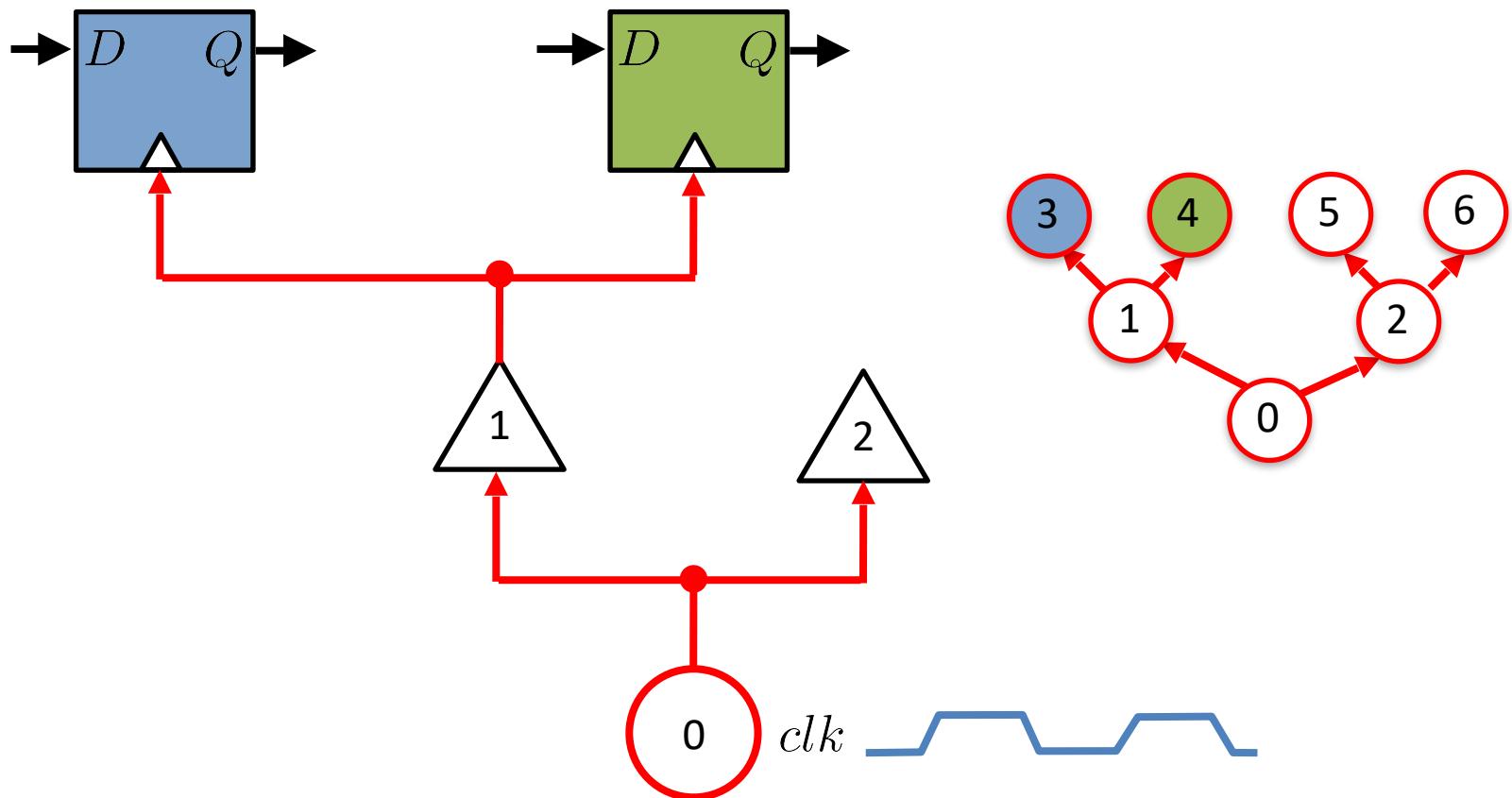


# Chapter 11

## Low-degree clock distribution networks

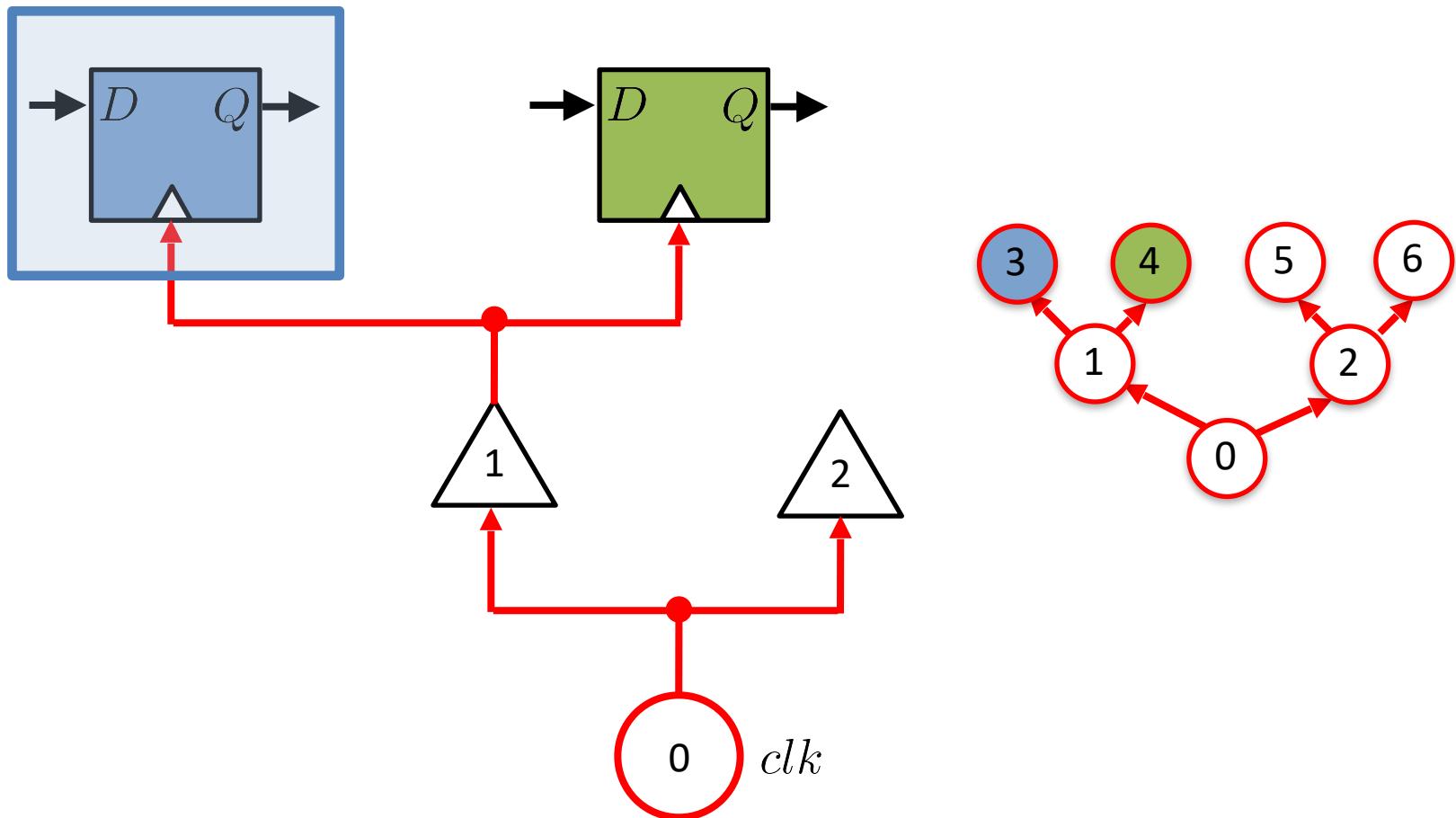
Matthias Fuegger and Christoph Lenzen

# Clock distribution network

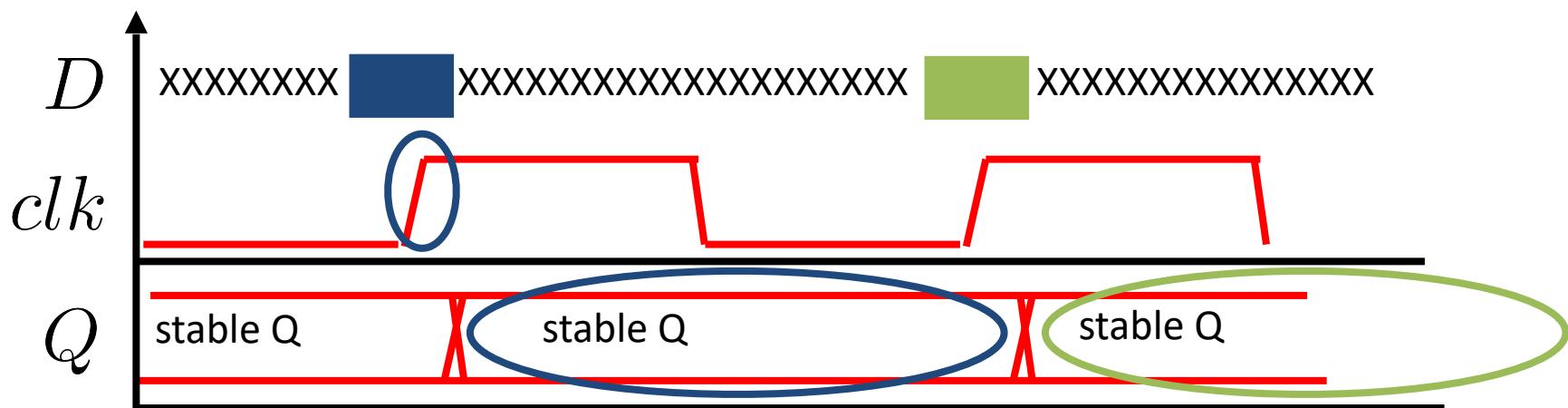
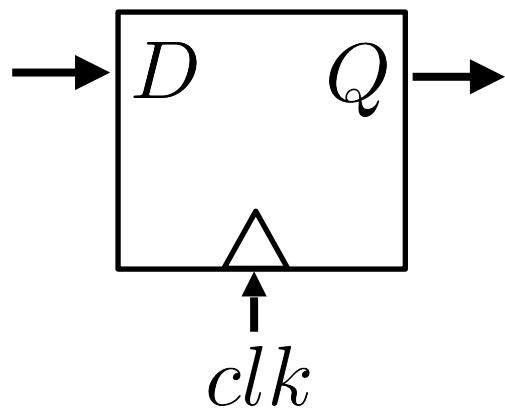


# Quick summary...

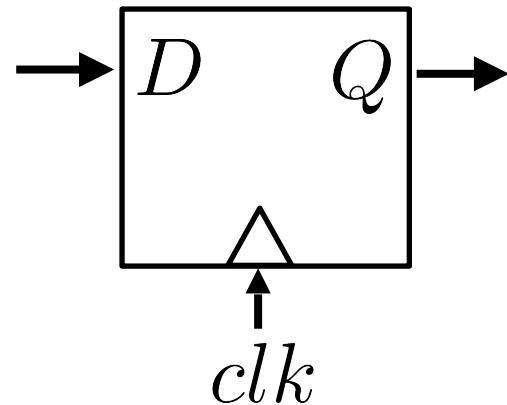
# Clock distribution network



# Flip-flop = edge triggered copy

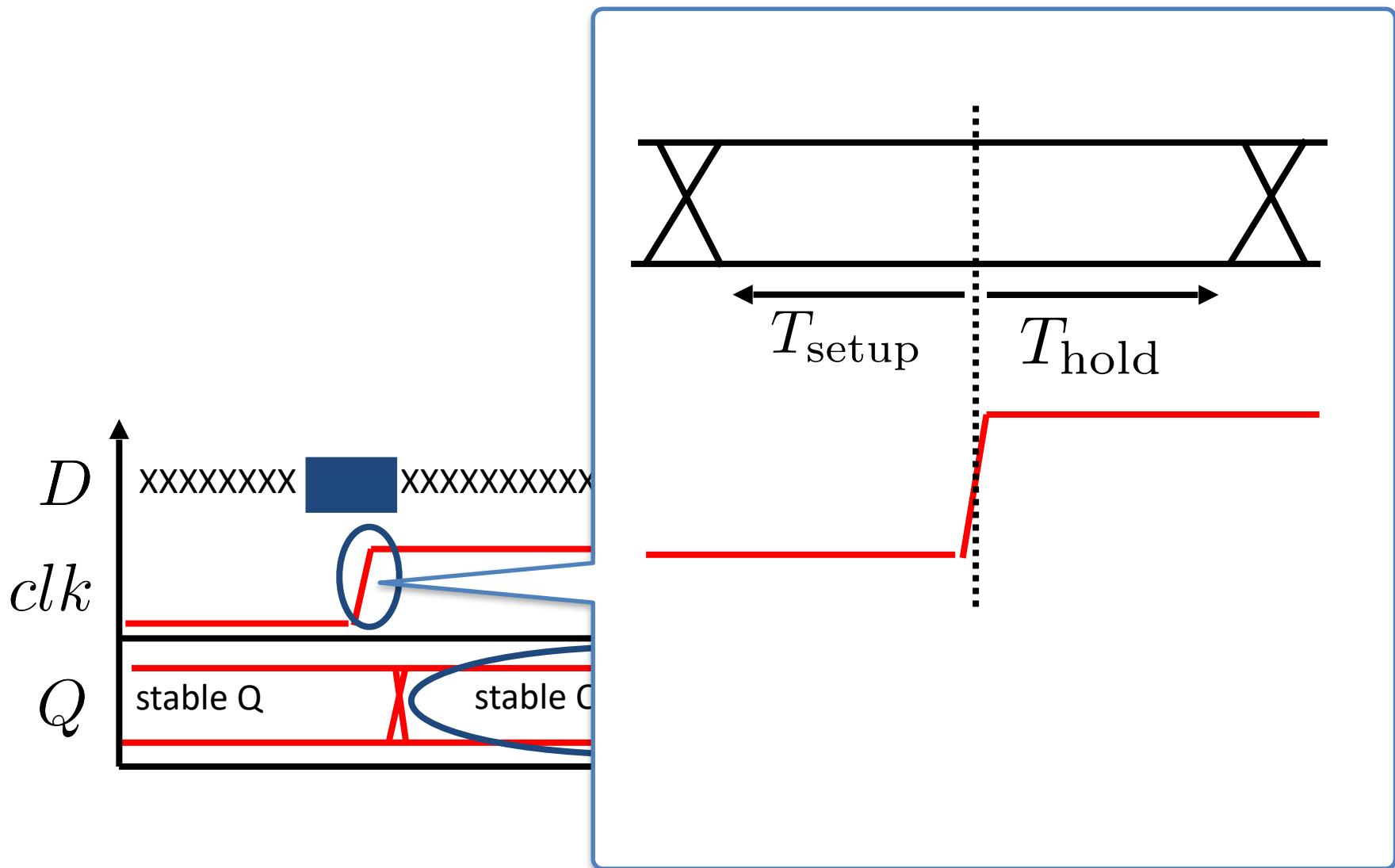


# Flip-flop = edge triggered copy

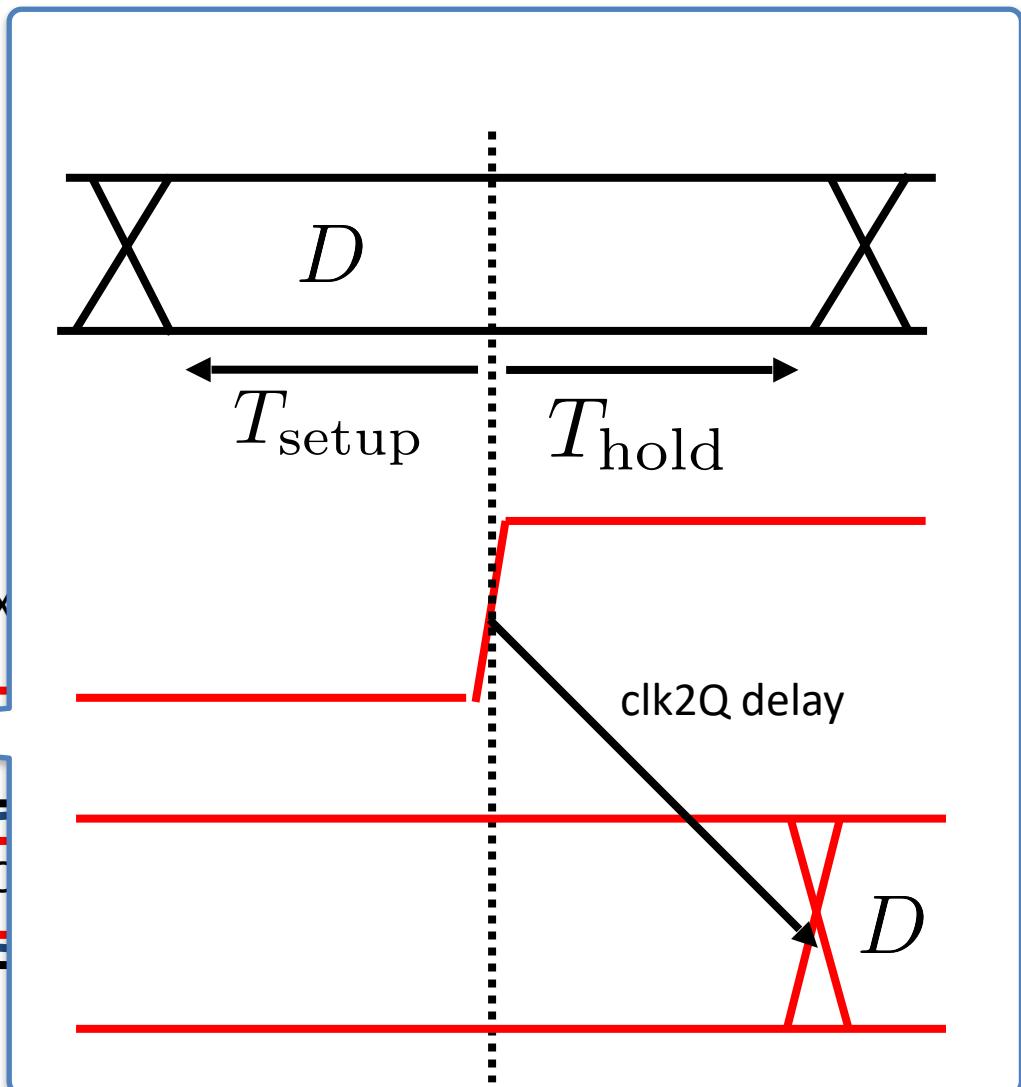
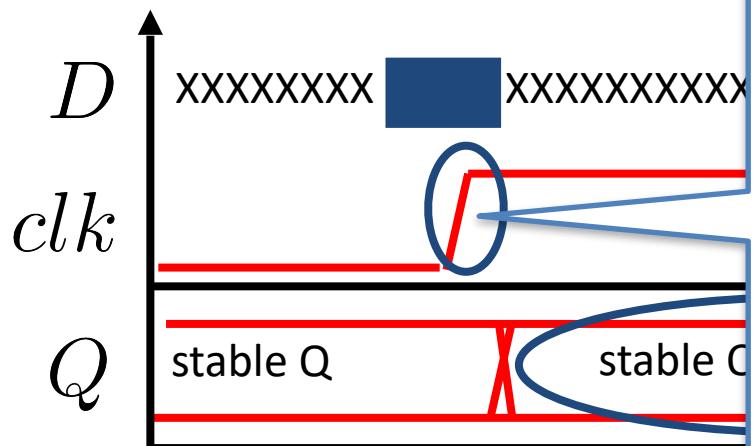


```
-- FF
FF: process (clk, D)
begin
    if (clk'event and clk = '1') then
        Q <= D;
    end if;
end process FF;
```

