Computational Geometry, SS 2020
Assignment 4
Due: Wednesday 24.6, 8:00 (morning!) CET

Send your solutions in pdf format to:

andre.nusser@mpi-inf.mpg.de

Problem 1. (8 points)

Computing a θ-graph in the plane reduces to finding for each point p in input set S its partner $k_u(p)$ for some direction u. (Recall that $k_u(p)$ is the point $q \in S \setminus \{p\}$ with $\angle(\vec{pq}, u) \le \phi = \theta/2$ and whose projection of onto the ray $R_u(p)$ is closest to p.)

Give an algorithm that given S and u produces for each $p \in S$ its partner $k_u(p)$ provided it exists. Your algorithm should run in $O(n \log n)$ time.

Hint: You may find it easier to compute the inverse of the function $k_u(\cdot)$.

Problem 2. (8+1 points)

For a set S of n points in $\mathbb{R}^d$ define $X(S)$ to be the sum of all interpoint distances, i.e., $X(S) = \sum_{\{p,q\} \subset S} \delta(p,q)$. Design an efficient algorithm for approximating $X(S)$, i.e., given S and parameter ε the algorithm should output a number Y with $X(S) \le Y \le (1+\varepsilon)X(S)$.

Show that you can compute the exact sum of all **squared** distances in less time.

Problem 3. (5+1 points)

Let T be the Delaunay triangulation of some point set S in the plane. Let D be some disk and let T_D be the subgraph of T that is induced by the vertex set $S \cap D$. ("Induced" here means that T_D contains exactly those edges of T that have both endpoints in $S \cap D$.) Prove that T_D is a connected graph.

Is the same statement true if instead of the Delaunay triangulation you are dealing with the graph formed by the vertices and edges of a Voronoi diagram?

Problem 4. (6 points)

In a point set $P \subseteq \mathbb{R}^2$ the nearest neighbor of $p \in P$ is the point $p' \in P \setminus \{p\}$ for which $\mathrm{dist}(p, p')$ is minimized. Given P, compute the nearest neighbor for each $p \in P$ in $O(n \log n)$ time.

Problem 5. (8 points)

In the L_1 metric (a.k.a. Manhattan or taxicab metric), the distance from (x, y) to (x', y') is $|x - x'| + |y - y'|$. Show that the L_1 Voronoi diagram of a set of n points (as a subdivision of $\mathbb{R}^2$ into polygons) has complexity $O(n)$.

Problem 6. (10 points)

Given a subdivision of $\mathbb{R}^2$ into convex shapes of total complexity n (with a doubly connected edge list), decide in $O(n)$ time if it is the Voronoi diagram of some point set P, and if so, output P.