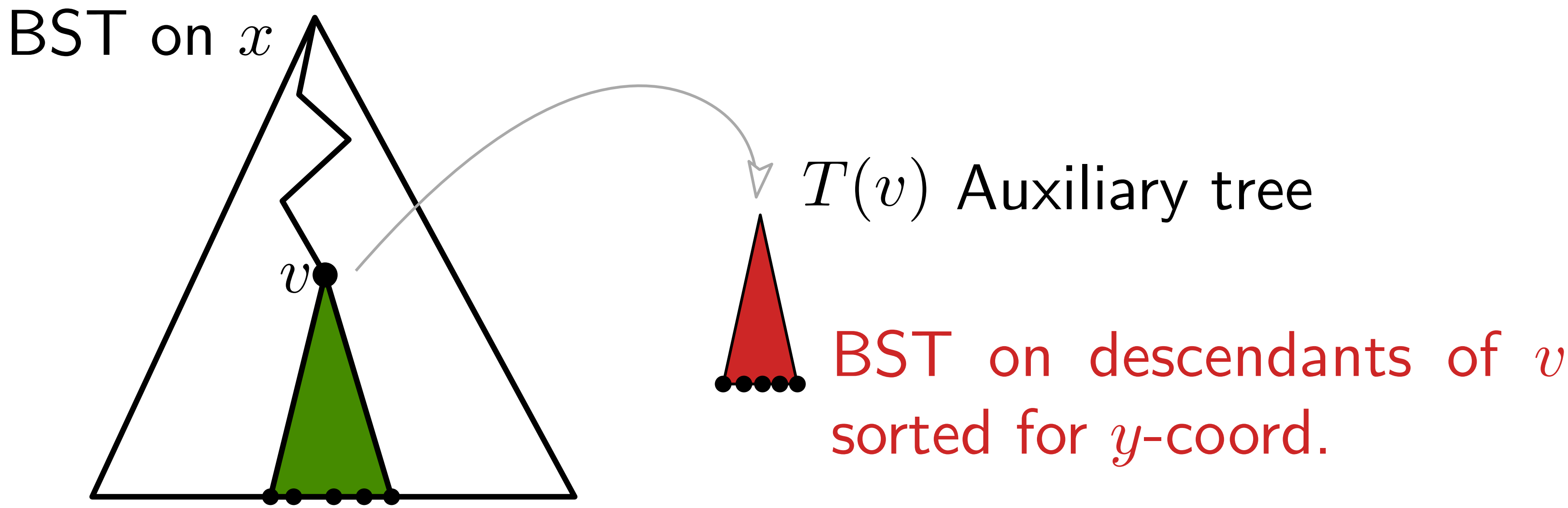

Construction:

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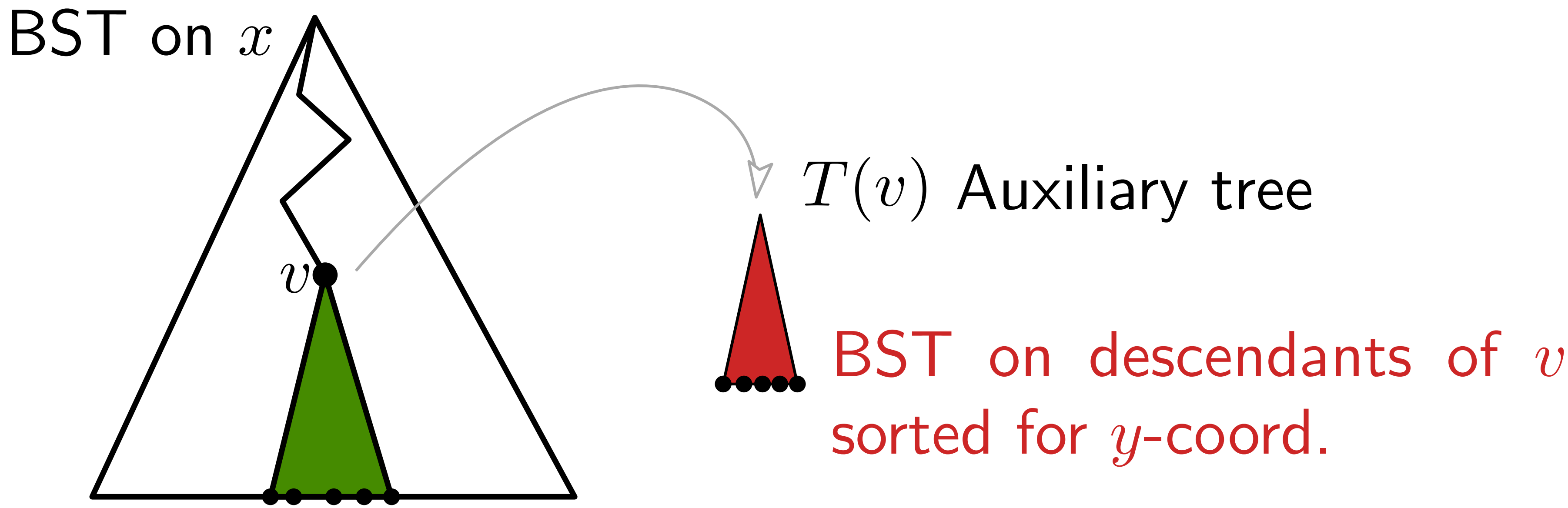

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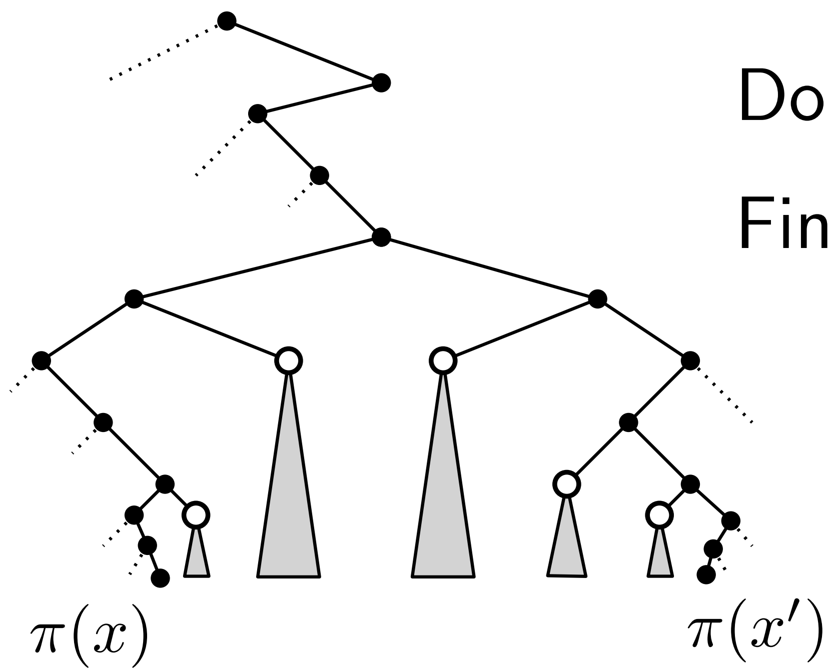
Querying a 2-dim range tree



Do 1-dim range query on x-coordiantes

Find $O(\log n)$ reported subtree roots

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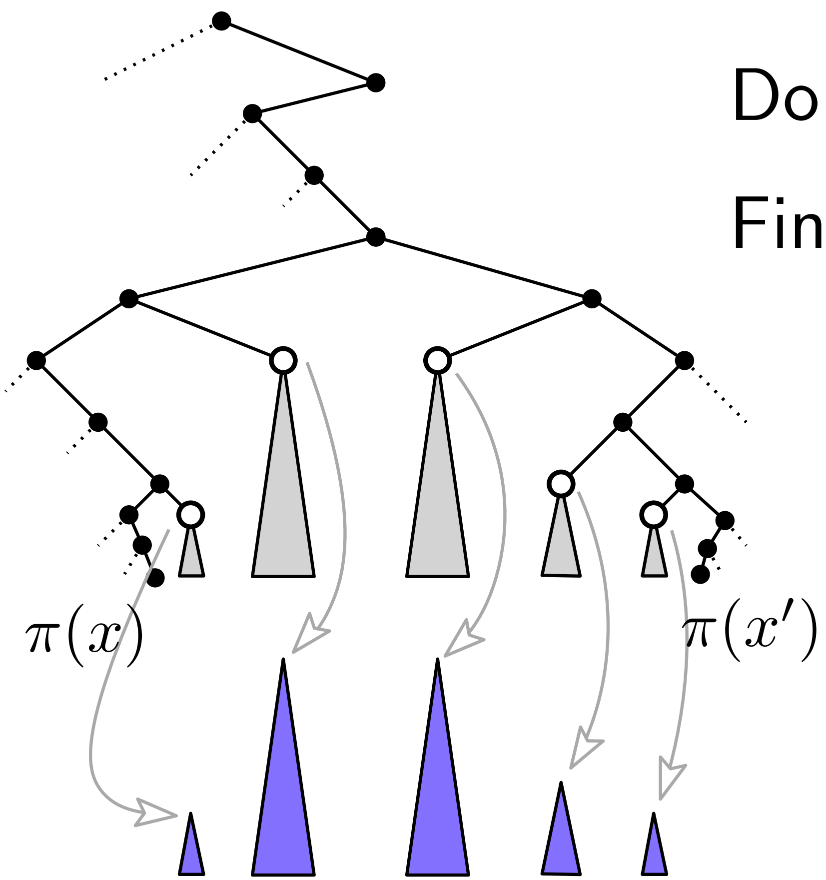