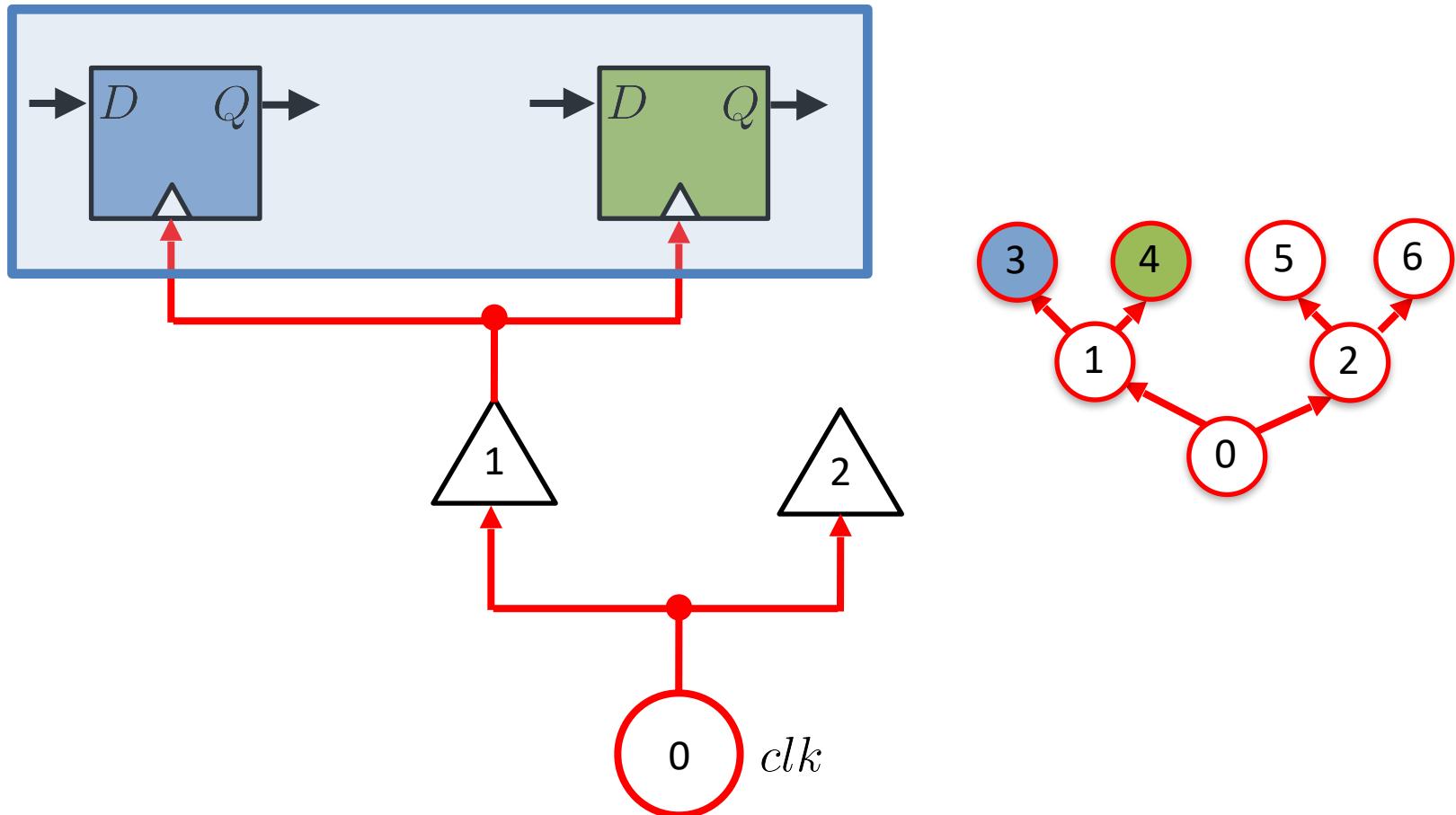
# Timing: constraints



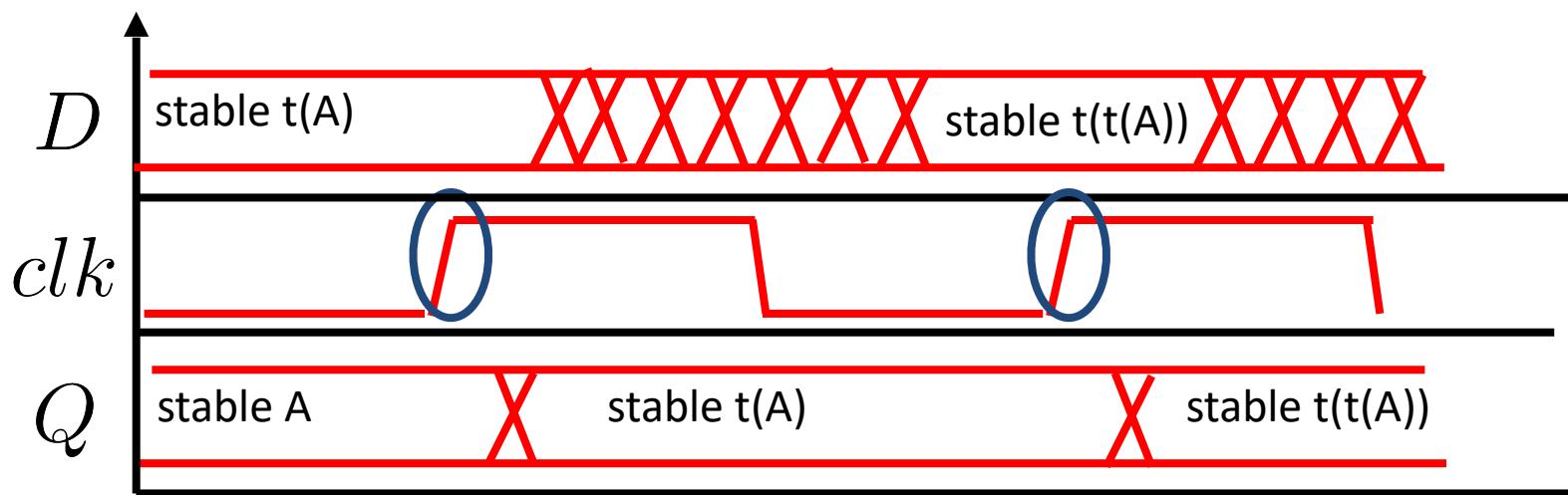
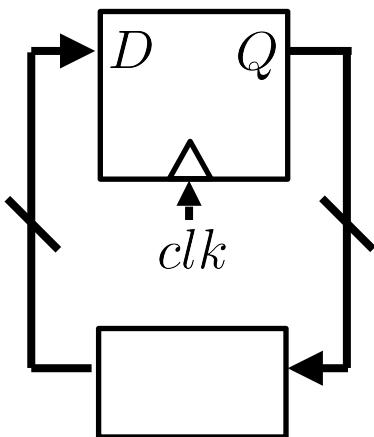
# Timing: guarantees

