Computational Geometry, SS 2020
Assignment 5
Due: Wednesday 8.7, 8:00 (morning!) CET

Send your solutions in pdf format to:

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Problem 1. (6 points)
The farthest point Delaunay triangulation of $P \subset \mathbb{R}^2$ consists of triangles whose circumcircle does not have any point of P in its exterior. Let $L(P)$ be the lifting of P to the paraboloid $z = x^2 + y^2$. Show that projecting the upper convex hull of $L(P)$ back to the $z = 0$ plane results in the farthest point Delaunay triangulation of P. (For no extra credit, convince yourself that the dual of this triangulation is the farthest point Voronoi diagram, where the cell of $p \in P$ consists of $x \in \mathbb{R}^2$ where $\text{dist}(x, p) \geq \text{dist}(x, p')$ for all $p' \in P \setminus \{p\}$).

Problem 2. (8 points)
Show that the greedy clustering does not give a constant-approximation for k-median in general metric spaces for any $k \geq 2$. (That is, for every $k \geq 2$ and for every $c > 0$, give a metric space where the cost of greedy clustering can be at least c times larger than the cost of the optimal k-median.)

Problem 3. (10 points)
Given a set of n disjoint *unit* disks in $\mathbb{R}^2$, show that there exists a horizontal or vertical line intersecting at most $3\sqrt{n}$ disks that is 4/5-balanced, i.e., each open halfspace defined by the line contains at most $\frac{4}{5}n$ disks.

Problem 4. (10 points)
In the Maximum Triangle-Free Subgraph problem, one is given a graph, and the goal is to find a maximum size subset S of vertices such that the subgraph induced by S does not contain any triangles. We will solve this problem in unit disk graphs, where the unit disks are given as input.

(a) Show that the solution set S of disks has a balanced square (or line) separator of size $O(\sqrt{k})$.

(b) Give an $n^{O(\sqrt{k})}$ exact algorithm.

(c) Give a PTAS, i.e., an algorithm that finds a vertex set inducing a triangle-free subgraph of size at least $(1 - \varepsilon)OPT$ within $n^{O(1/\varepsilon)}$ time.

Problem 5. (7 points)
Let $S = (X, \mathcal{R})$ be a range space of VC-dimension d. Define a new range space $\widetilde{S} = (X, \mathcal{R} \cup \overline{\mathcal{R}})$, where $\overline{\mathcal{R}}$ contains all the complements of the ranges in $\mathcal{R}$.

What can you say about the VC-dimension of $\widetilde{S}$?

Problem 6. (7 points)

Let $S = (X, \mathcal{R})$ be a range space of VC-dimension d. Prove that its dual range space $S^* = (\mathcal{R}, X^*)$ has VC-dimension at least $\lfloor \log_2 d \rfloor$. Here $X^* = \{R_p \,|\, p \in X\}$ with $R_p = \{r \in \mathcal{R} \,|\, p \in r\}$.

Hint: Try to represent a finite range space as a 0-1-matrix ...