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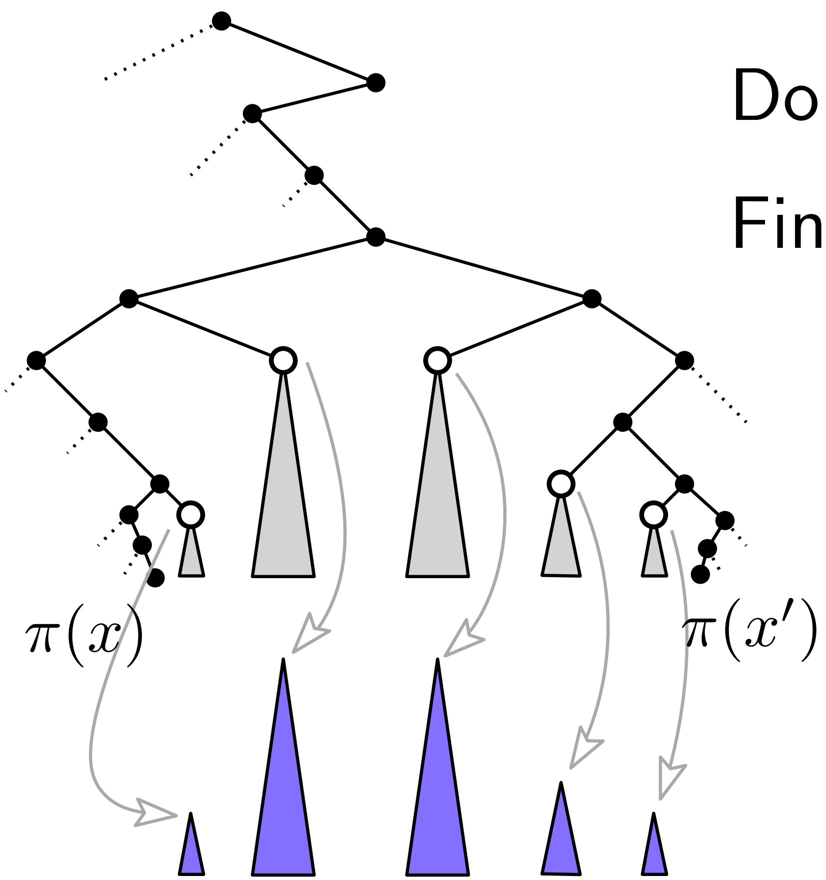

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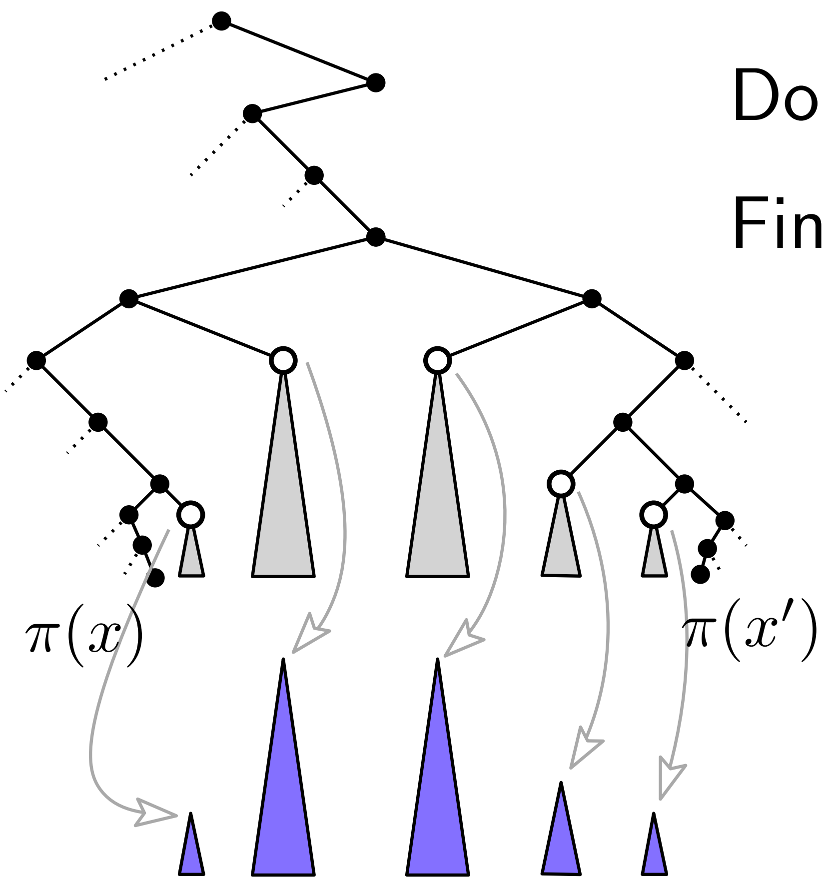
Total query time:

$$\sum_v O(\log n + k_v) = O(\log^2 n + k)$$

Querying a 2-dim range tree



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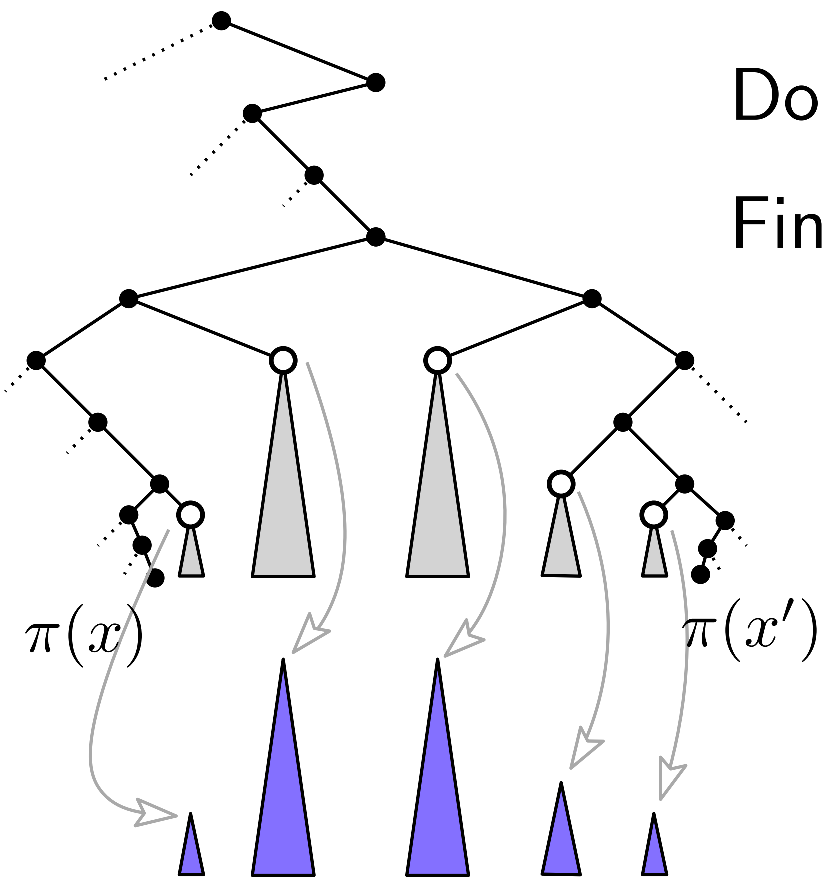

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Each level stores chunks of a $(d-1)$-dim range tree

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In $\mathbb{R}^d$, it gives $O(\log^d n + k)$ $\boxed{\text{But! space is up to } \Theta(n \log^{d-1} n)}$

Handling degeneracies

Composite numbers: $a, b \in \mathbb{R} \rightarrow (a|b)$
Sorted lexicographically.