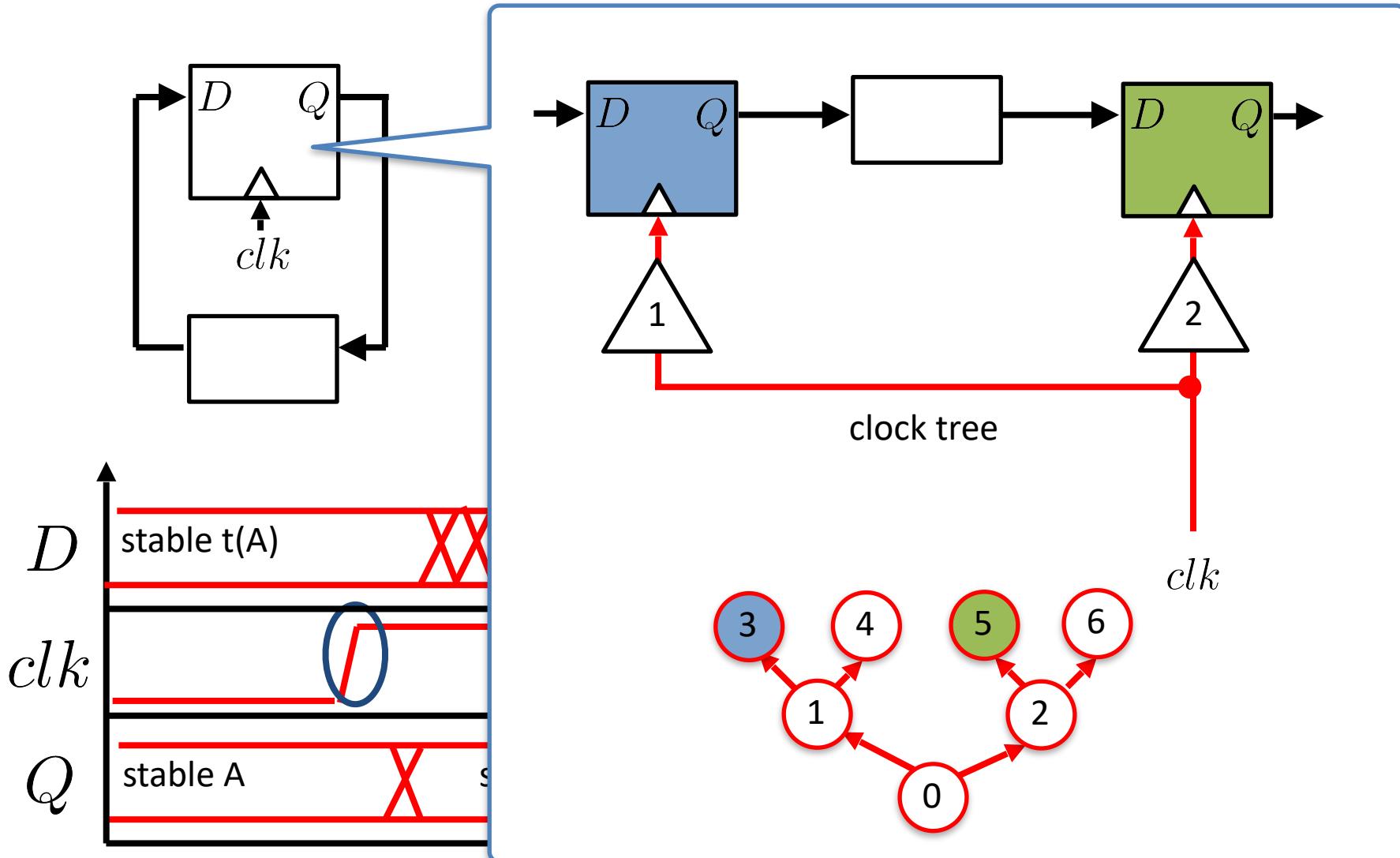


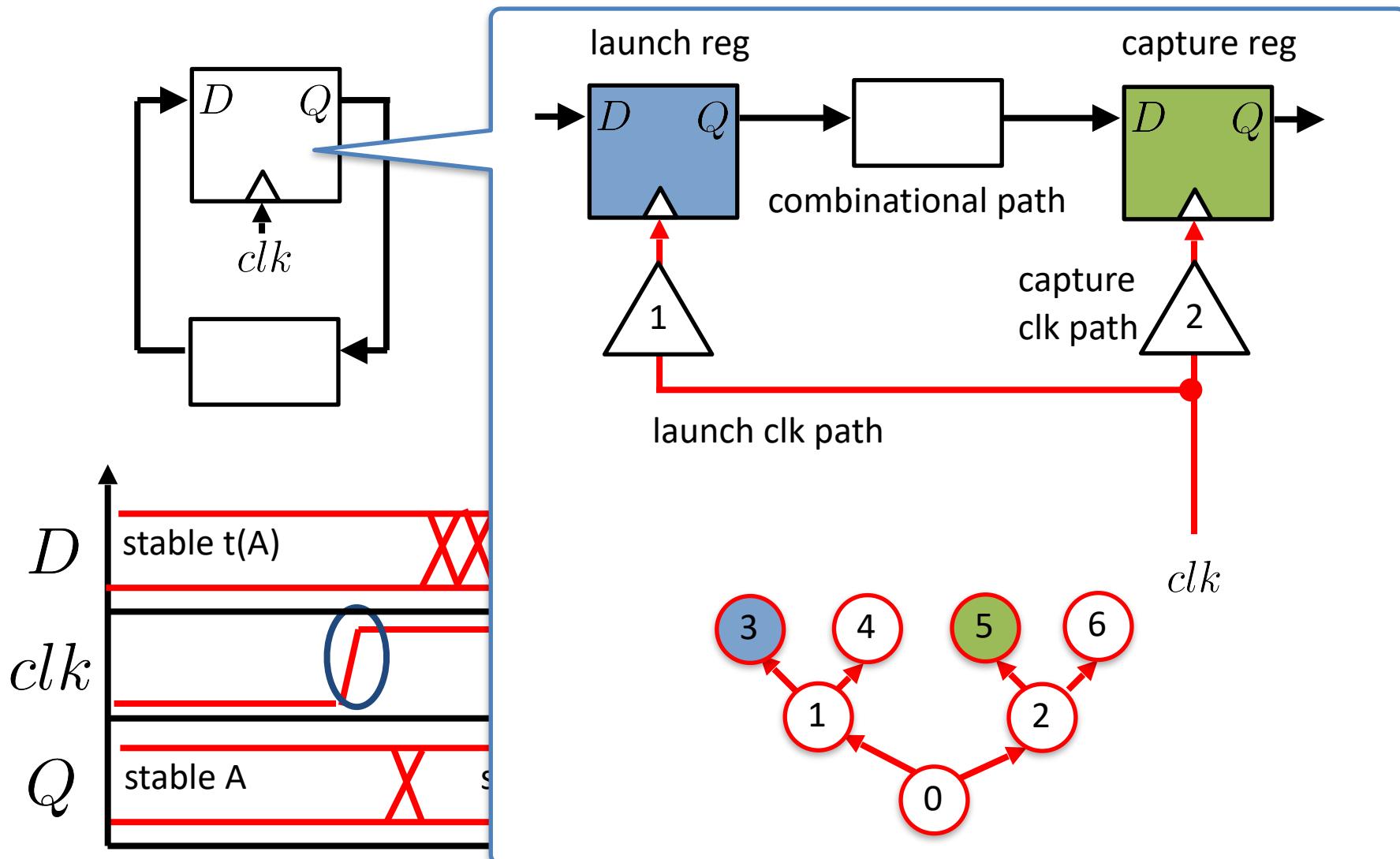
# Goal: small skew

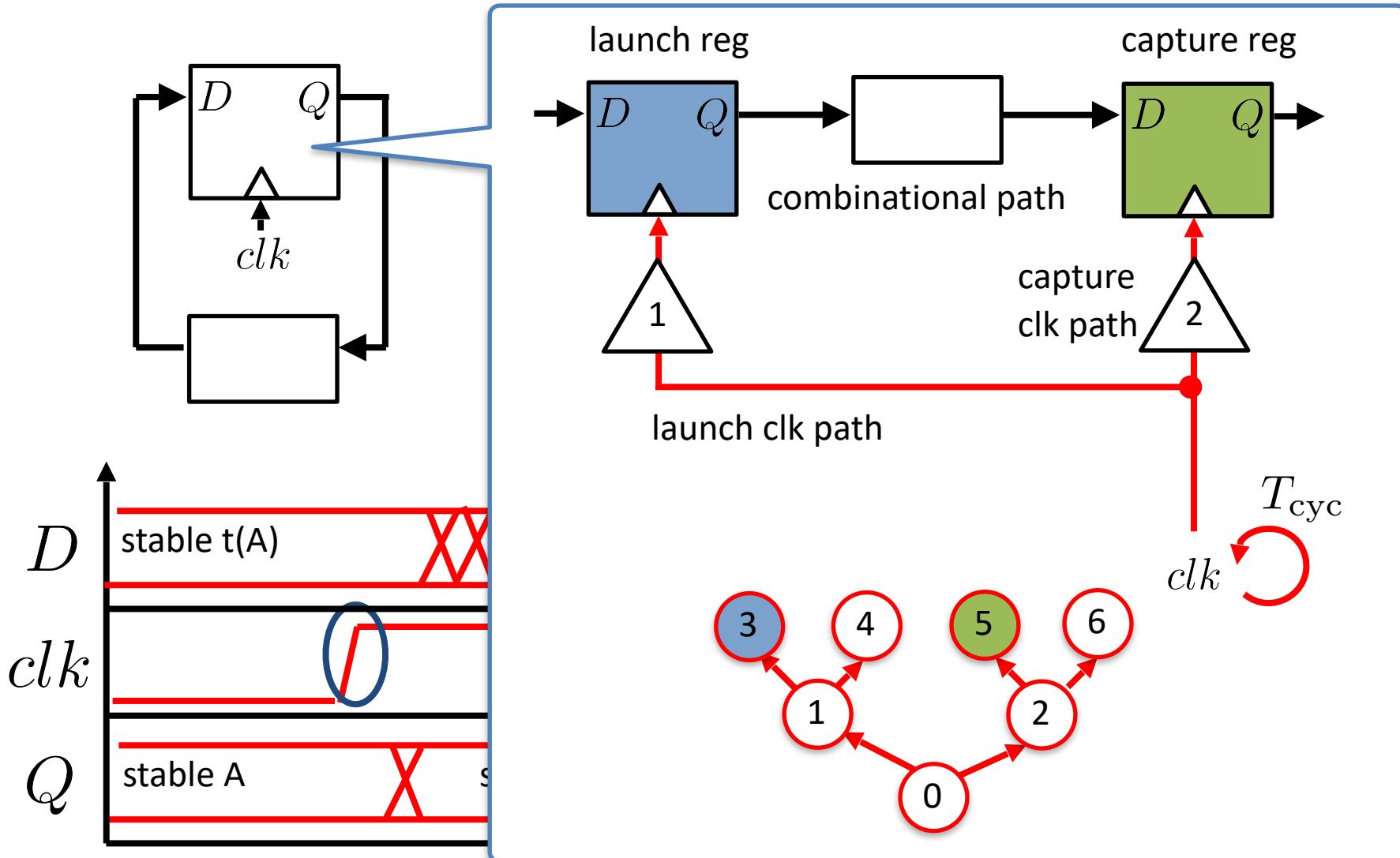


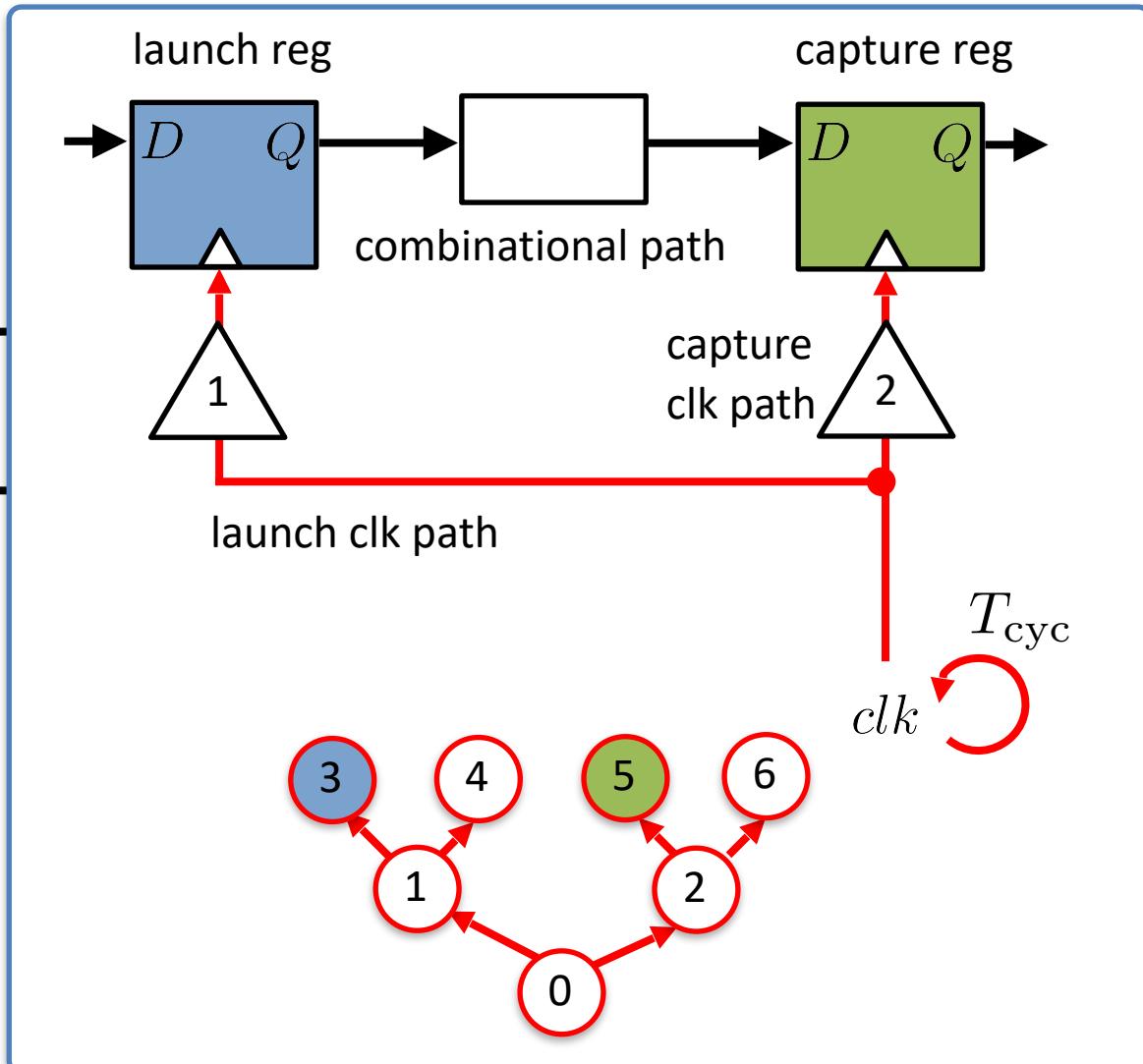
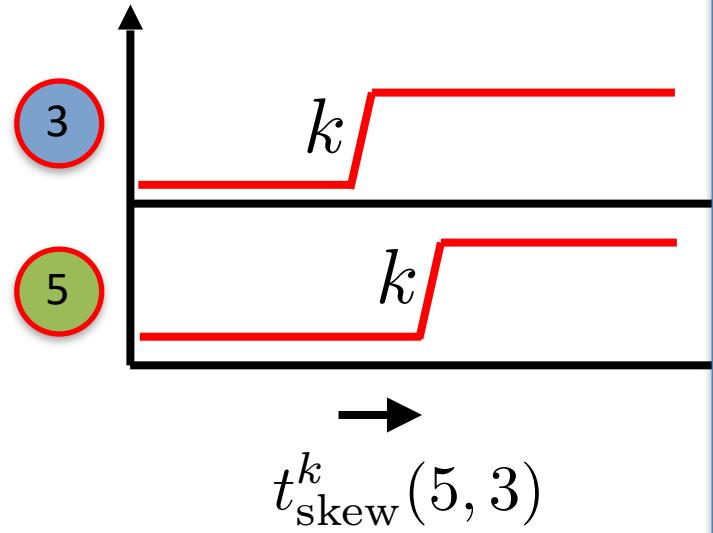
# Clocked Design

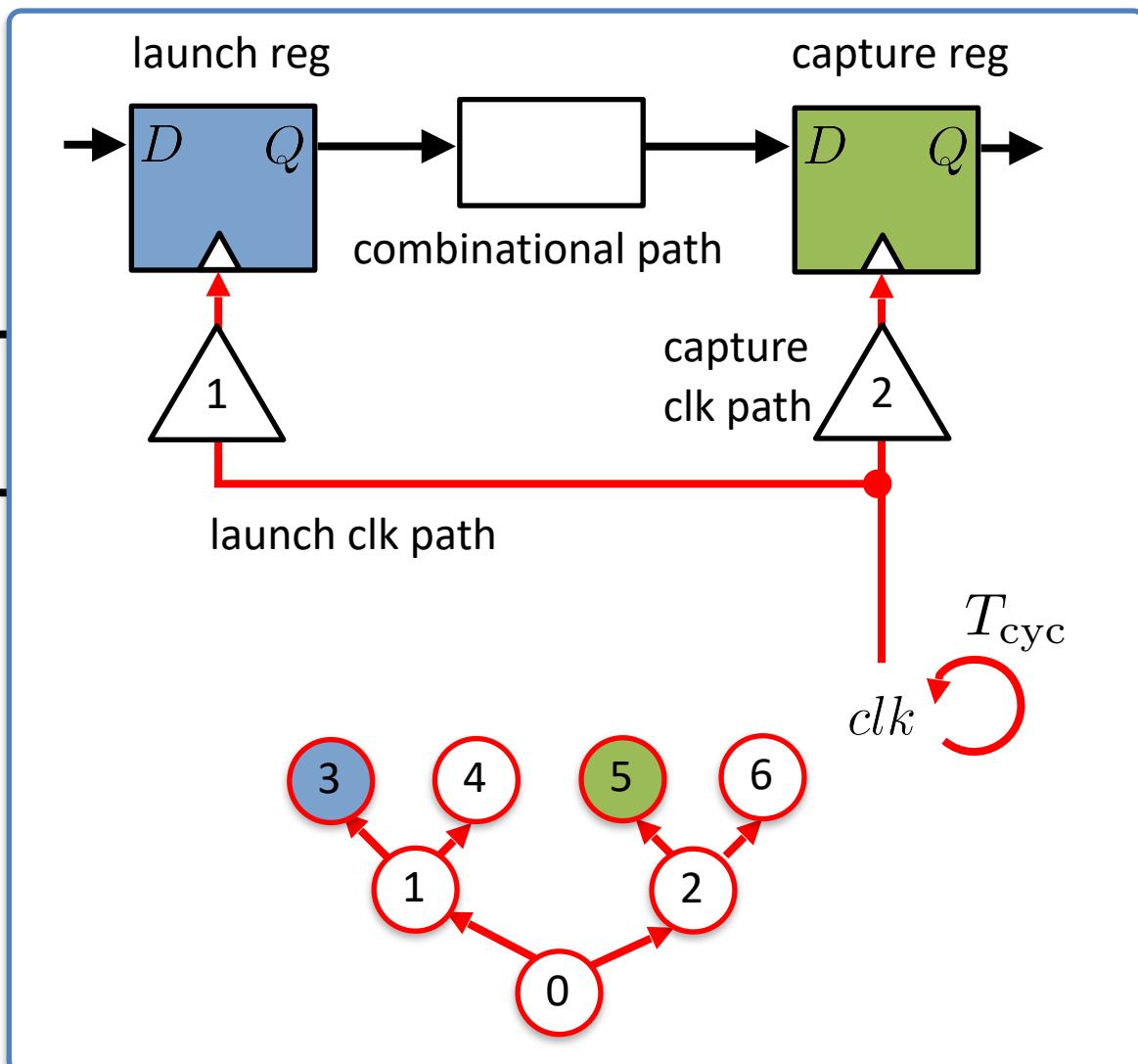
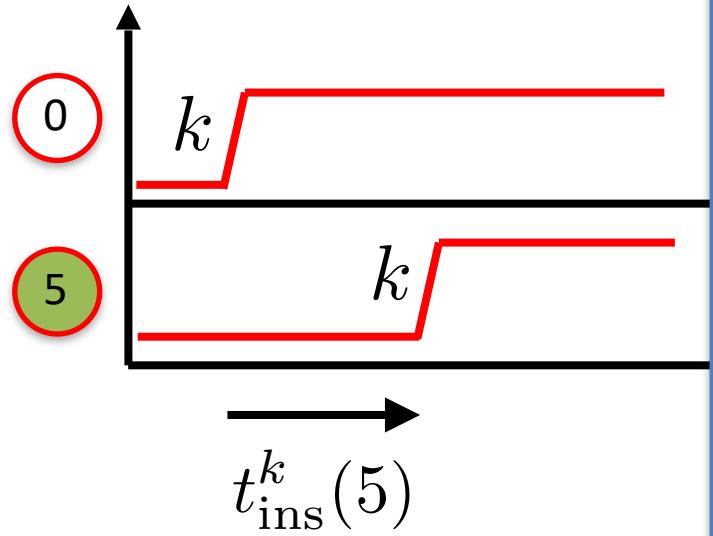


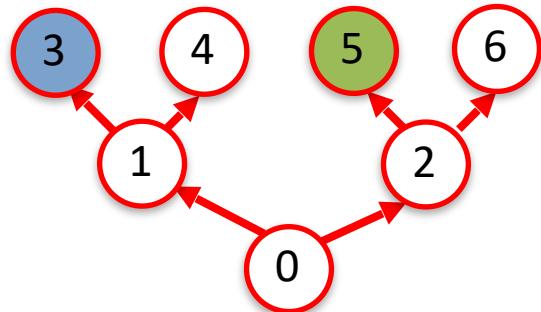
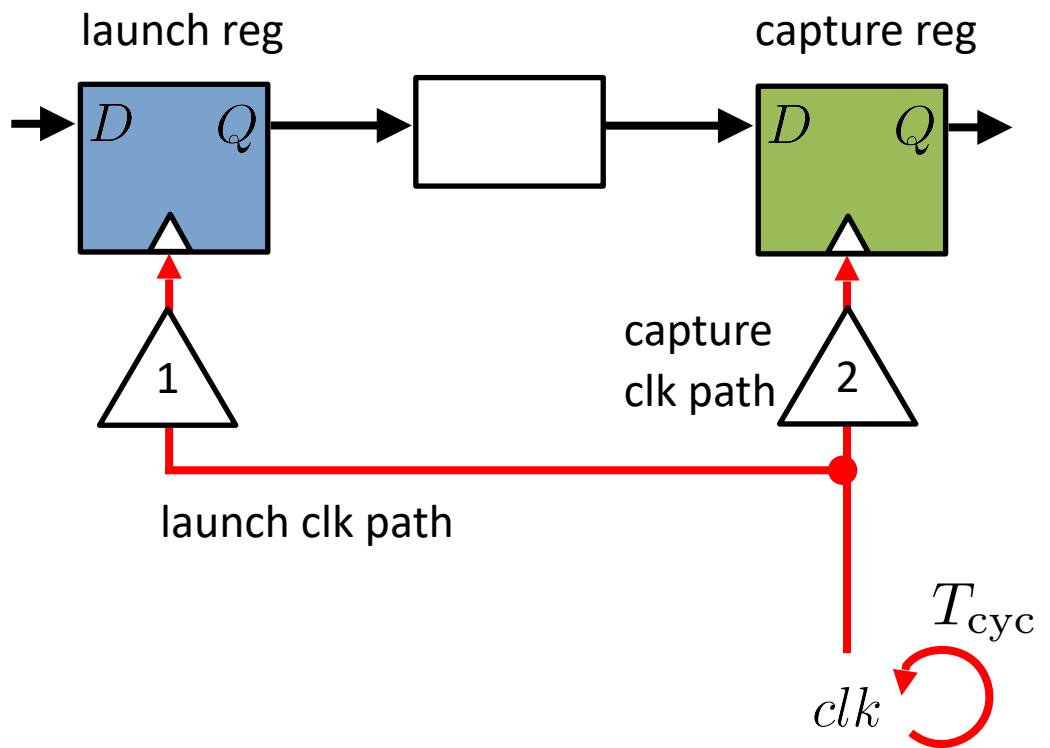




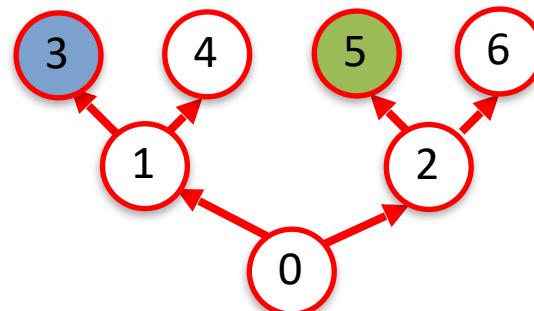
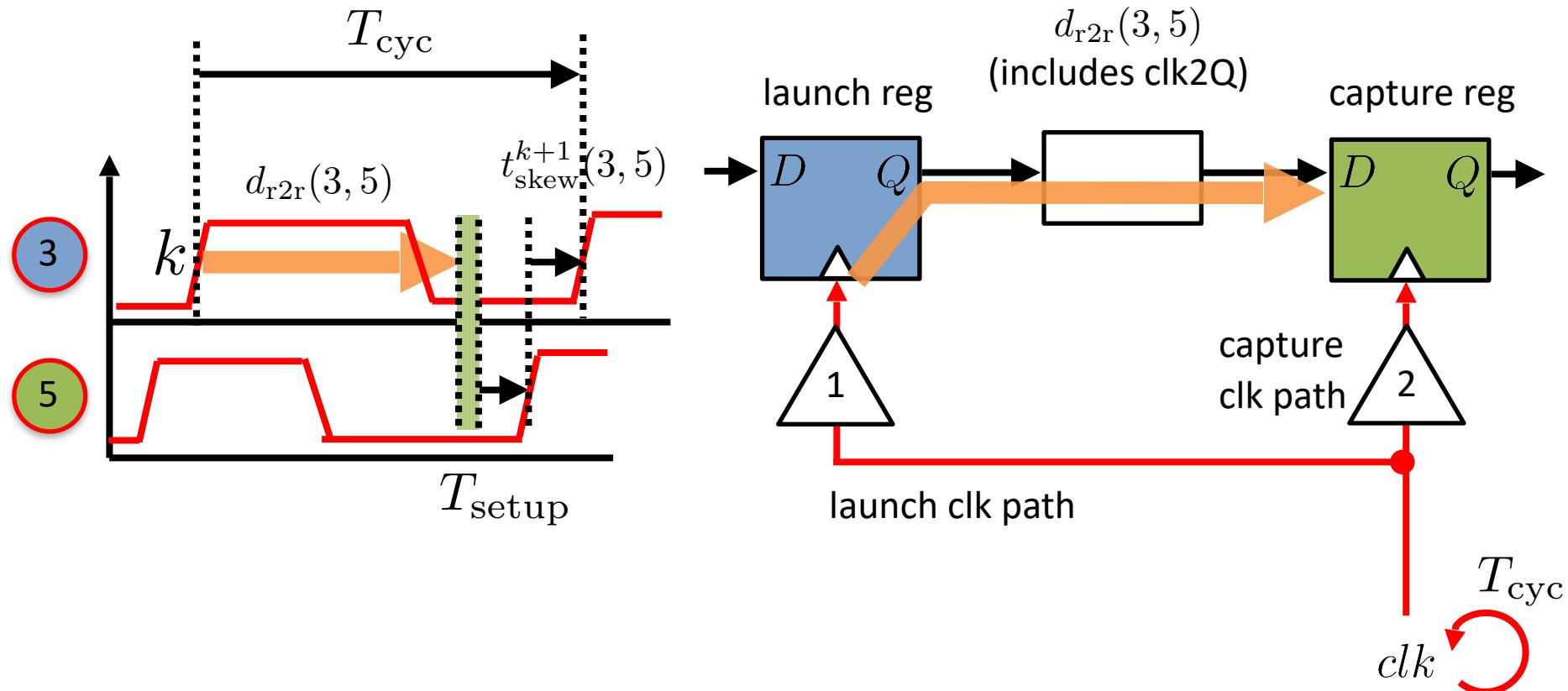




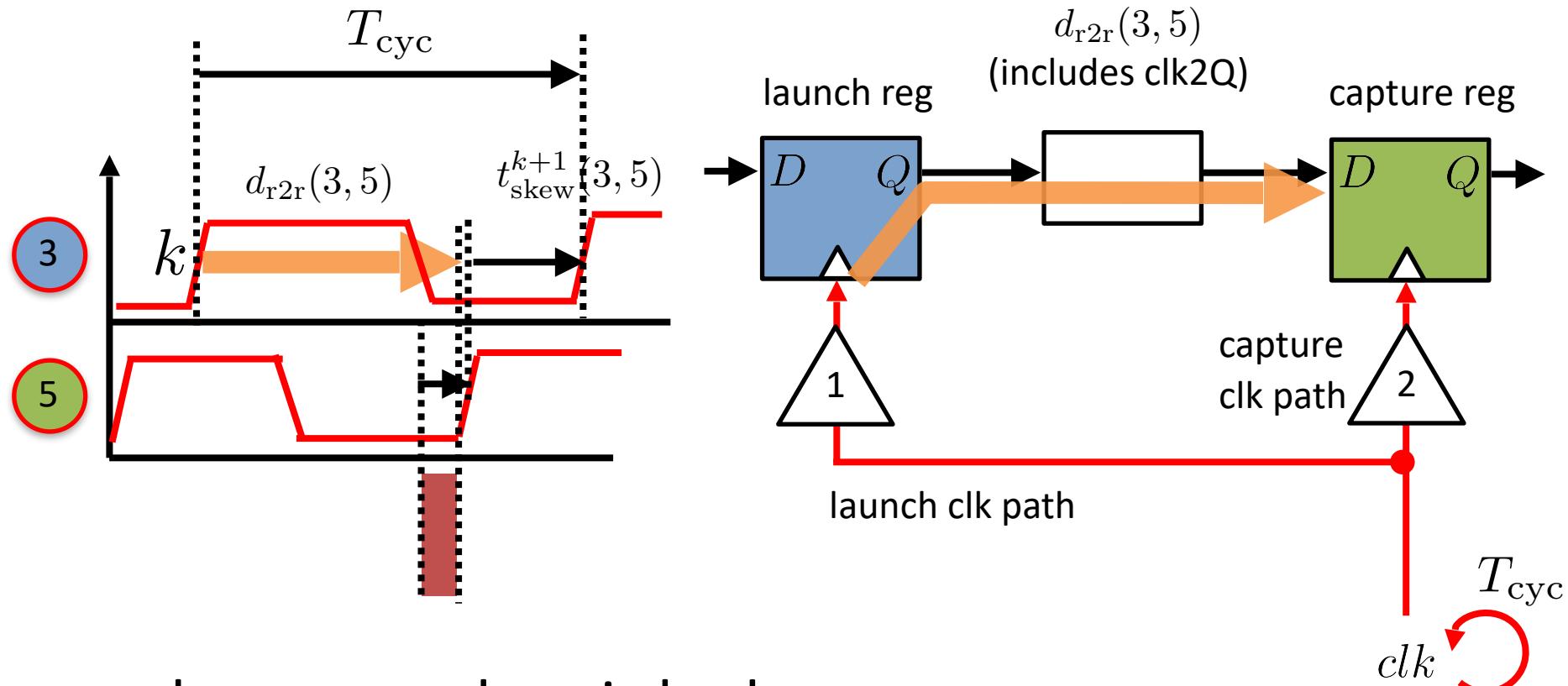




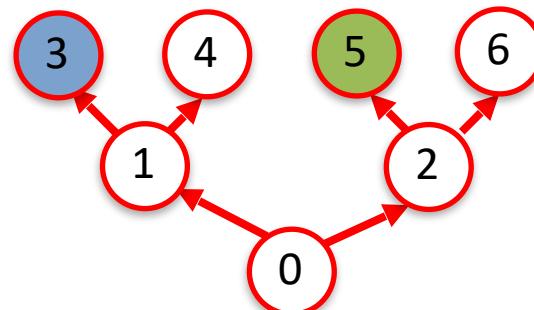
# The setup constraint



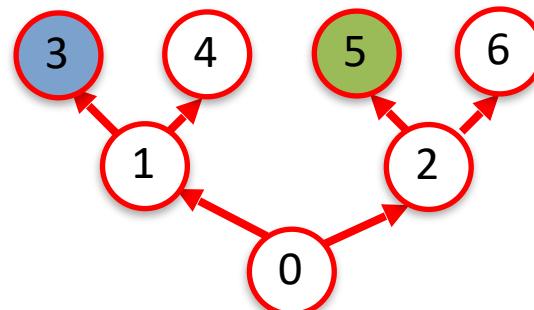
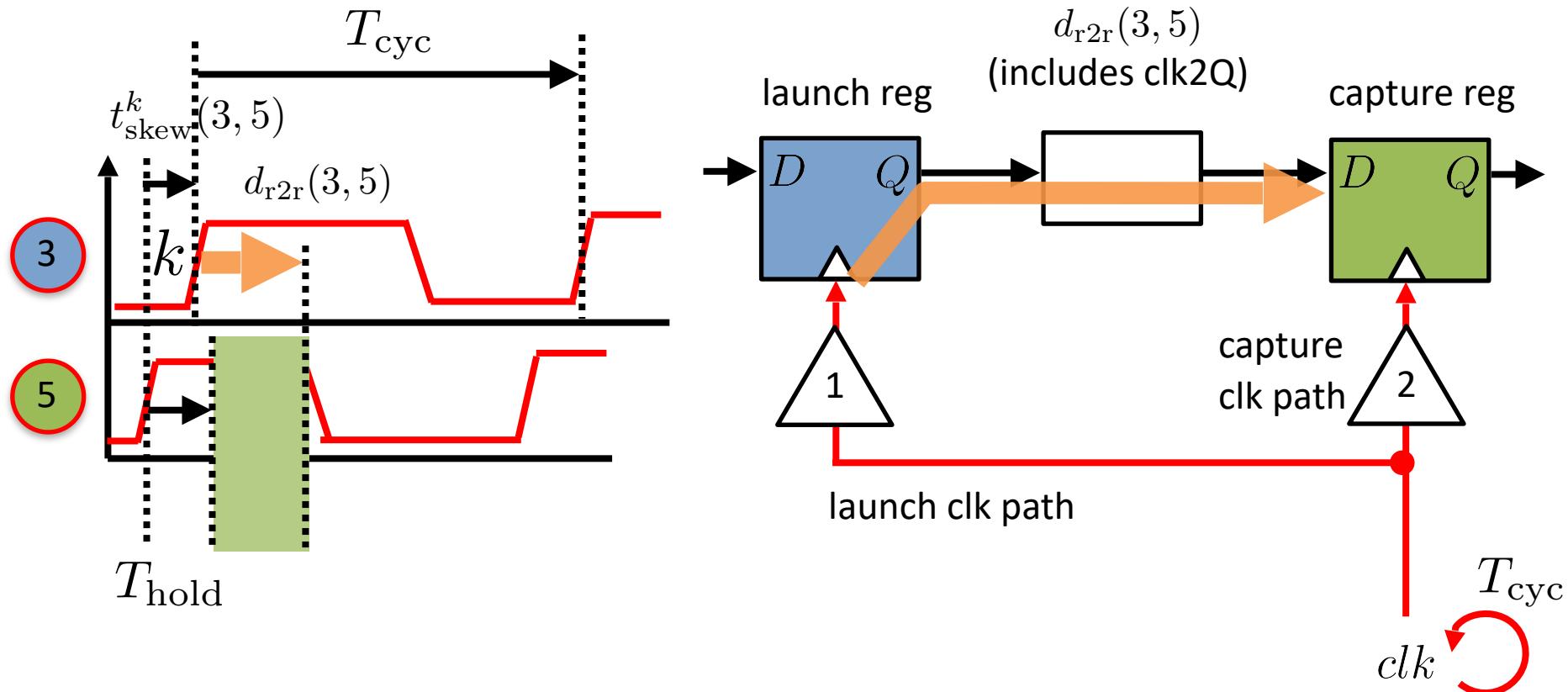
# The setup constraint



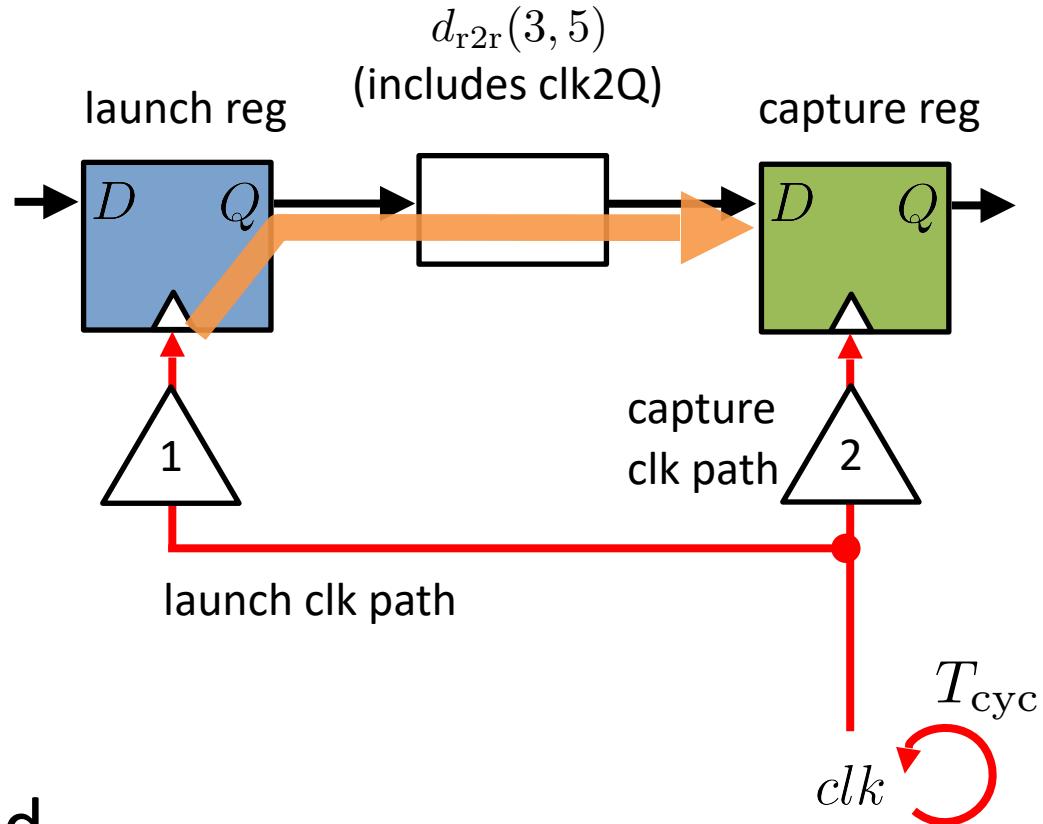
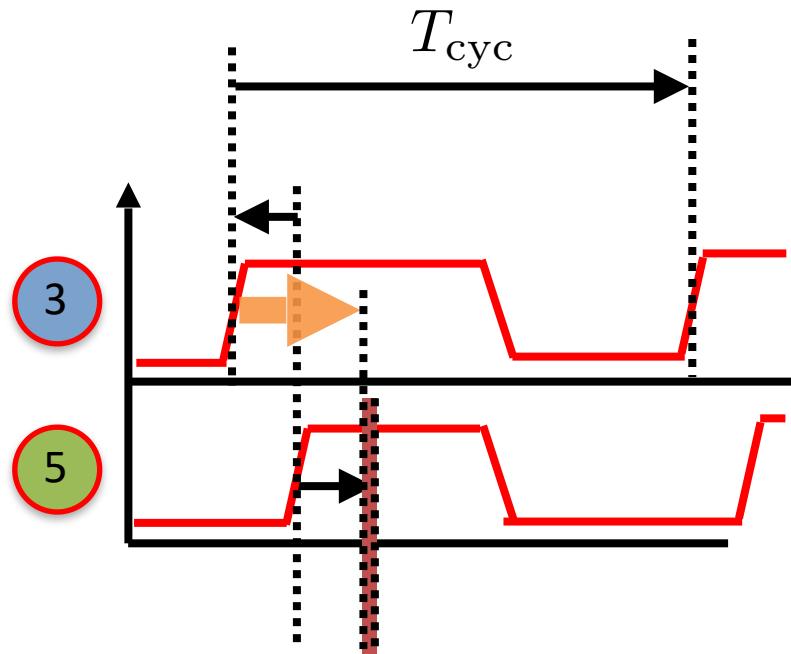
large pos. skew is bad