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$\Rightarrow$ all first and second coords are distinct

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Given query $Q = [x, x'] \times [y, y']$, replace by

$$Q^* = \left[(x| -\infty),\ (x'|\infty)\right] \times \left[(y| -\infty),\ (y'|\infty)\right]$$

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Observation.

$$p \in Q \Leftrightarrow p^* \in Q^*$$

(Special) Fractional cascading

Lueker 1978, Willard 1978
Chazelle and Guibas 1986

Speeding up binary searches on subsets

Array and subarray, query $[4, 45]$
$\Rightarrow$ two binary searhces?

A	2	3	5	8	13	21	34	55	89	92	95	99

B	2	8	13	34	55	92	95

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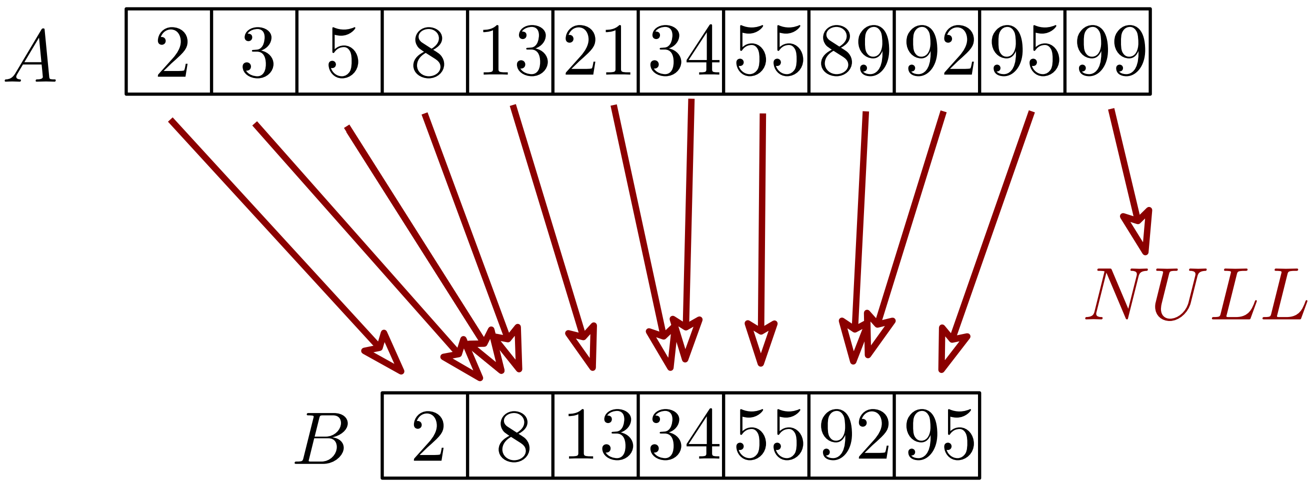

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Array and subarray, query $[4, 45]$
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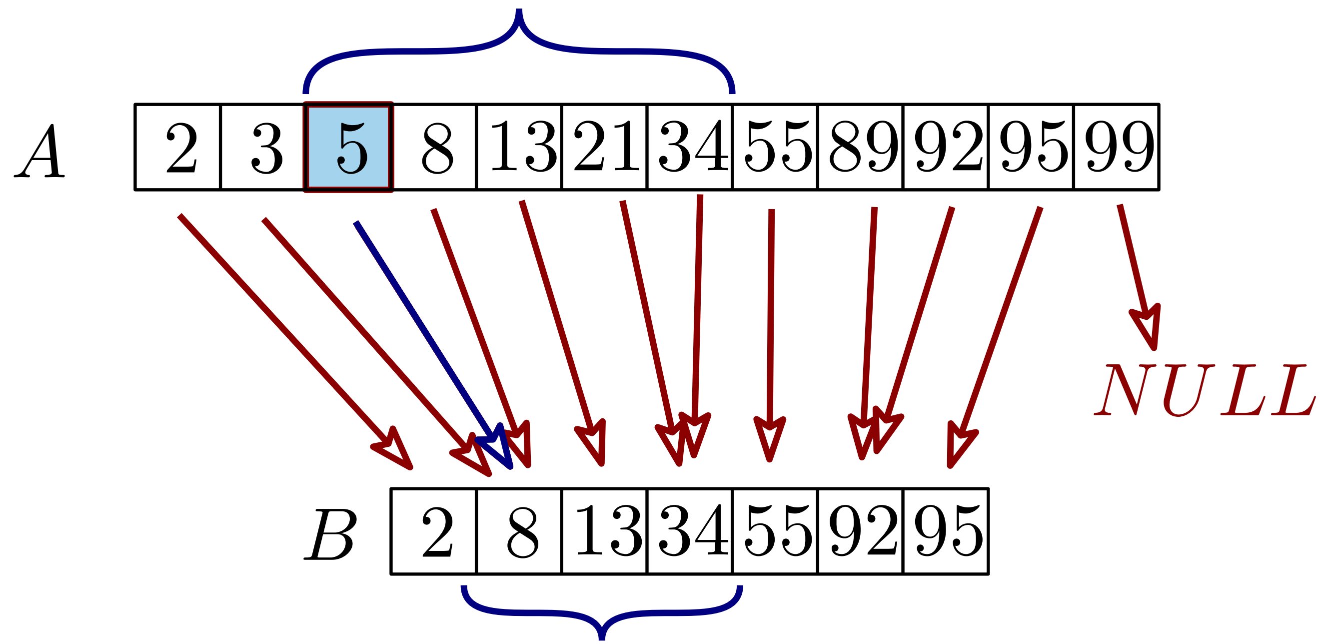

Add pointer from $A[x]$ to the smallest in B larger than $A[x]$

Binary search A, use pointer of smallest reported

Layered range trees in $\mathbb{R}^2$

Idea:

Replace 2nd level BSTs for y-coords w/ sorted arrays A_v
$+$ 2 pointers per entry

At node v, array A_v has:

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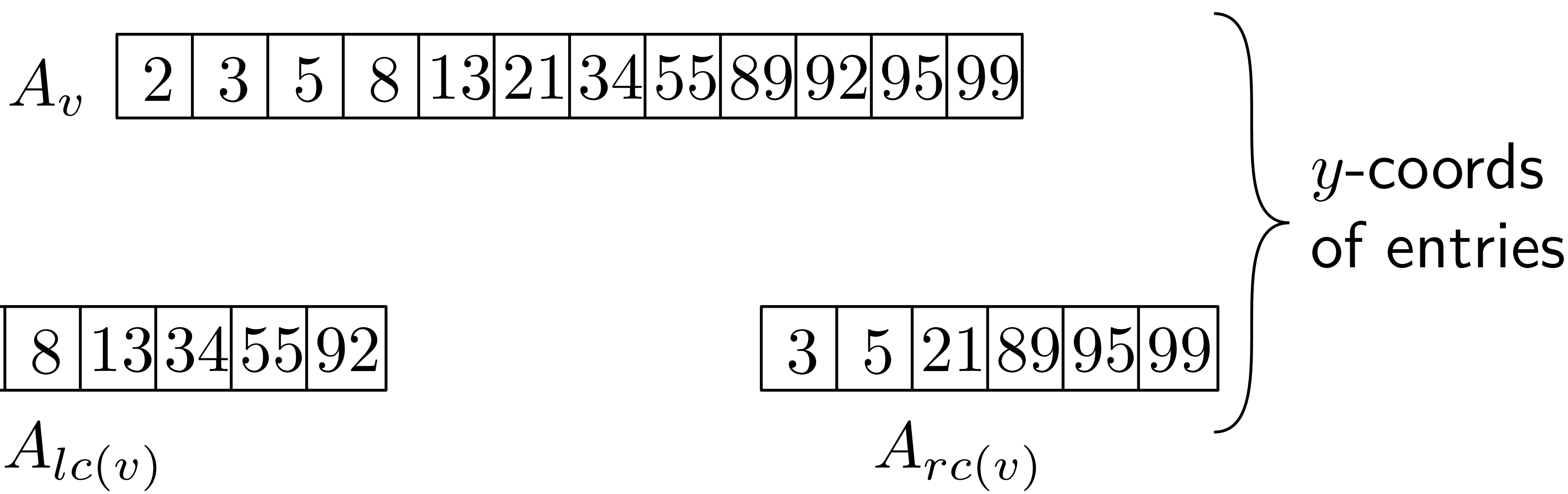