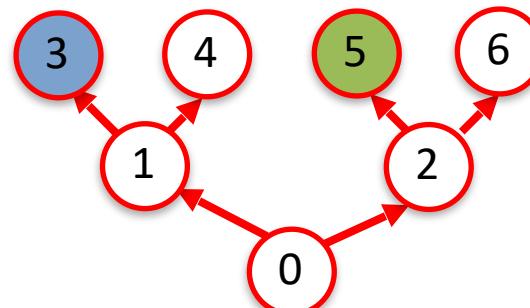
# The hold constraint



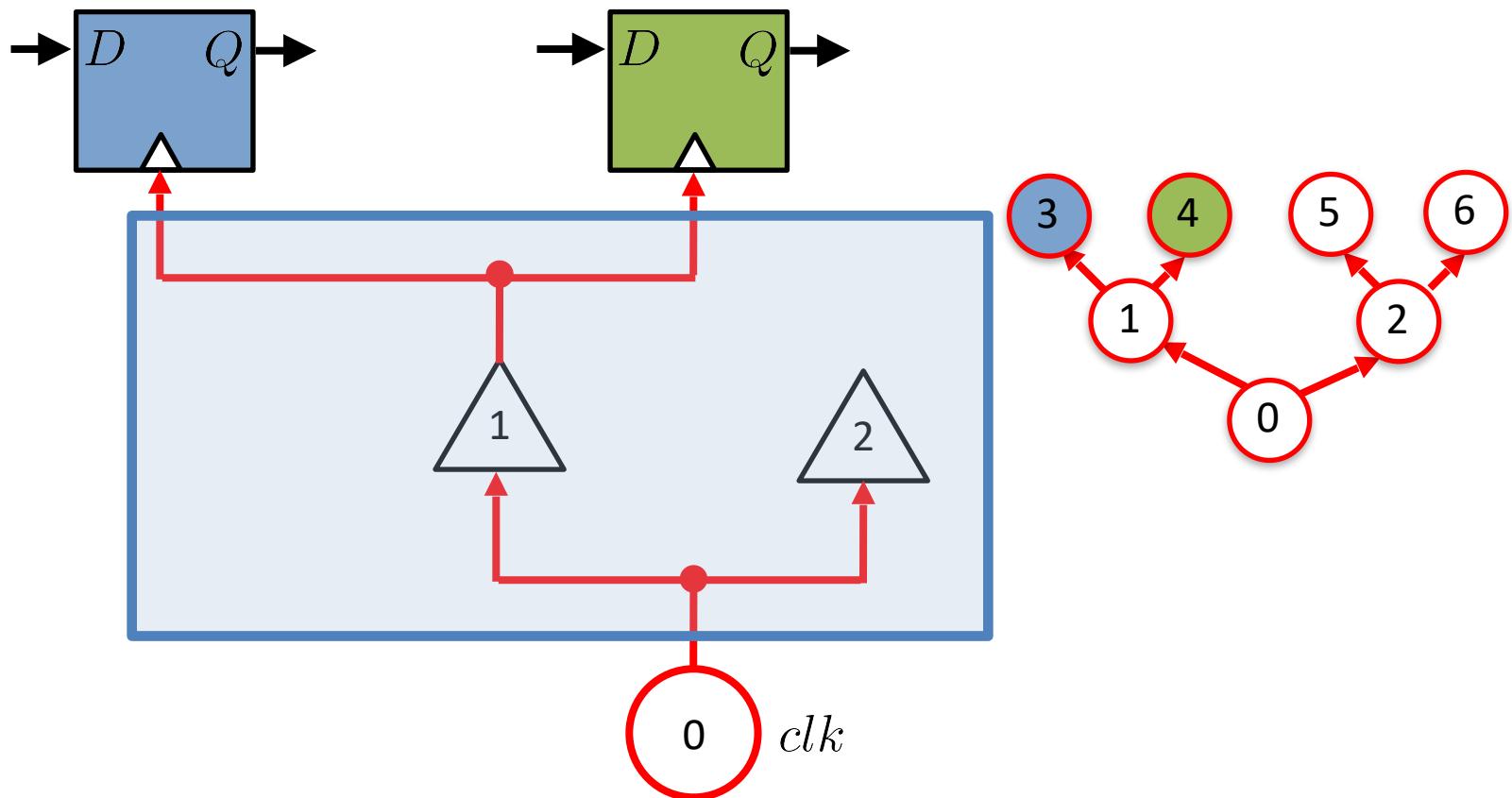
# The hold constraint



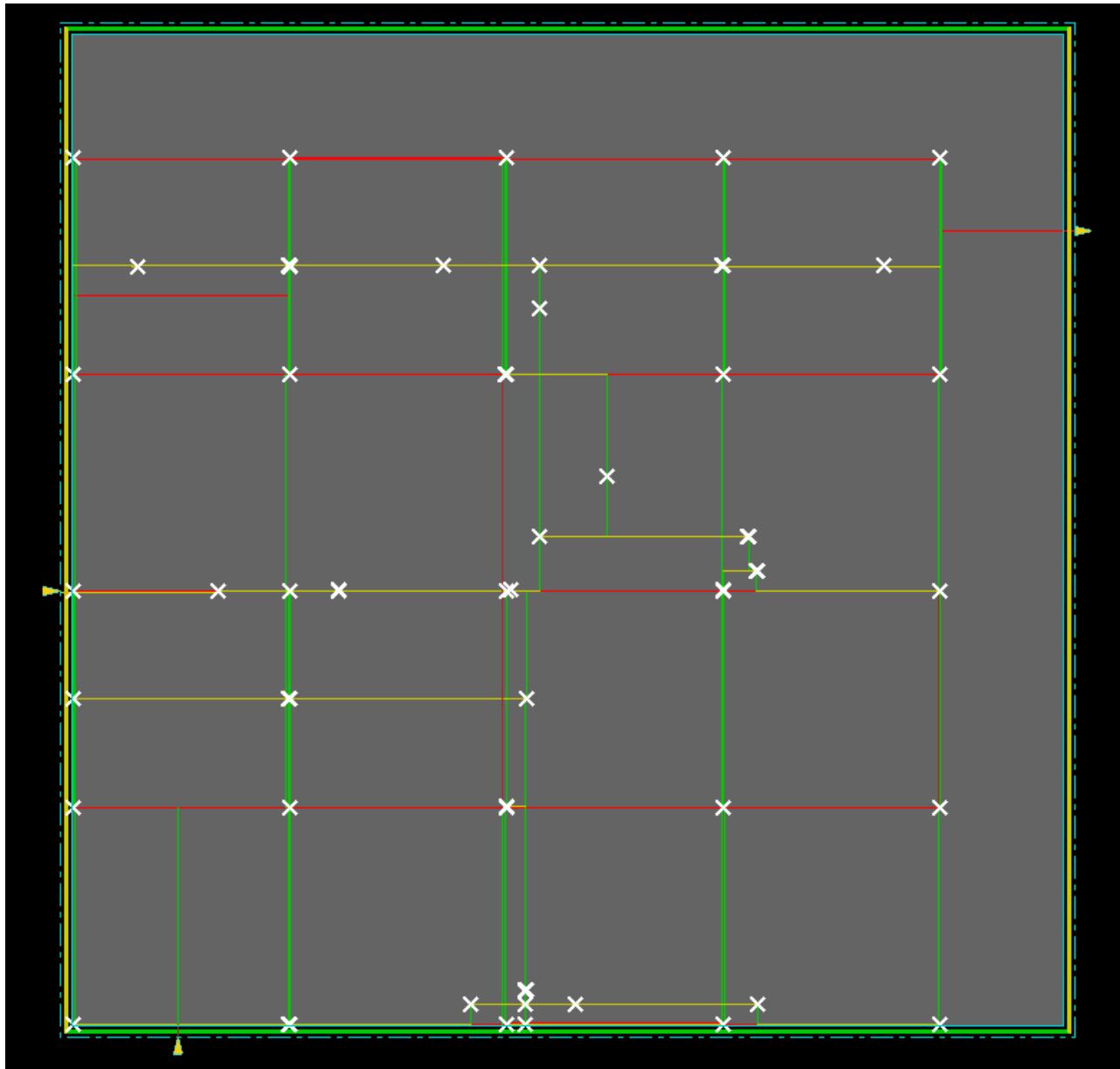
large neg. skew is bad



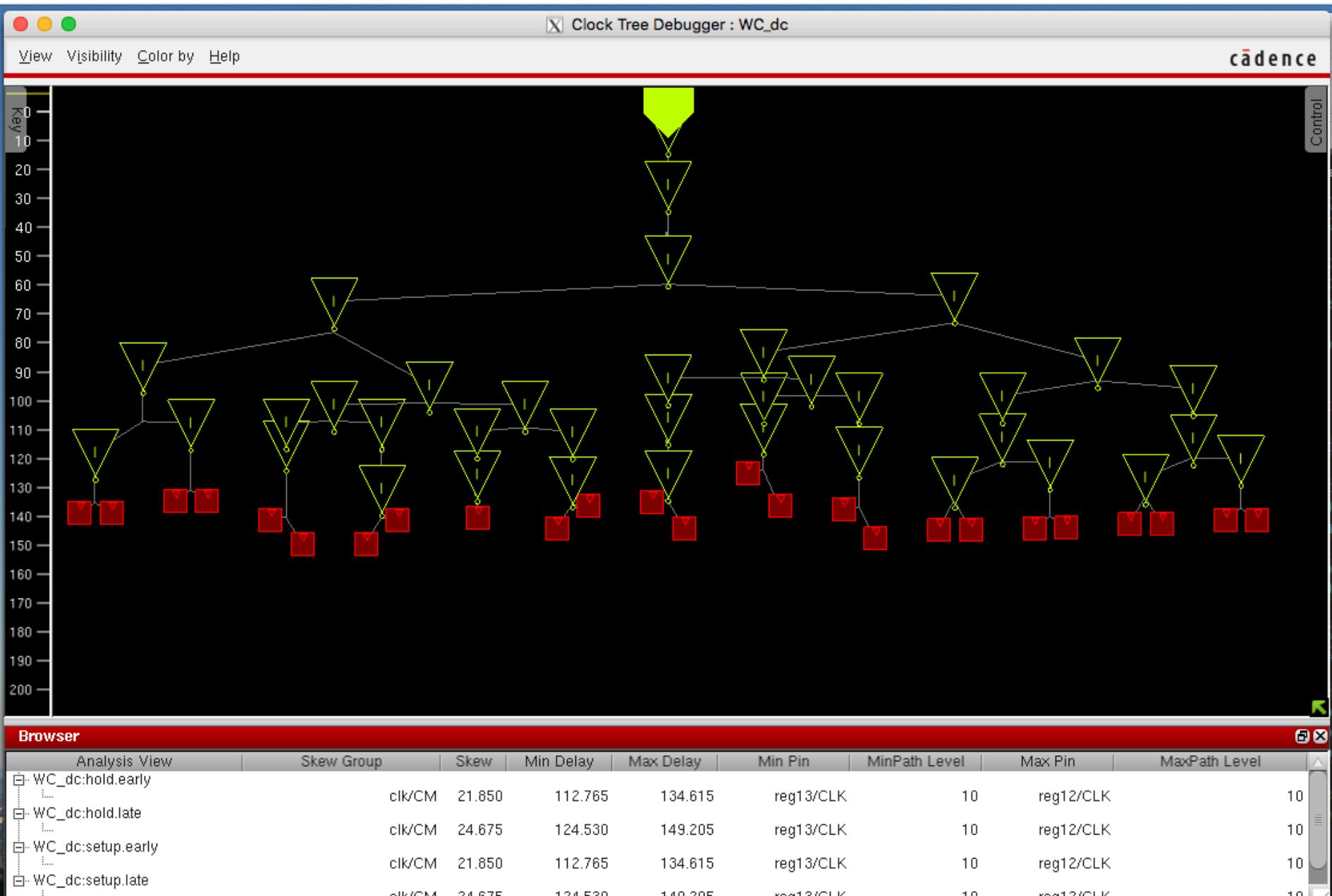
# Clock distribution network



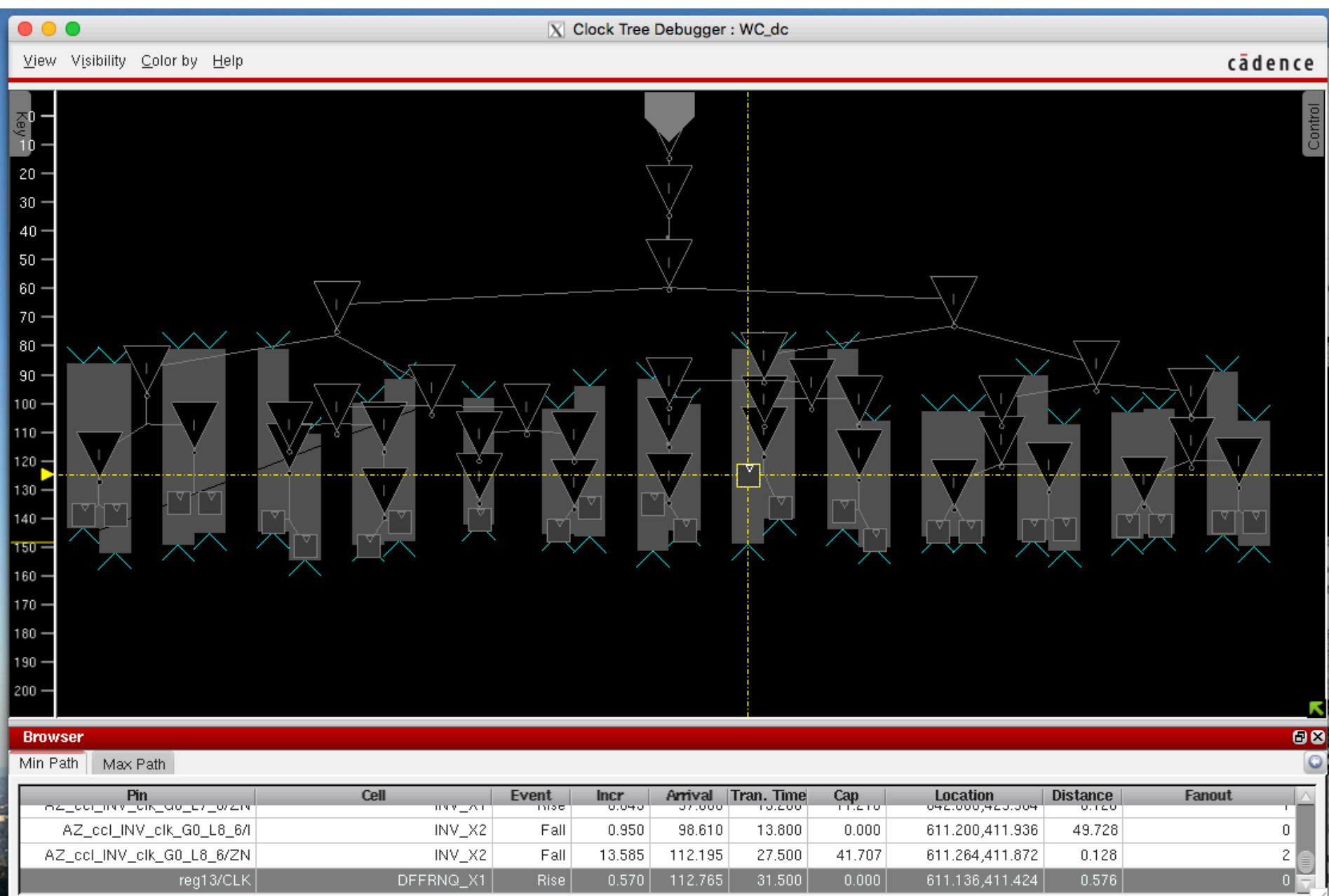
# The clock



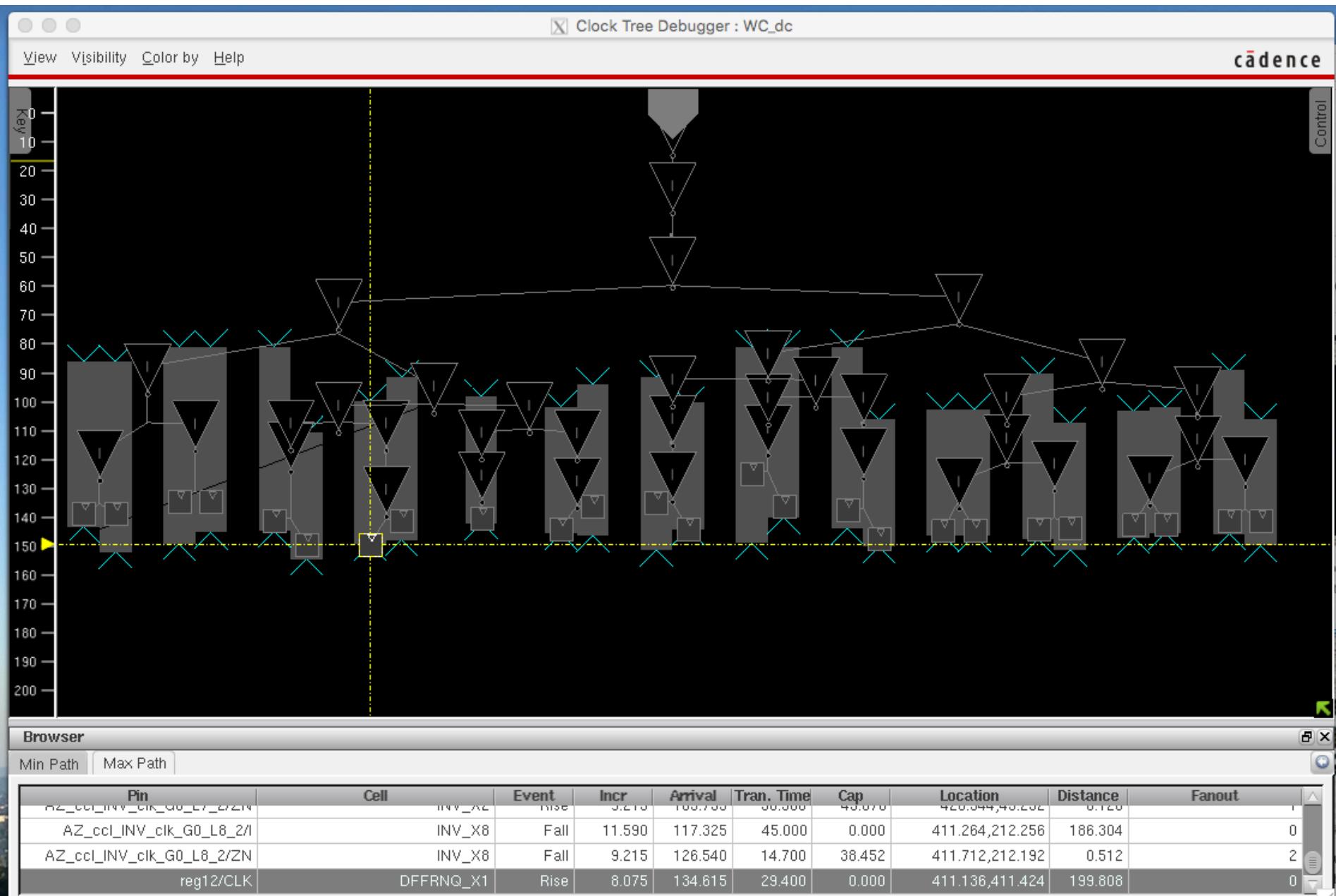
# Its skew



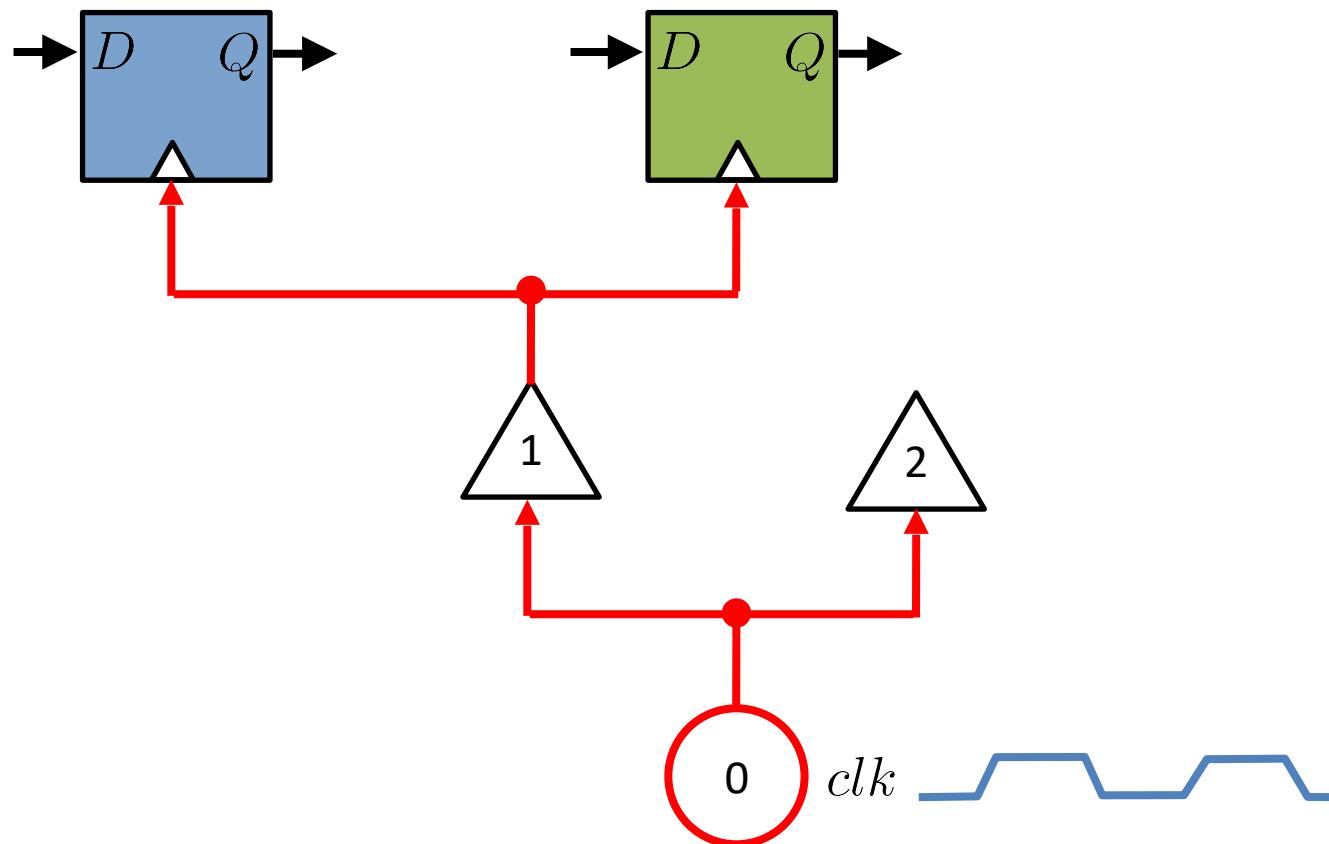
# Minimum path



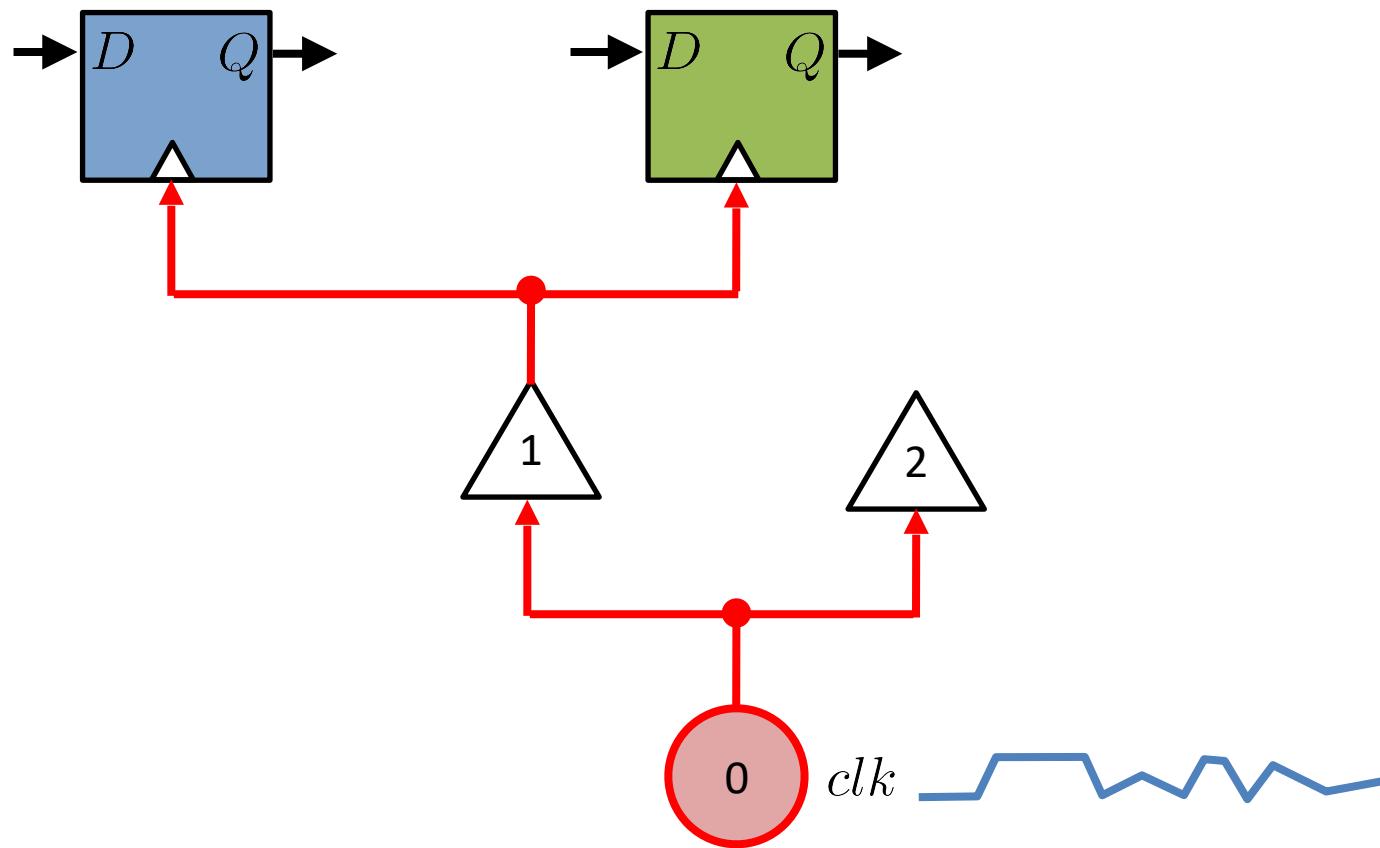
# Maximum path



# But ...

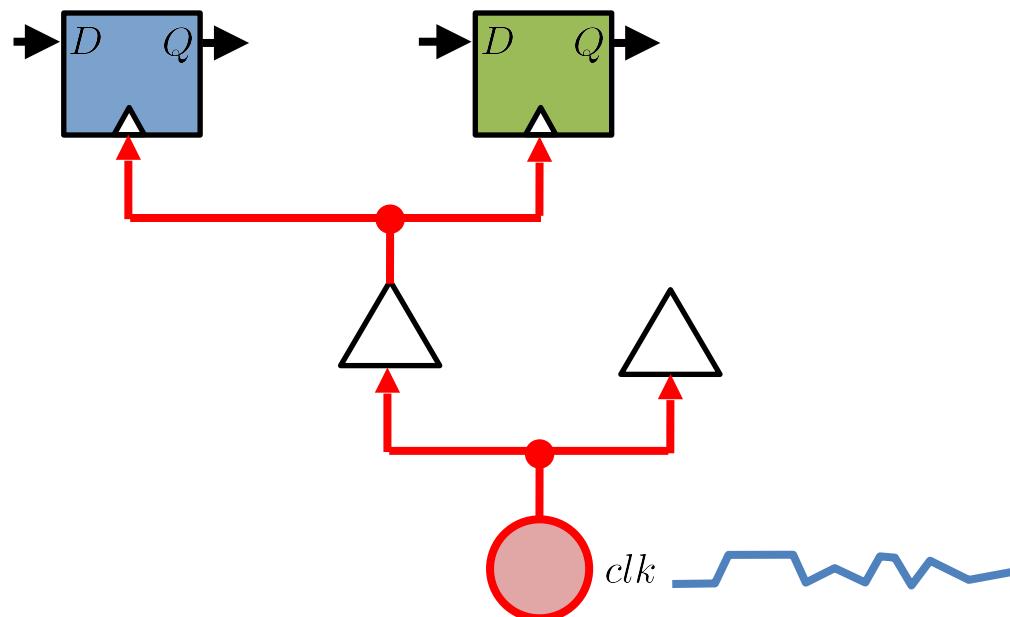


... faults



# New requirements

Guarantee skew among some clock outputs despite faults



# Pulse Synchronization

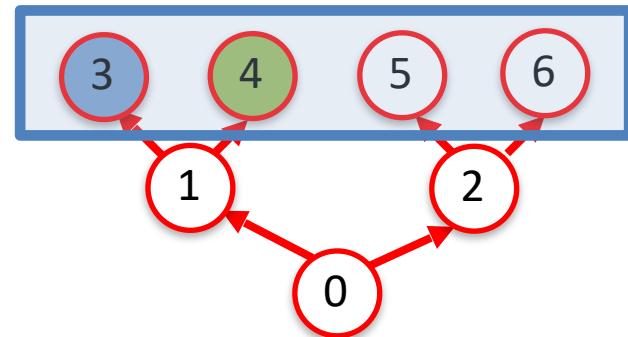
In pulse synchronization, for each  $i \in \mathbb{N}$ , every (correct) node  $v \in V_g$  generates pulse  $i$  exactly once.

Let  $p_{v,i}$  denote the time when  $v$  generates the  $i$ -th pulse. We require that there are  $\mathcal{S}, P_{\max}, P_{\min} \in \mathbb{R}_{>0}$  satisfying

1. skew:  $\sup_{i \in \mathbb{N}, u, w \in V_g} \{|p_{v,i} - p_{w,i}| \} = \mathcal{S}$

2. per-1:  $\inf_{i \in \mathbb{N}} \left\{ \min_{v \in V_g} p_{v,i+1} - \max_{v \in V_g} p_{v,i} \right\} \geq P_{\min}$

3. per-2:  $\sup_{i \in \mathbb{N}} \left\{ \max_{v \in V_g} p_{v,i+1} - \min_{v \in V_g} p_{v,i} \right\} \leq P_{\max}$



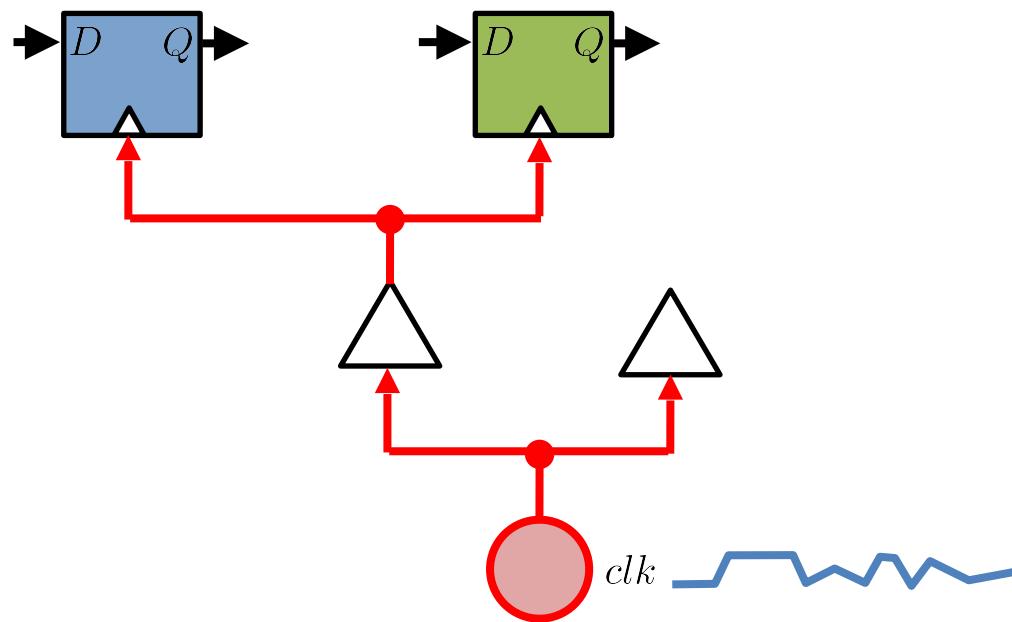
# Lower bounds

*Number of faults  $f$ . Then necessarily:*

*Global:  $n > 3f$*

*Local: degree  $> 2f$*

# Ideas?



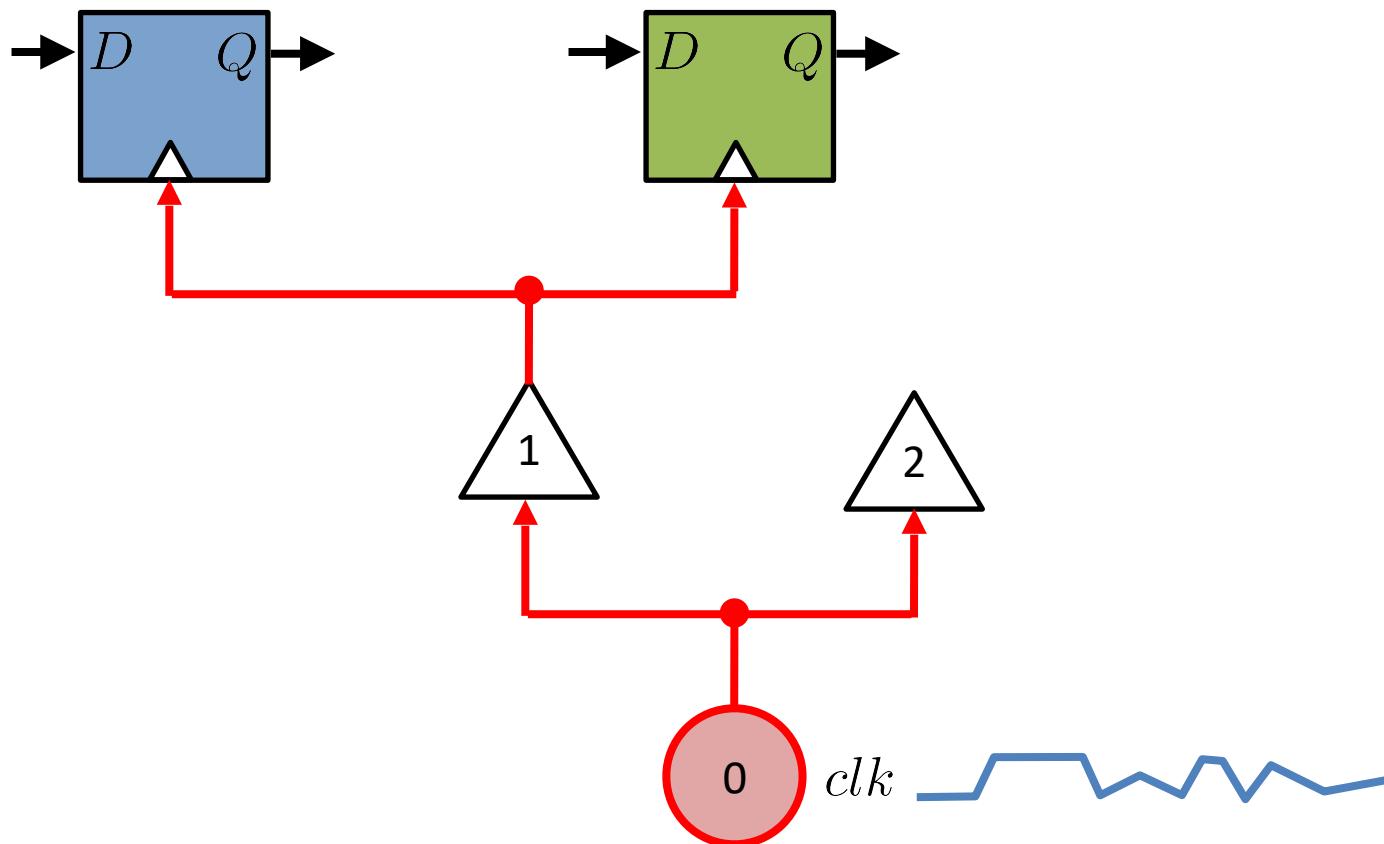


# Chapter 11 (2)

## Low-degree clock distribution networks

Matthias Fuegger and Christoph Lenzen

# Faults



# Pulse Synchronization

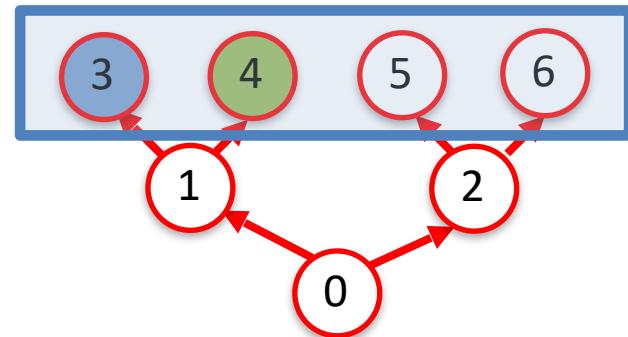
In pulse synchronization, for each  $i \in \mathbb{N}$ , every (correct) node  $v \in V_g$  generates pulse  $i$  exactly once.

Let  $p_{v,i}$  denote the time when  $v$  generates the  $i$ -th pulse. We require that there are  $\mathcal{S}, P_{\max}, P_{\min} \in \mathbb{R}_{>0}$  satisfying

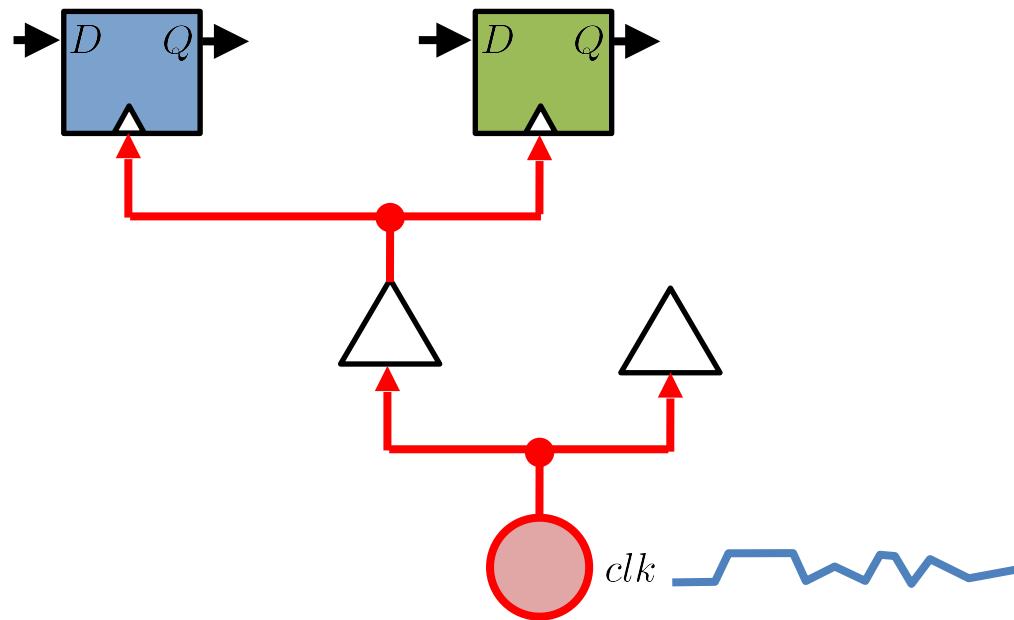
1. skew:  $\sup_{i \in \mathbb{N}, u, w \in V_g} \{|p_{v,i} - p_{w,i}| \} = \mathcal{S}$

2. per-1:  $\inf_{i \in \mathbb{N}} \left\{ \min_{v \in V_g} p_{v,i+1} - \max_{v \in V_g} p_{v,i} \right\} \geq P_{\min}$

3. per-2:  $\sup_{i \in \mathbb{N}} \left\{ \max_{v \in V_g} p_{v,i+1} - \min_{v \in V_g} p_{v,i} \right\} \leq P_{\max}$

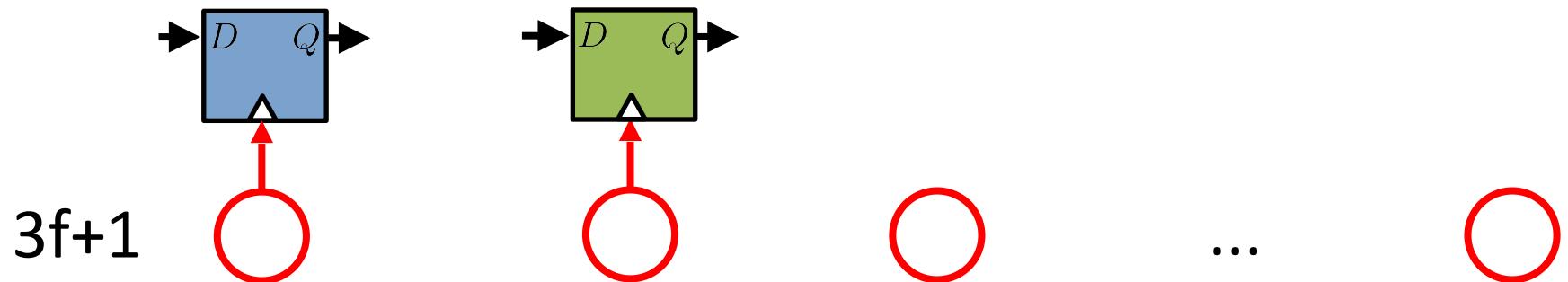


# Last time: ideas session



# Idea 1: only LW algorithm

no tree, only LW



# Idea 1

5x5 grid

properties:

- **fault tolerance?**



- cost?



- skew?

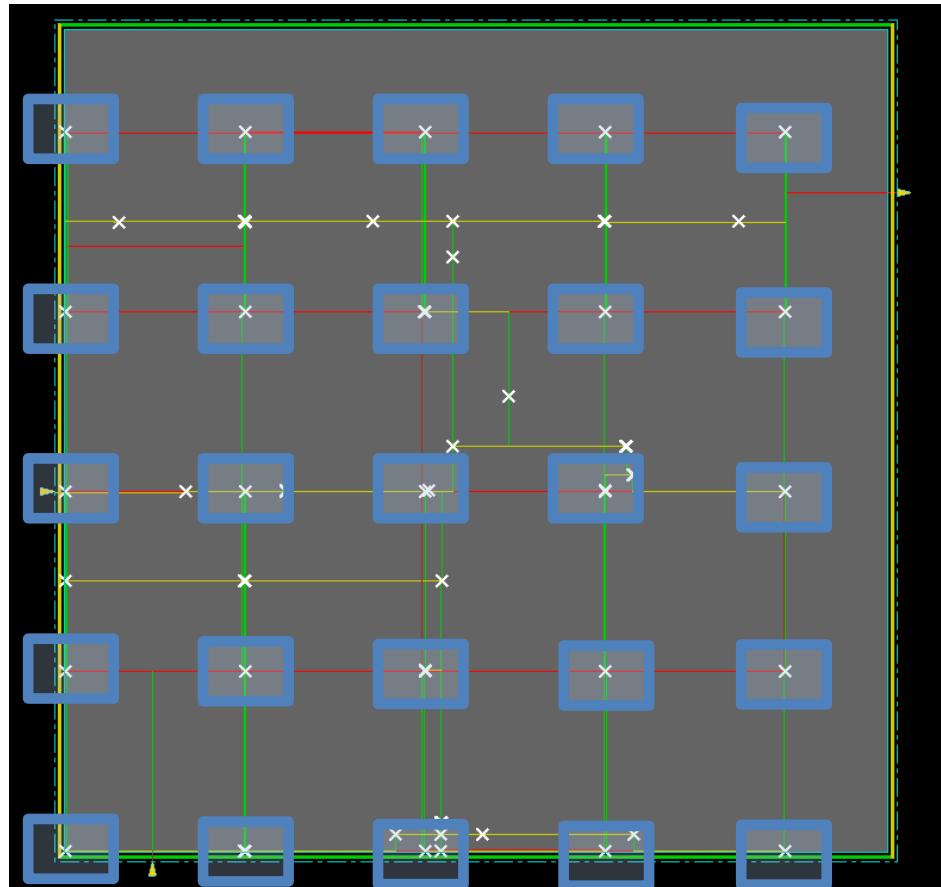


# Idea 1

5x5 grid

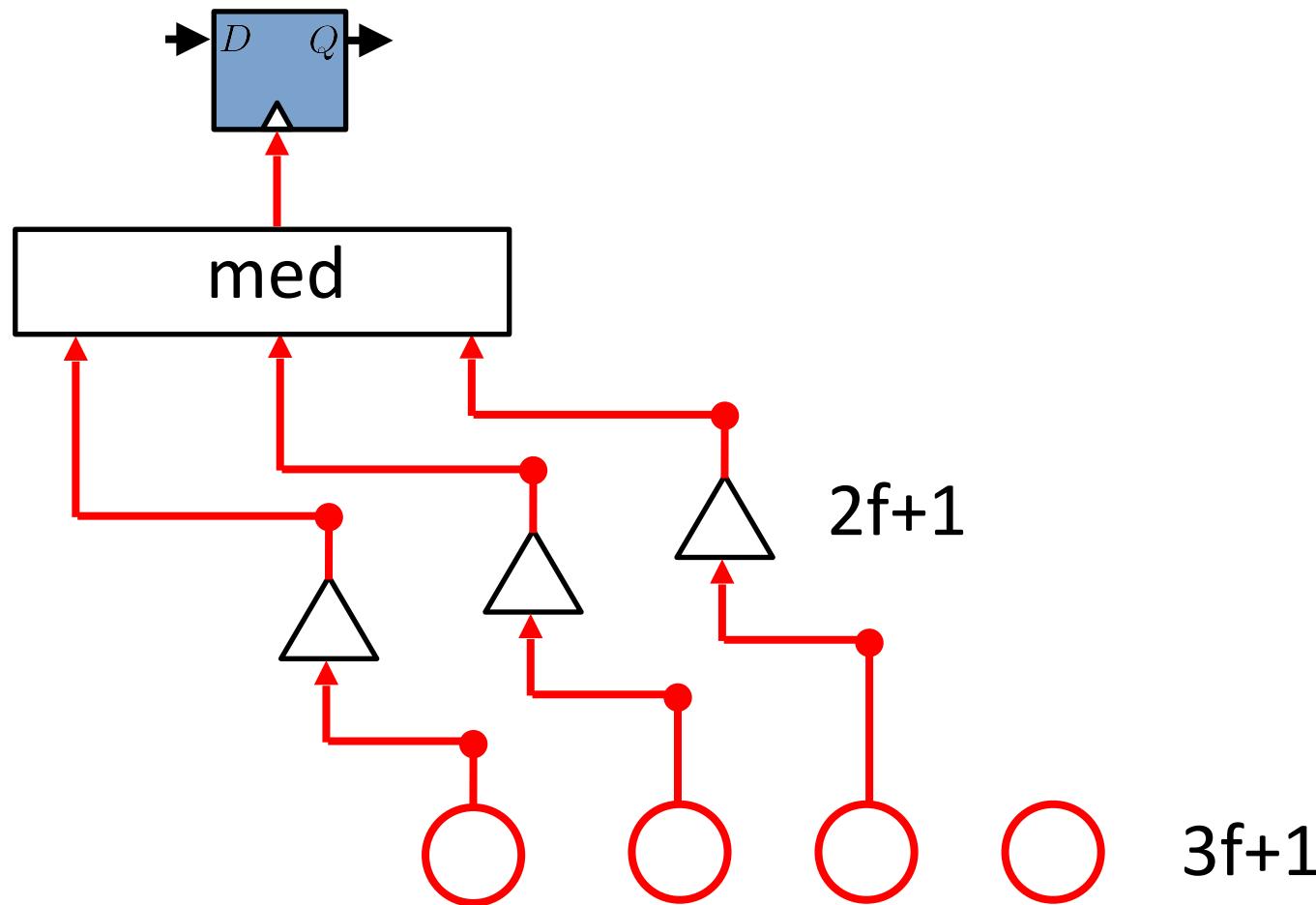
properties:

- fault tolerance?
- cost?
- skew?



# Idea 2: redundant trees

replicate the clock source, vote on output



# Idea 2

5x5 grid

properties:

- fault tolerance?



- skew?

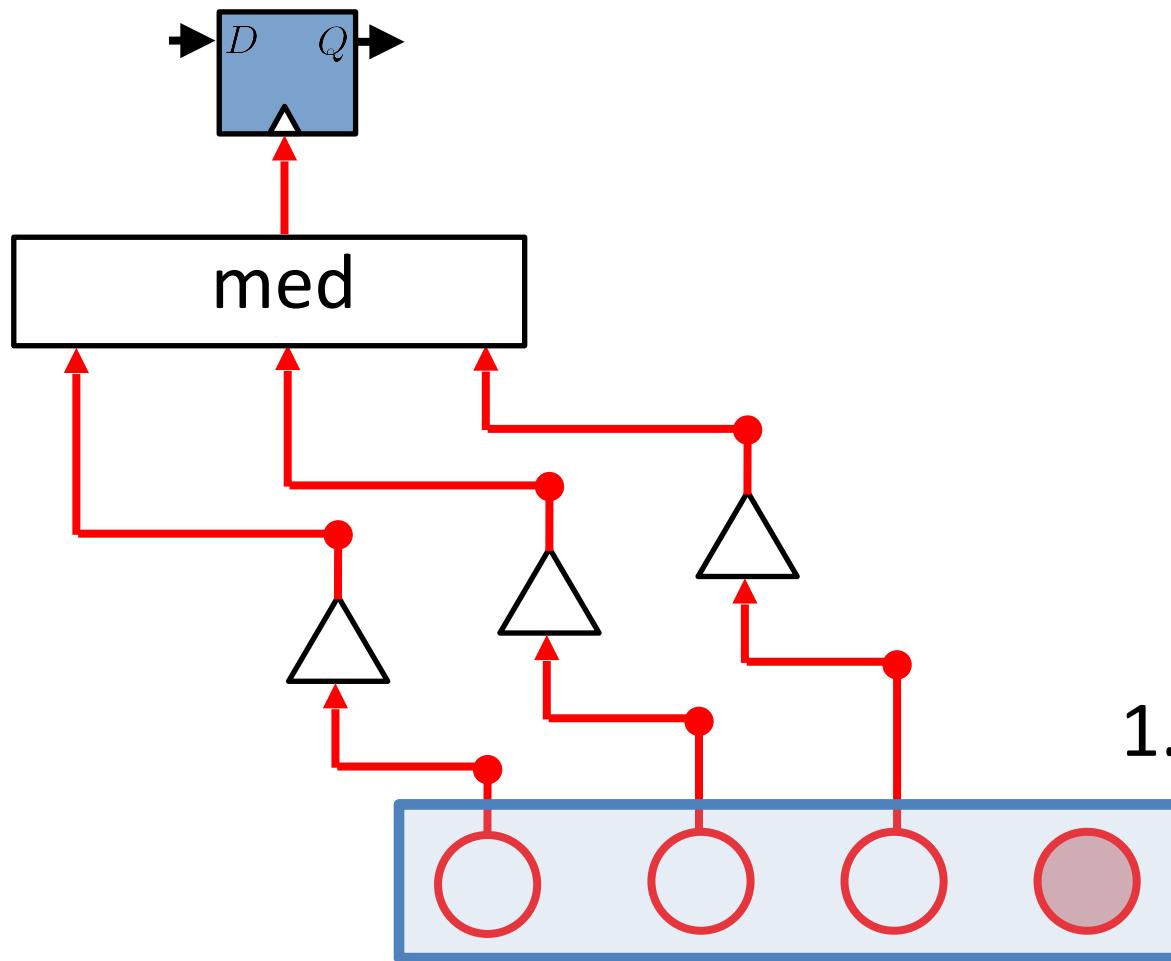


- cost?