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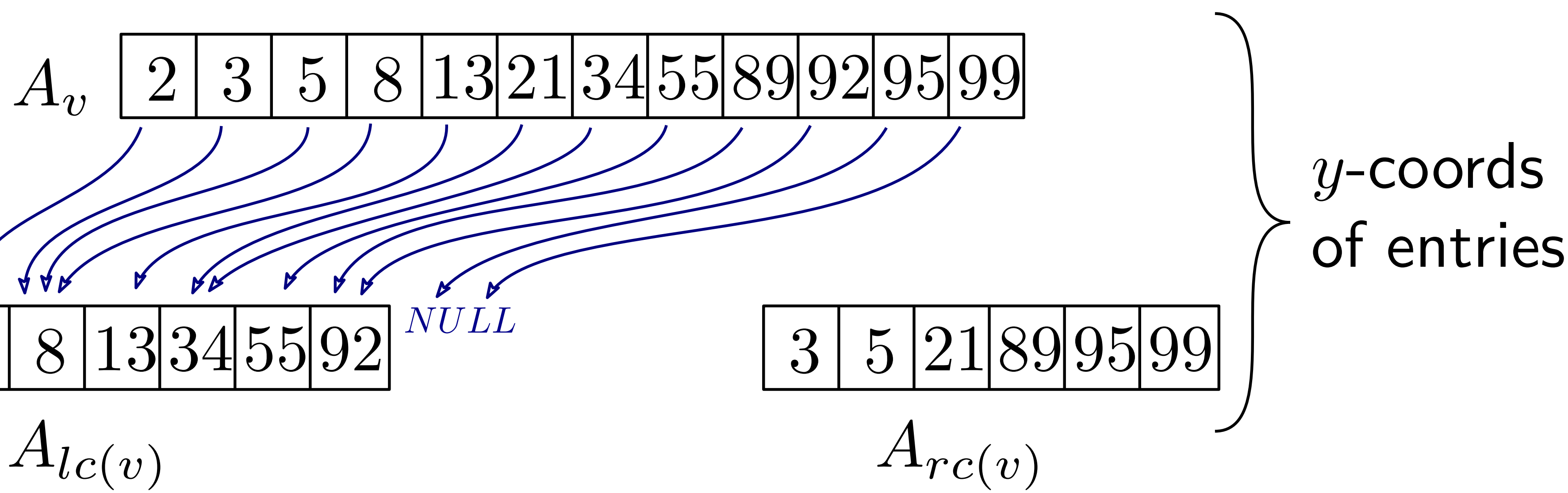

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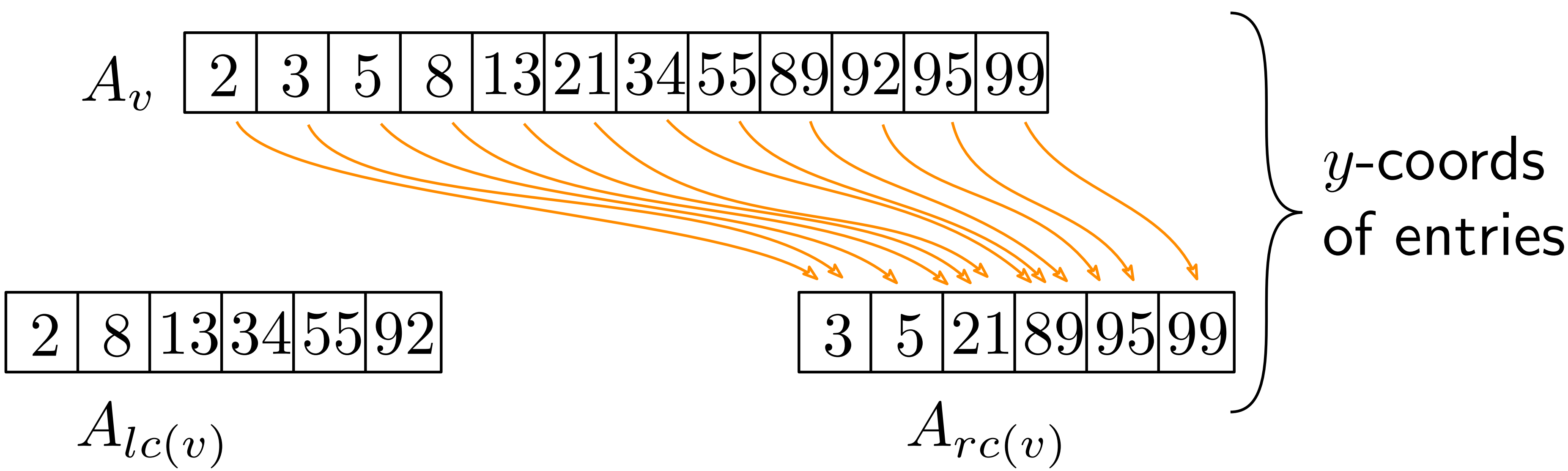

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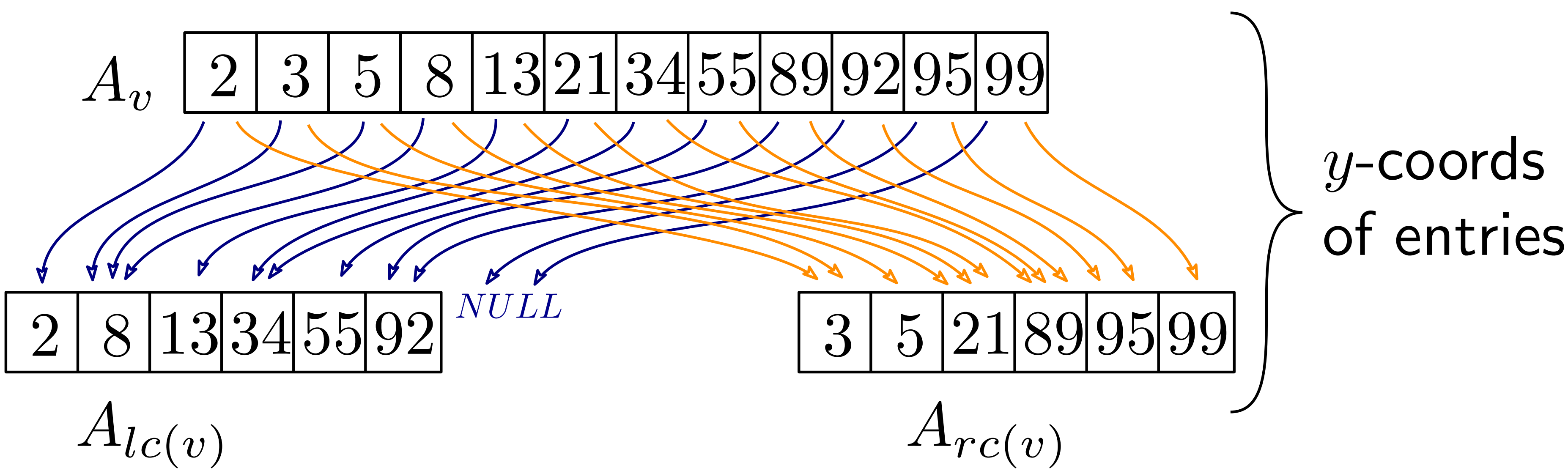

Querying a layered range tree

Query: $[x, x'] \times [y, y']$

Search top (x-)tree for x and x'.
At $v = Split(x, x')$, binary search for y in A_v $\quad \rightarrow A_v.find(y)$

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Reporting from $T(w)$ can be done from A_w
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Priority search trees
McCreight 1985

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Priority queue (insert, pop max)

$+$

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- root r stores point with max y-coord
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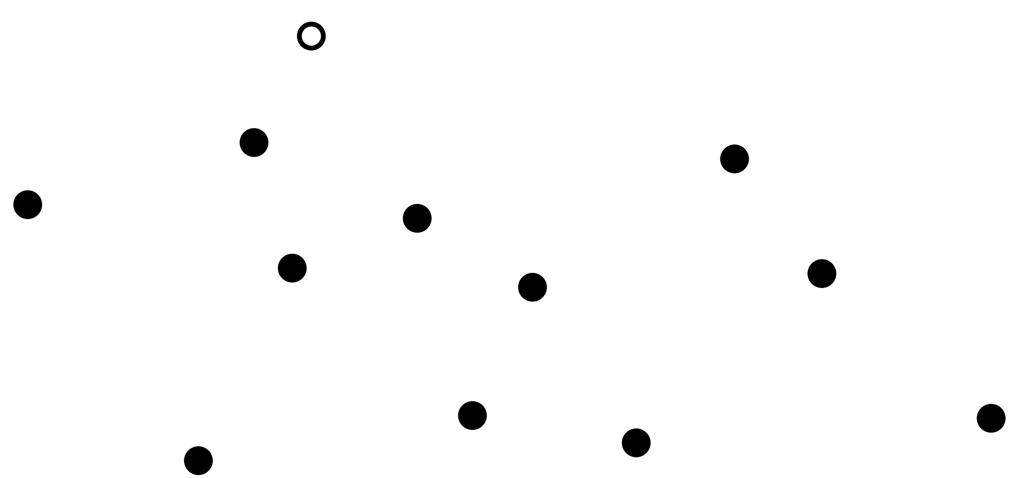