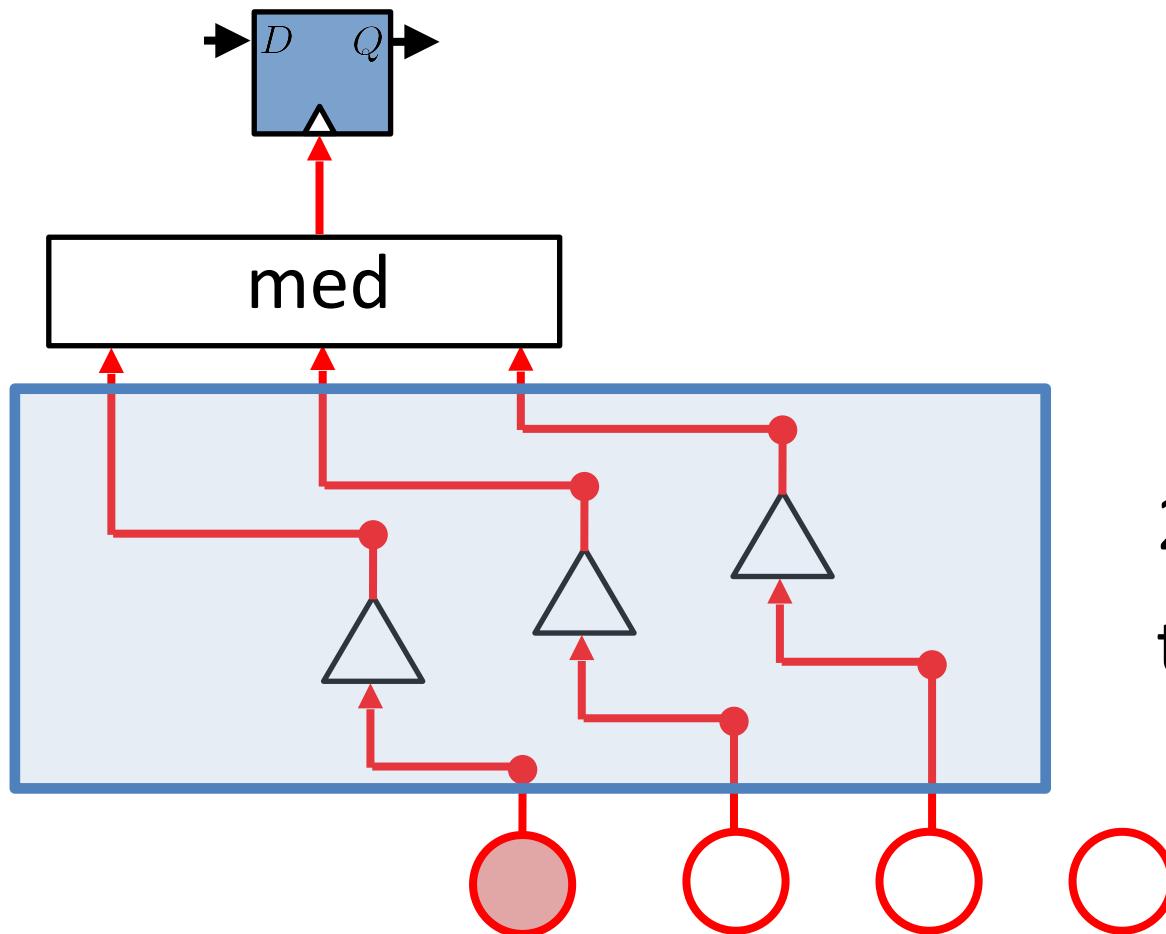
# Fault tolerance

replicate the clock source, vote on output



# Fault tolerance

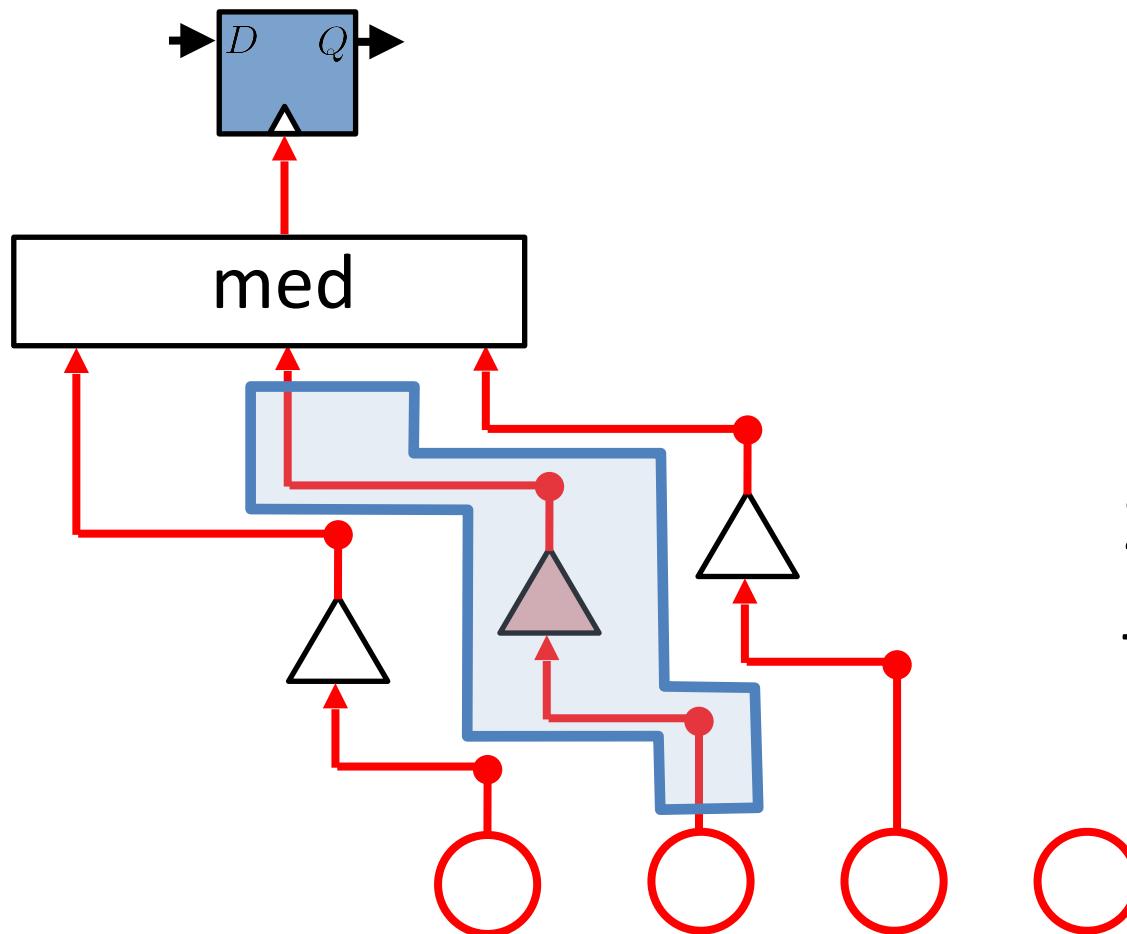
replicate the clock source, vote on output



2. at most  $f$   
trees fail

# Fault tolerance

replicate the clock source, vote on output



2. at most  $f$   
trees fail

# Idea 2

5x5 grid

properties:

- fault tolerance?



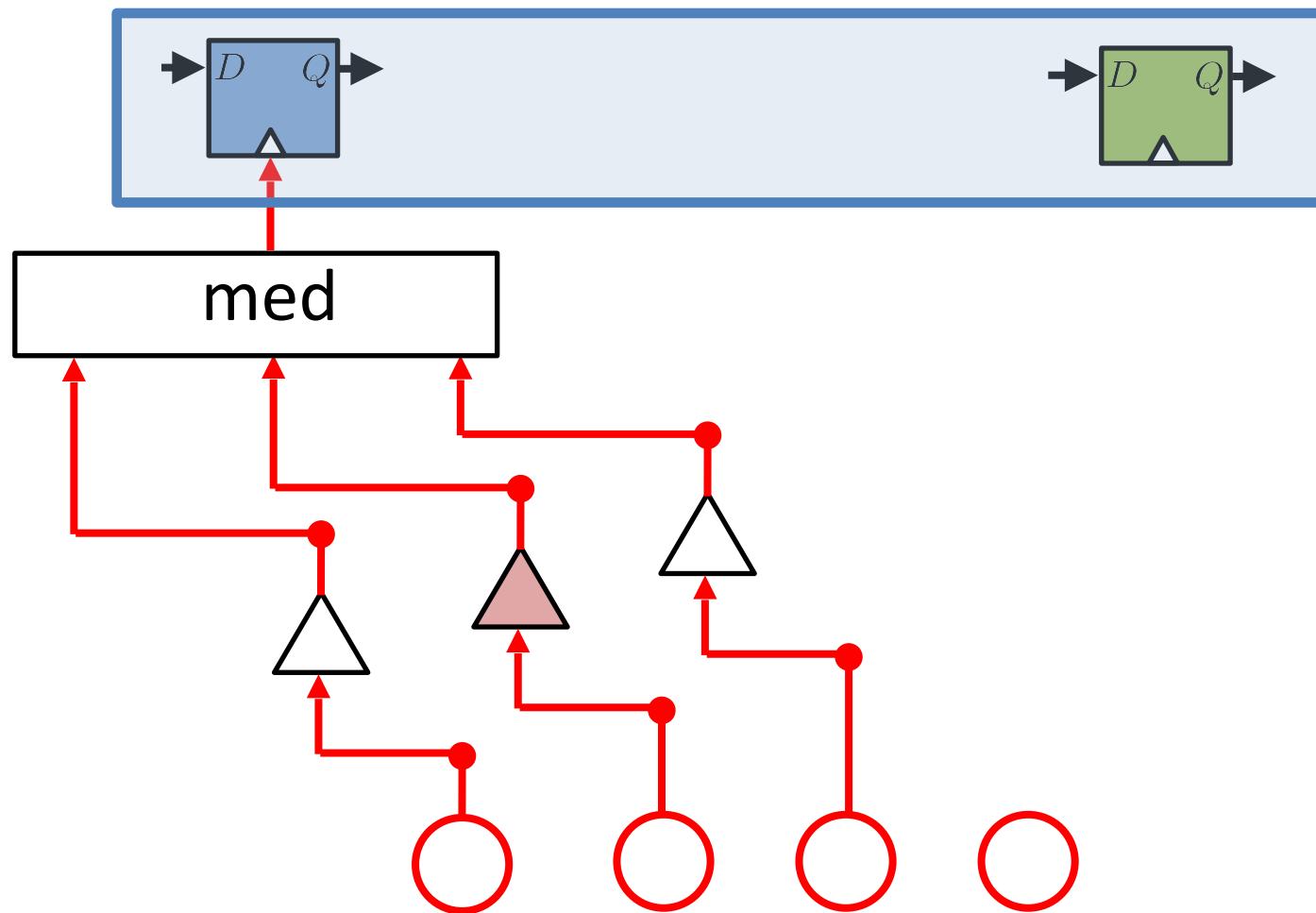
- skew?



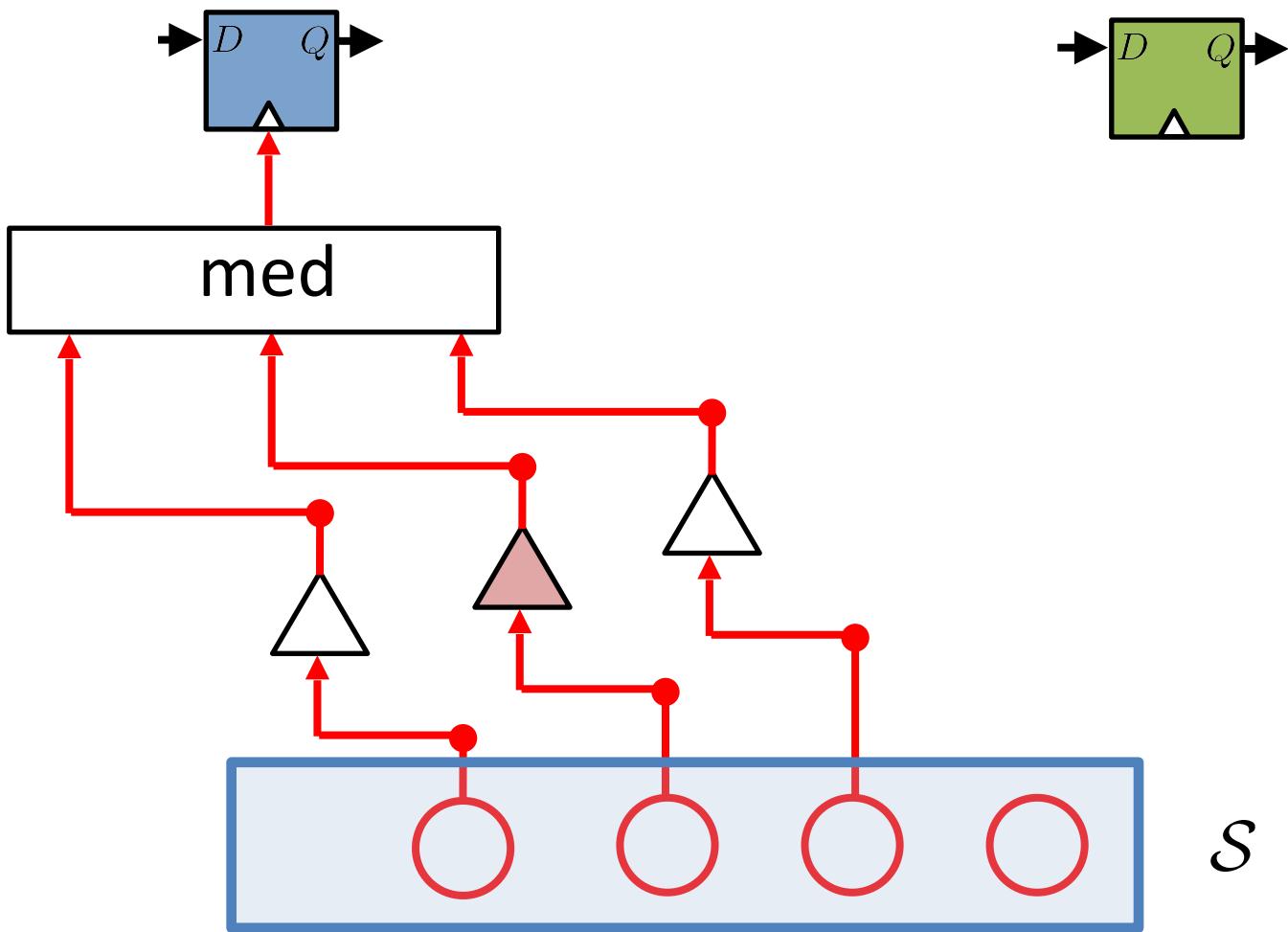
- cost?