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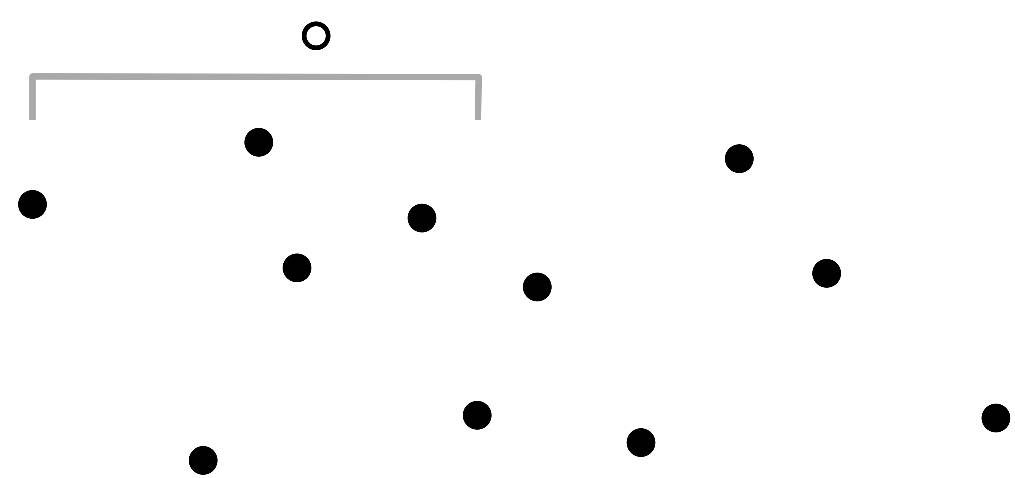

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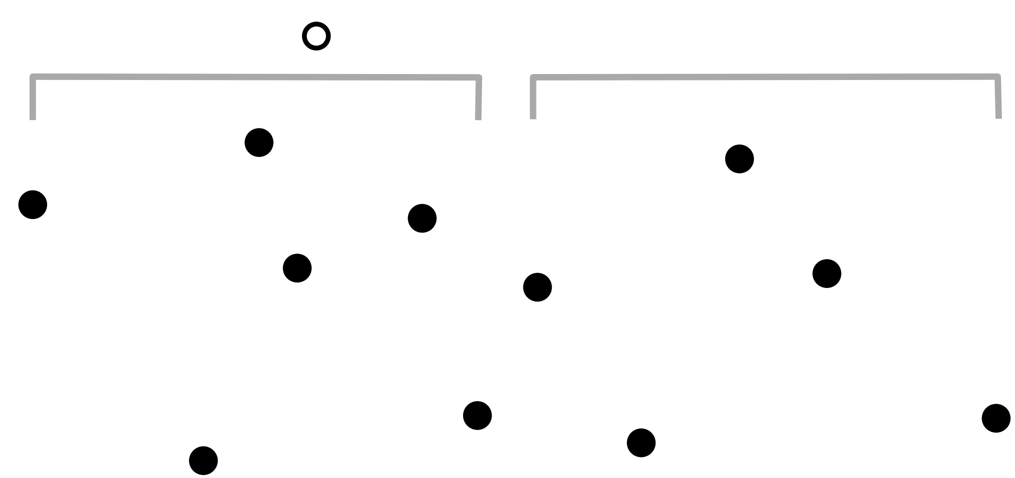

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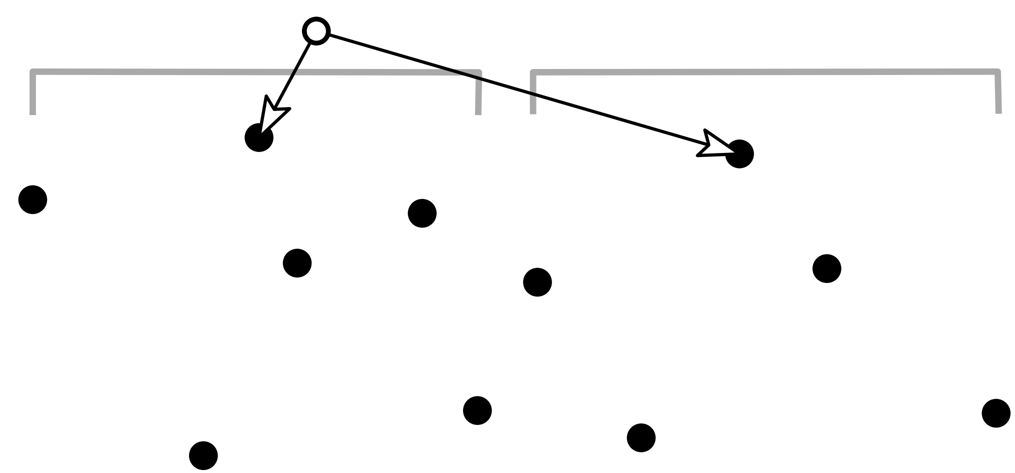

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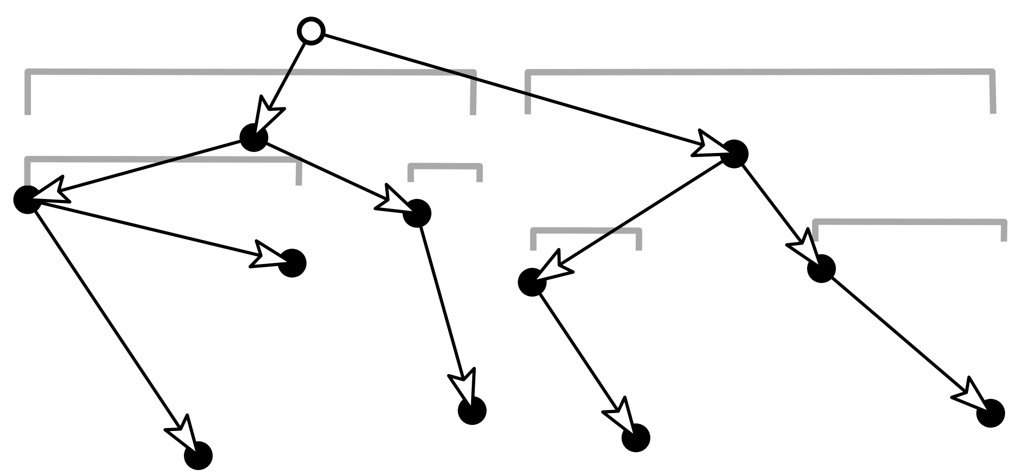

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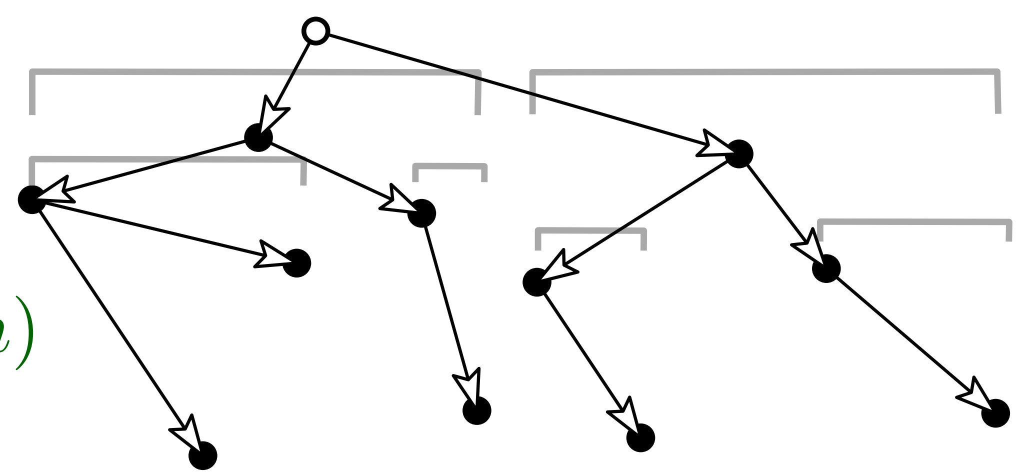
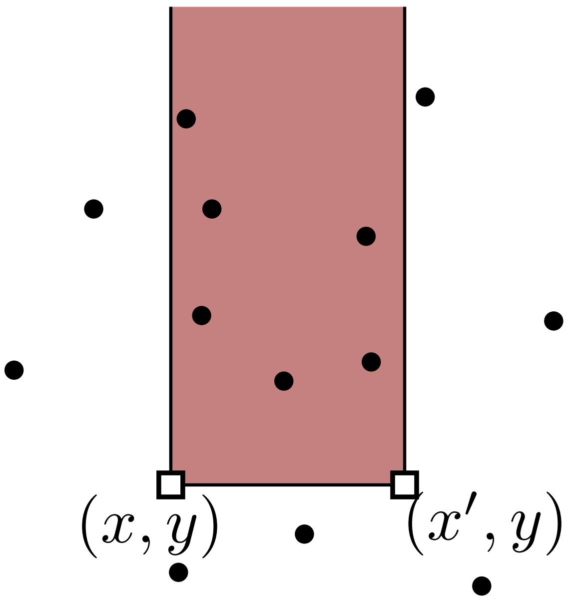


Answering 3-sided queries

Wlog queries of the type $[x, x'] \times [y, \infty]$



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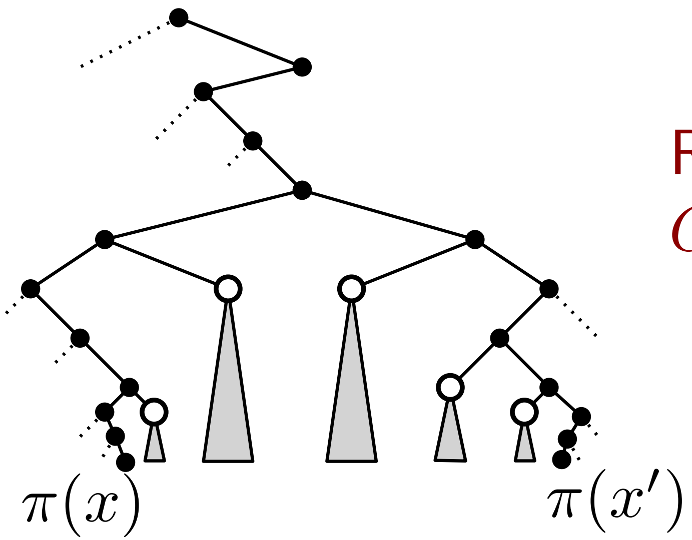

Report subtree of v:
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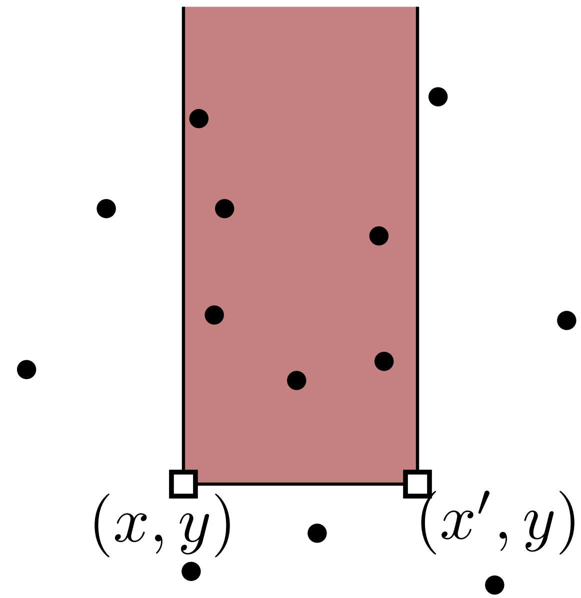

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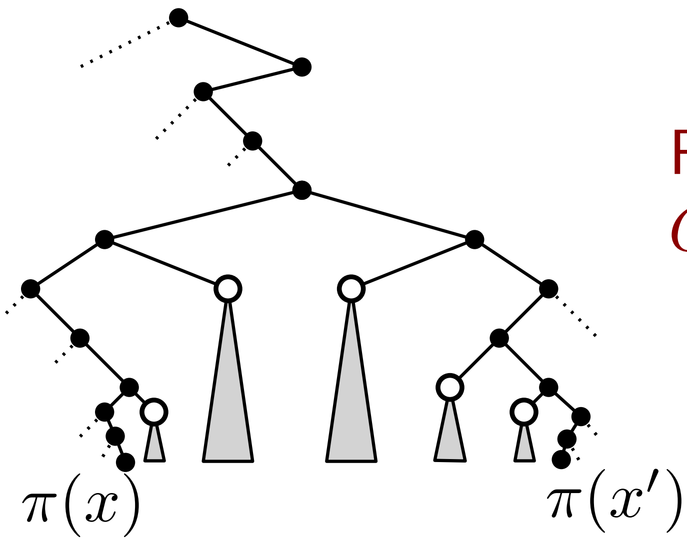

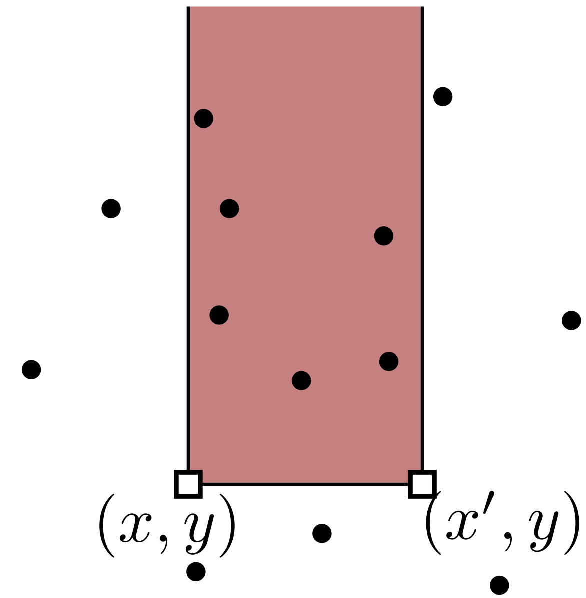

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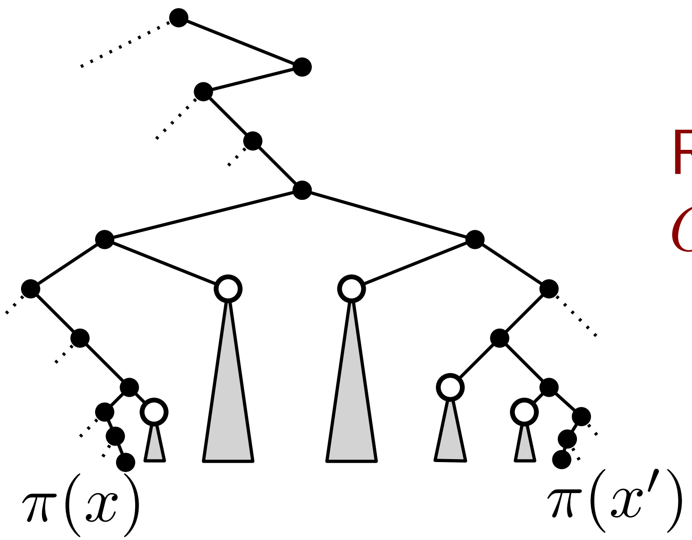

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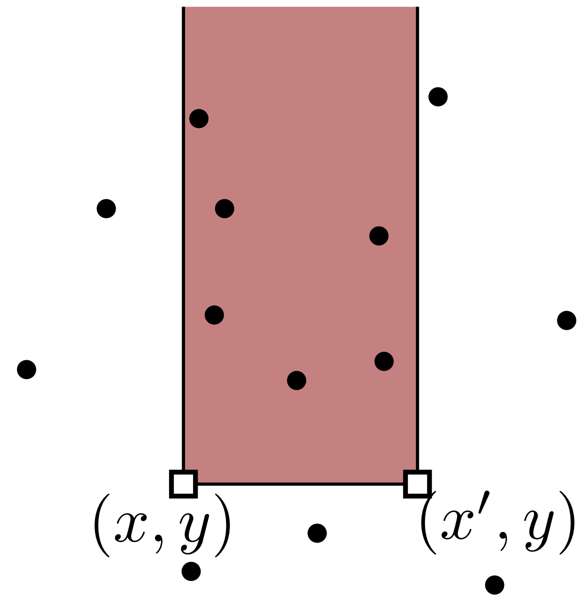

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Query time: $O(\log n + k)$

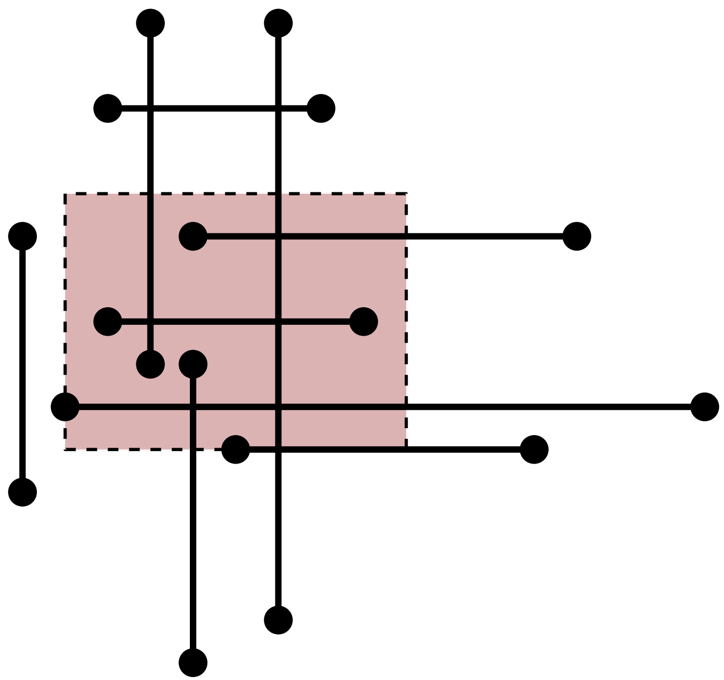
Current best data structures

Assume coordinates fit in machine words on w bits.
Multiply all by $\log w \simeq \log\log n$.

amortized

Static	Dynamic
s:$O(n)$ q:$O(\log^{\varepsilon} n + k \log^{\varepsilon} n)$ or s:$O(n \log\log n)$, q:$O(\log\log n + k \log\log n)$ or s:$O(n \log^{\varepsilon} n)$, q:$O(\log\log n + k)$	s:$O(n \log^{2/3+o(1)} n)$ q:$O(\frac{\log n}{\log\log n} + k)$ u:$O(\log^{2/3+o(1)} n)$
Chan–Larsen–Pătrașcu	Chan–Tsakalidis

Range searching axis-parallel intervals



$\rightarrow$ intervals ending in Q:
range searching on endpoints

$\rightarrow$ intervals "crossing" Q?

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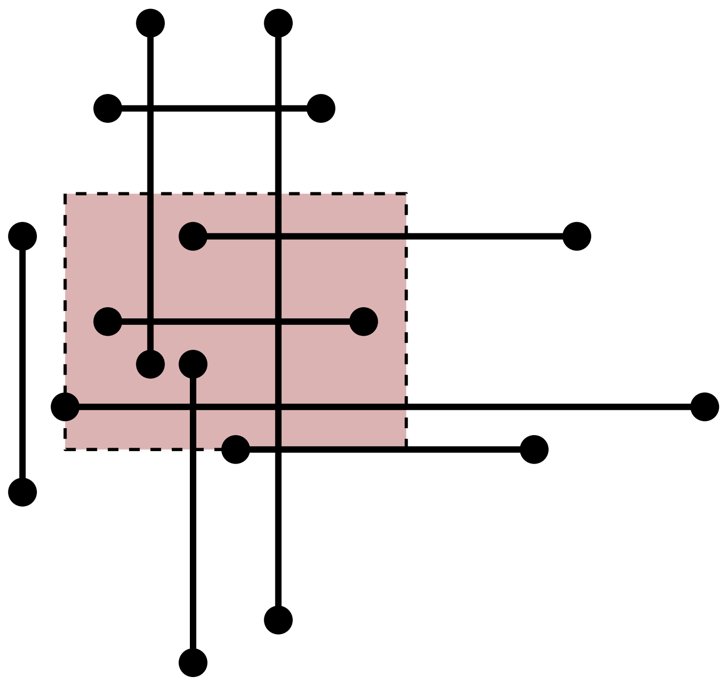

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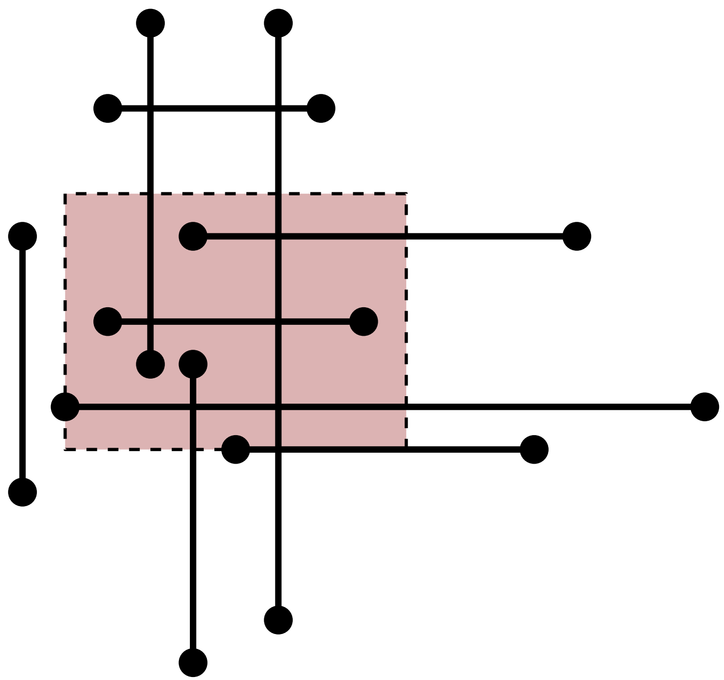

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Given a vertical query segment, report all horizontal segments it is crossing.

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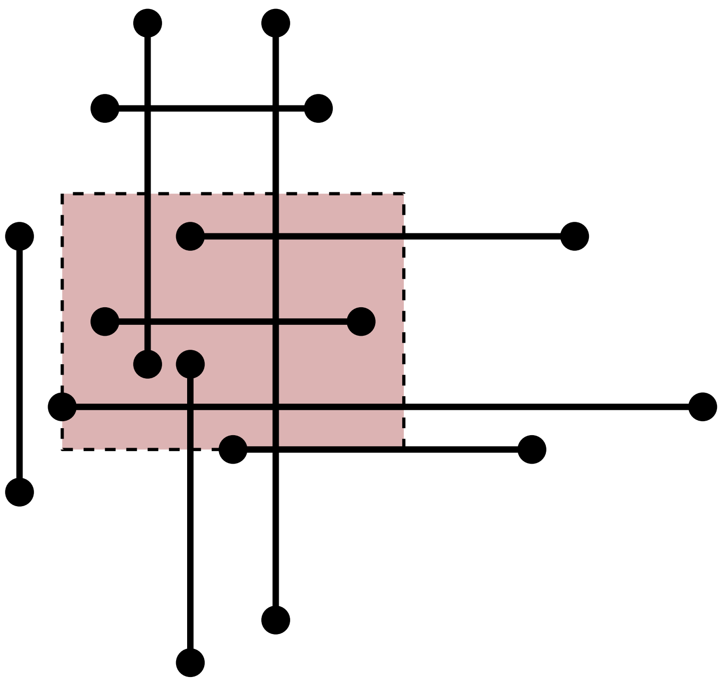

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Given a vertical query ~~segment~~ line, report all horizontal segments it is crossing.

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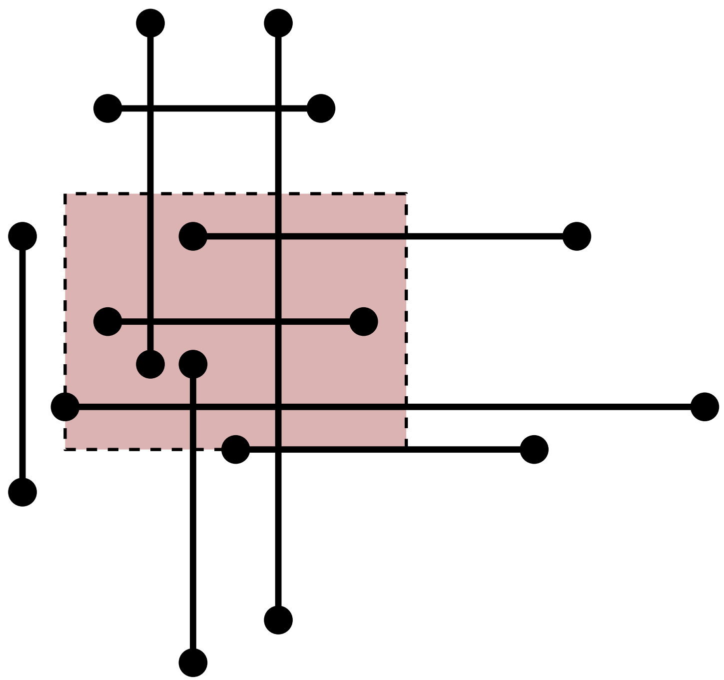

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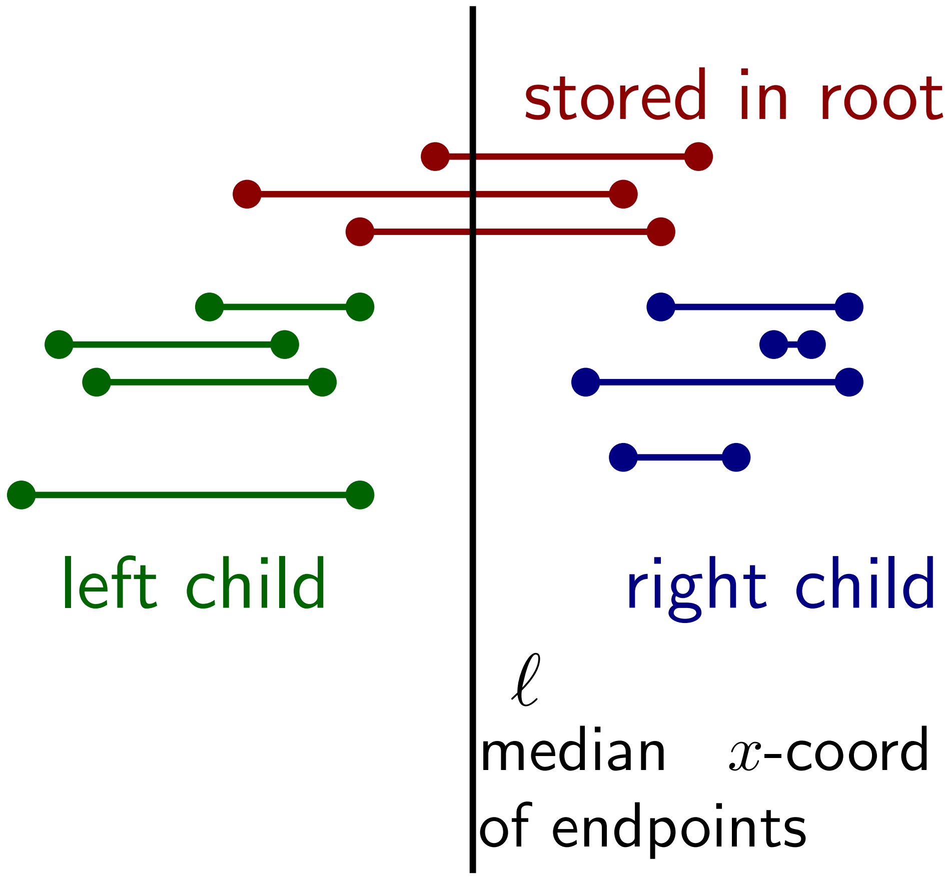
$\Leftrightarrow$ in $\mathbb{R}^1$, which intervals contain query point q?

Interval trees (Edelsbrunner 1980 McCreight 1980)

root has intervals intersected by ℓ sorted for left endpoints in a list (same for right)



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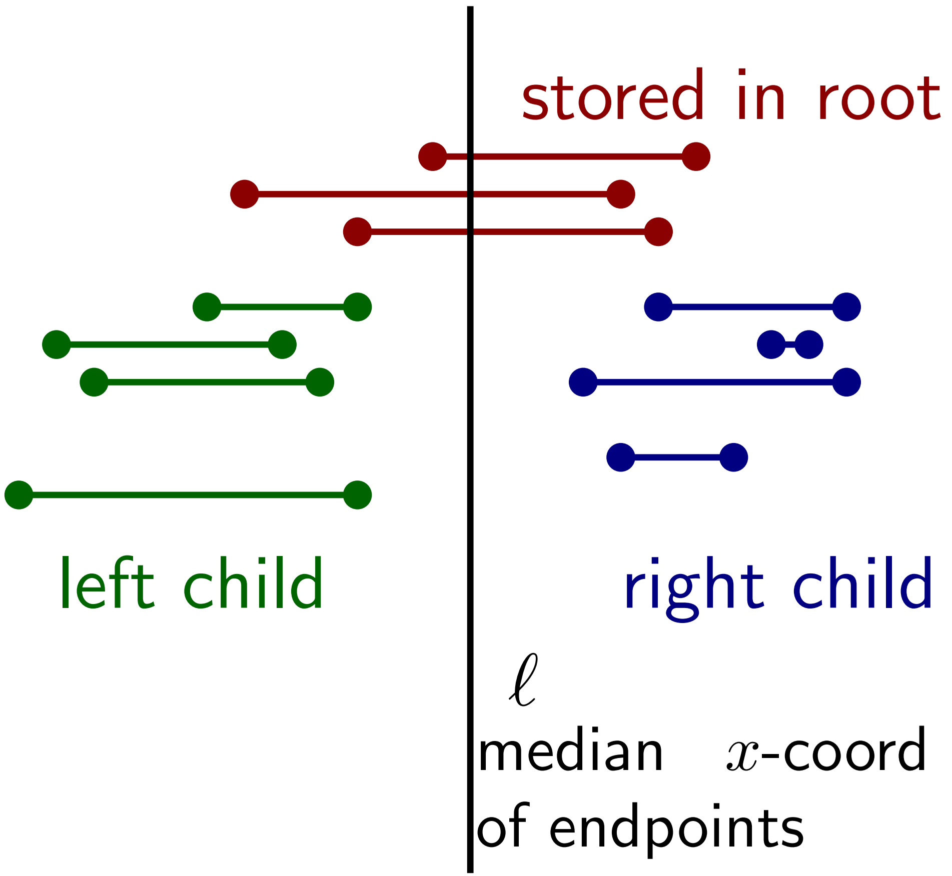

Preprocess: $O(n \log n)$

Space: $O(n)$

Query ($\mathbb{R}^1$): $O(\log n + k)$

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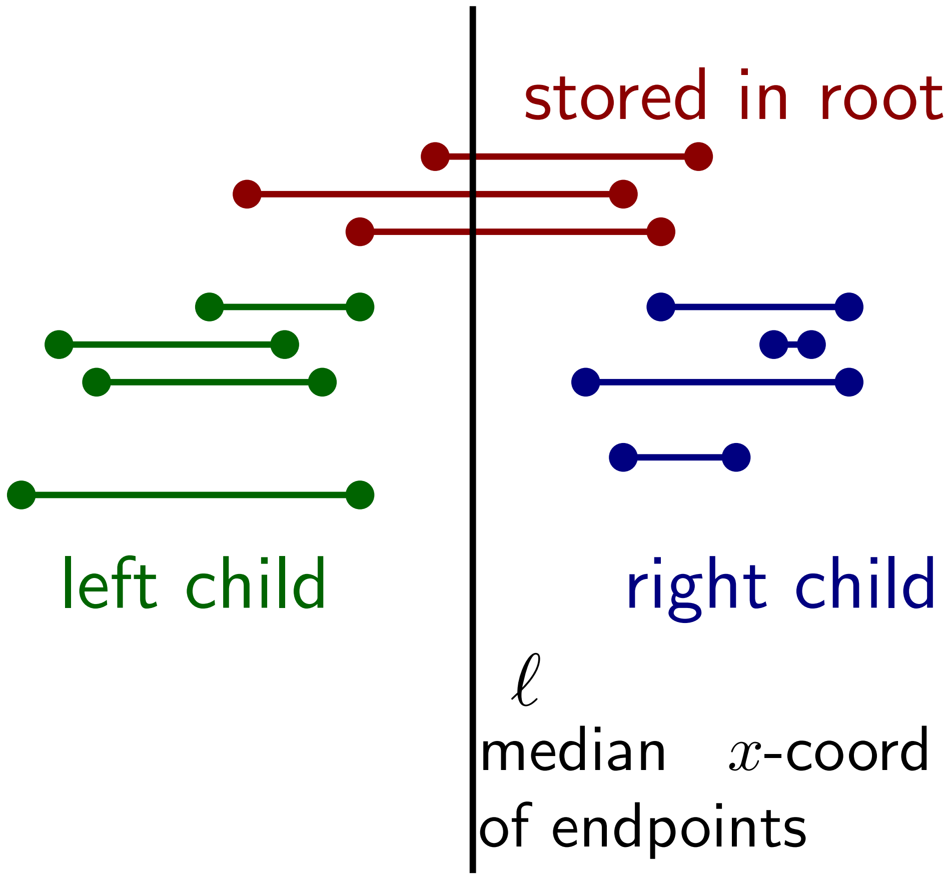

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If segment query:
use priority search tree instead of list