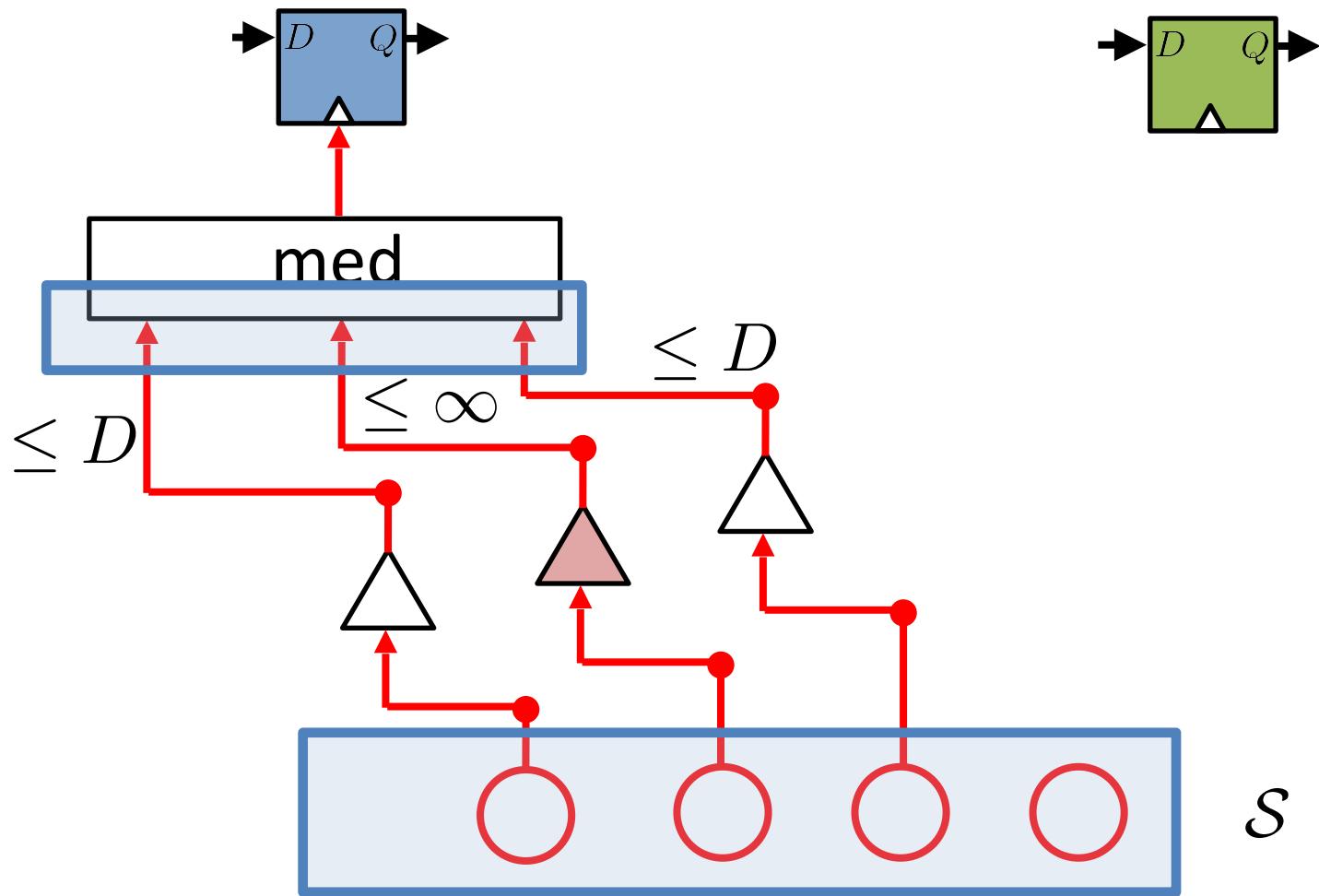
# Skew



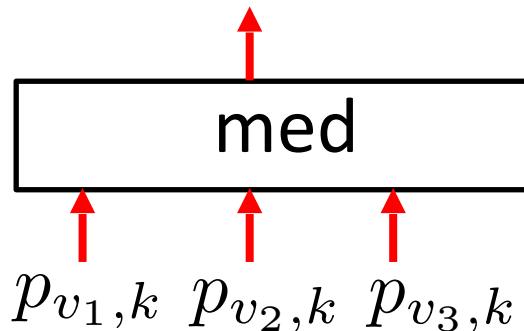
# Skew



# Skew



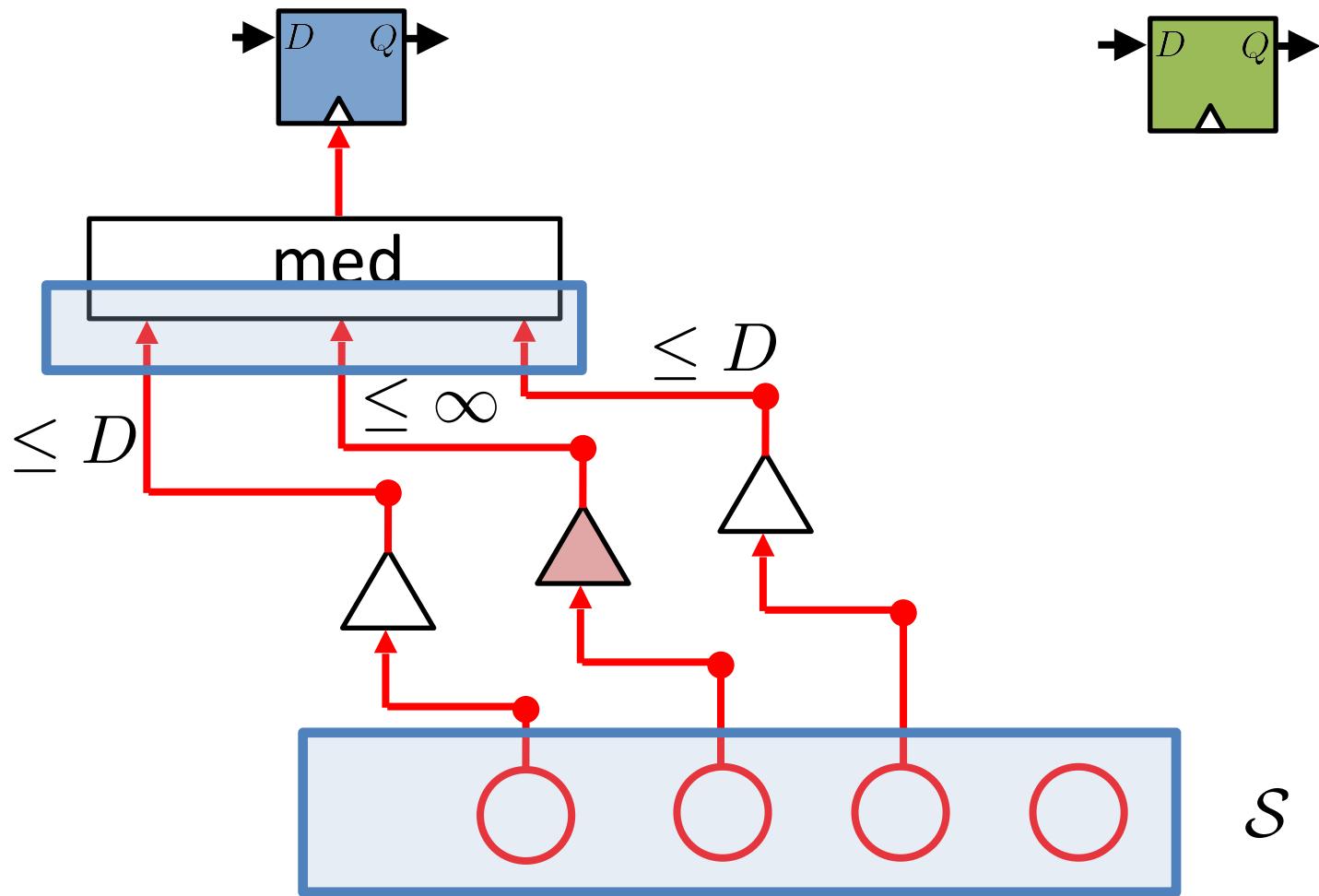
# Skew



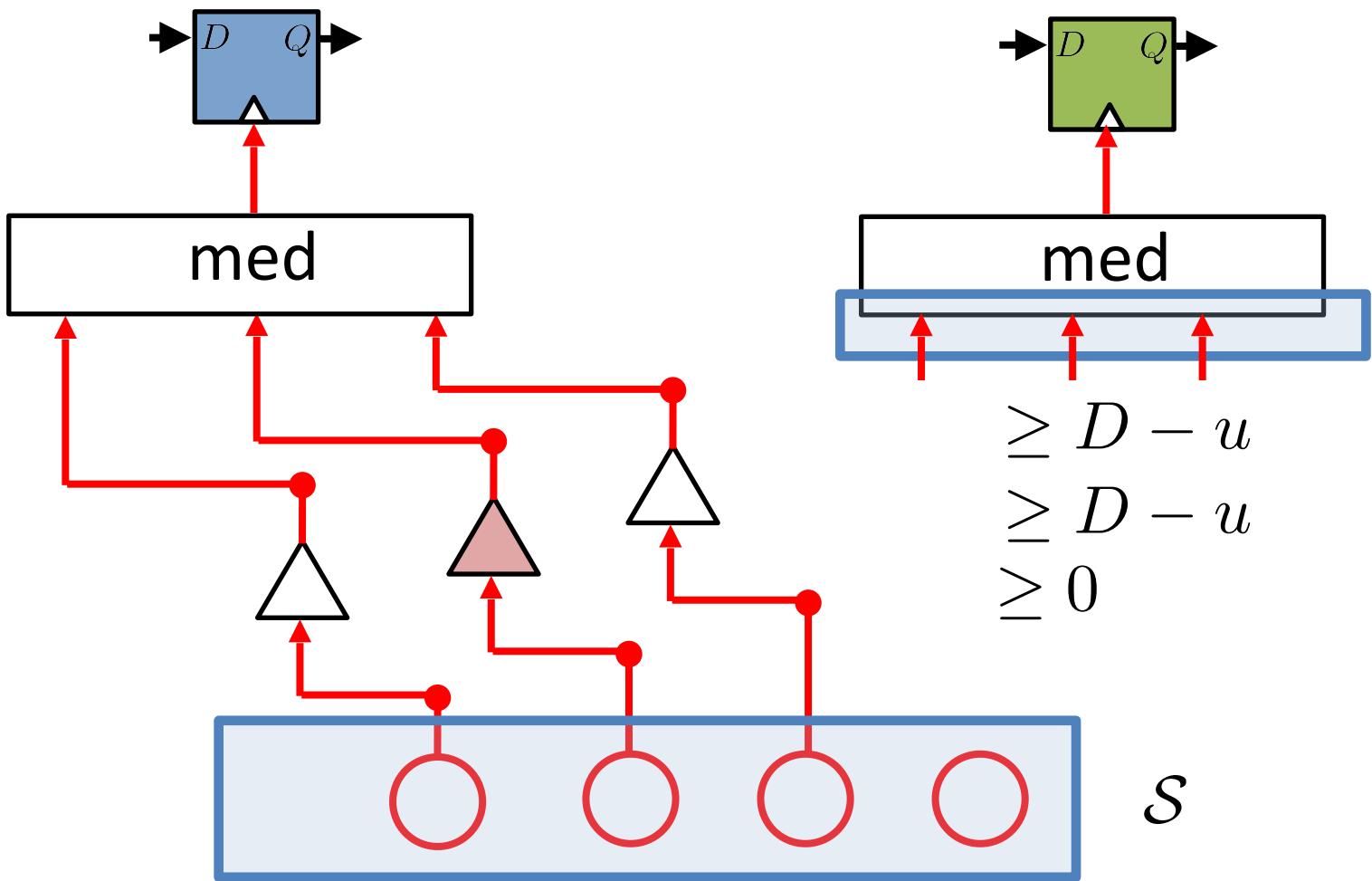
$$\begin{aligned}\text{med}(p_{v_1,k}, p_{v_2,k}, p_{v_3,k}) &\leq \text{med}(p_{r_1,k} + D, p_{r_2,k} + D, \infty) \\ &\leq \max(p_{r_1,k} + D, p_{r_2,k} + D) \\ &\leq \max(p_{r_1,k}, p_{r_2,k}) + D\end{aligned}$$

# Skew

$$\leq \max(p_{r_1,k}, p_{r_2,k}) + D$$

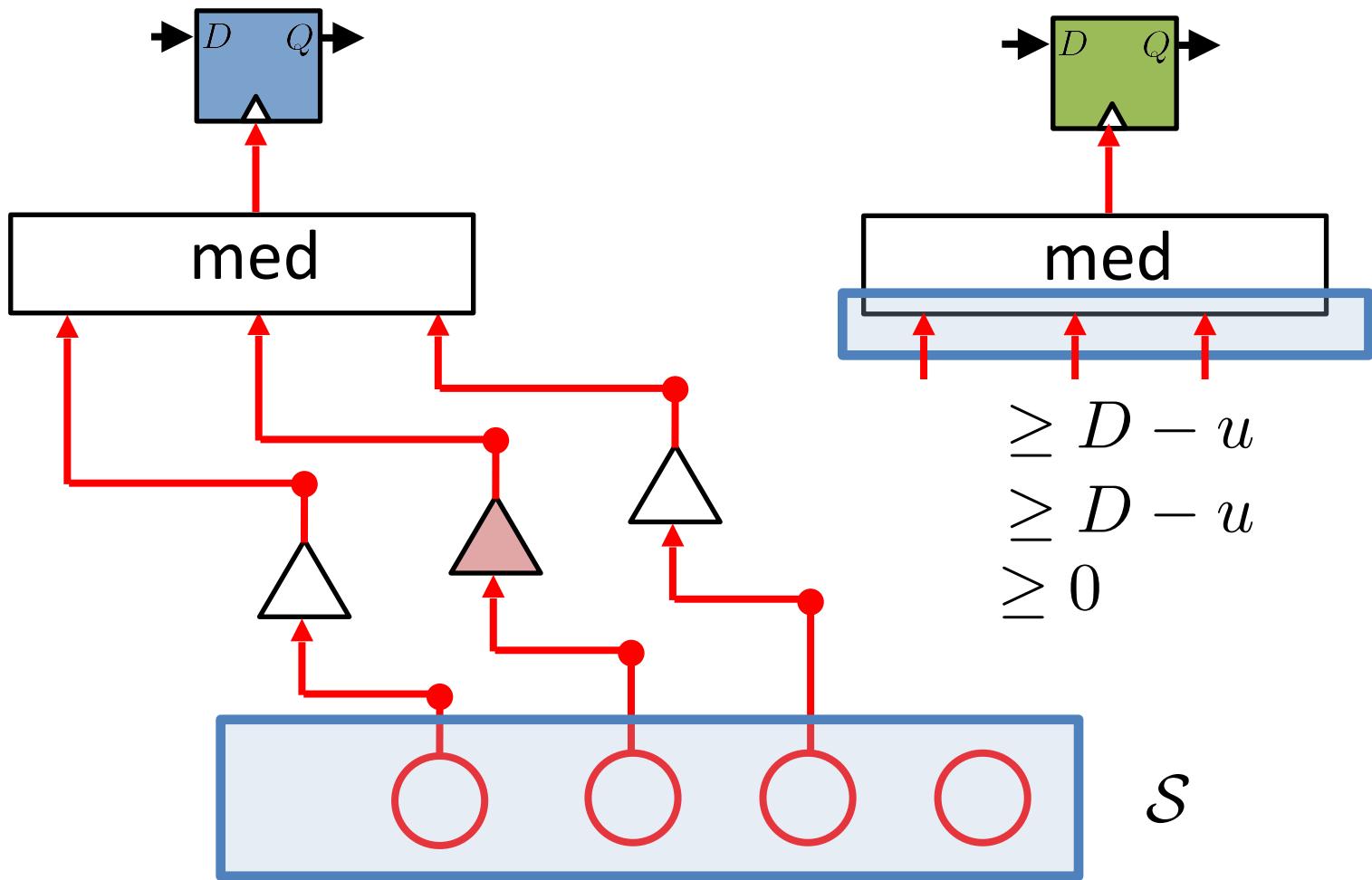


# Skew



# Skew

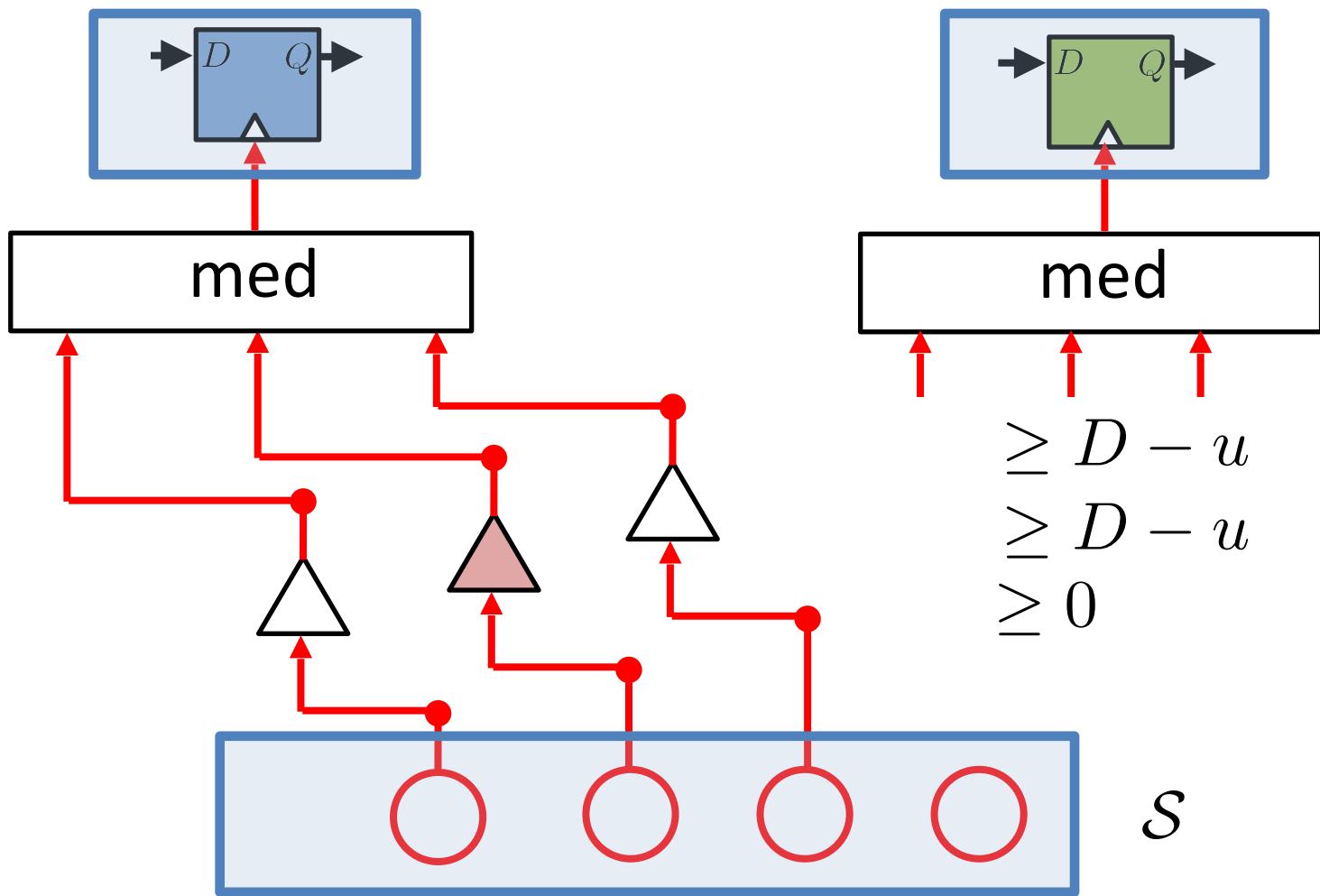
$$\geq \min(p_{r_1,k}, p_{r_2,k}) + D - u$$



# Skew

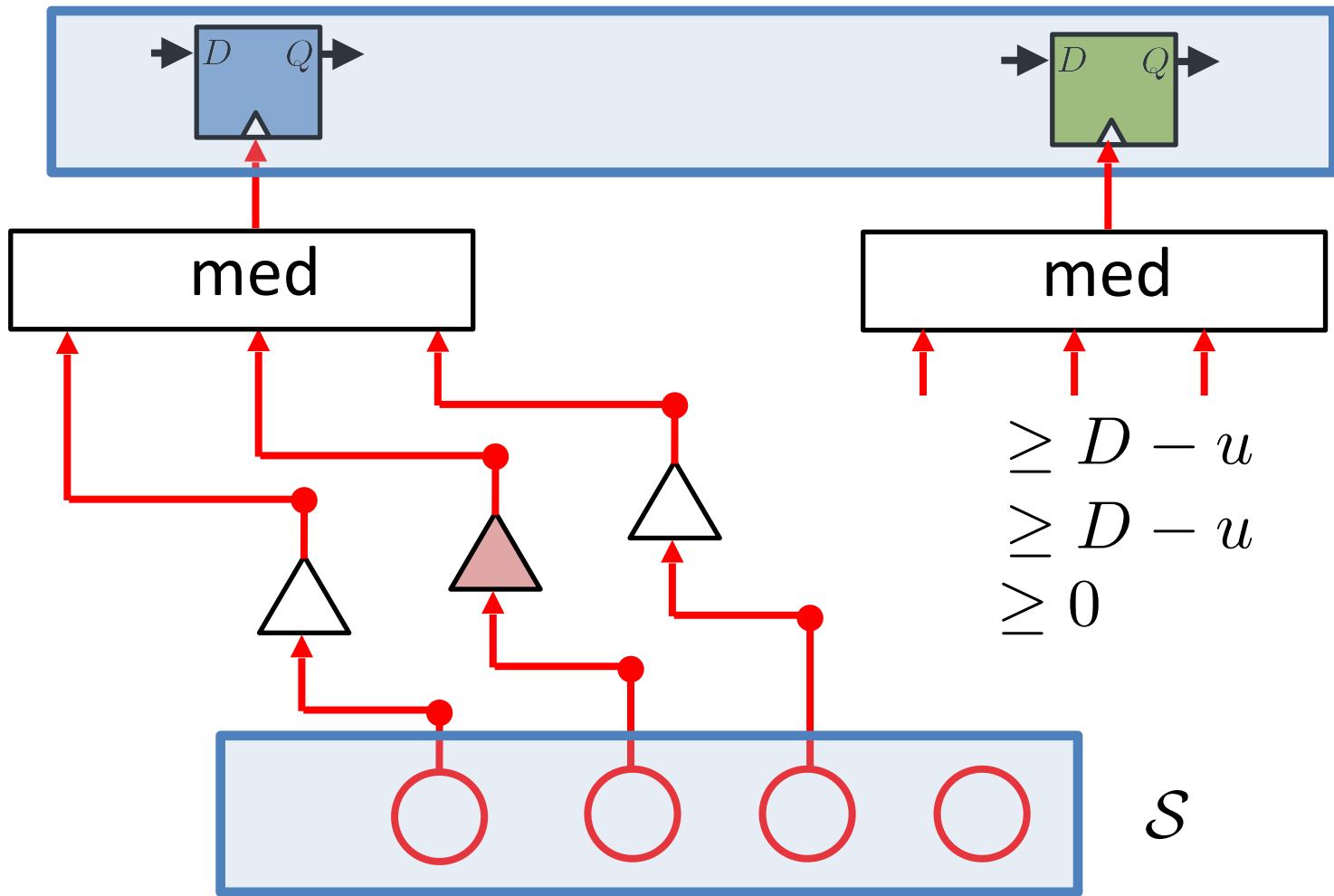
$$\leq \max(p_{r_1,k}, p_{r_2,k}) + D$$

$$\geq \min(p_{r_1,k}, p_{r_2,k}) + D - u$$



# Skew

$$|\cdot| \leq \mathcal{S} + u$$



# Idea 2

5x5 grid

properties:

- fault tolerance?



- skew?

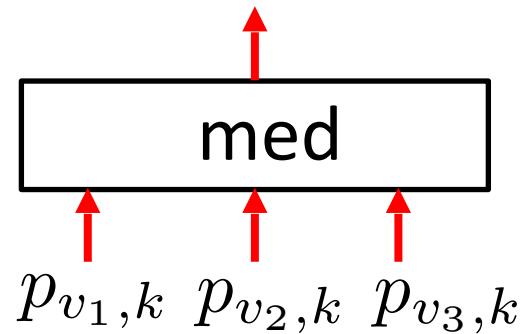


- **cost?**

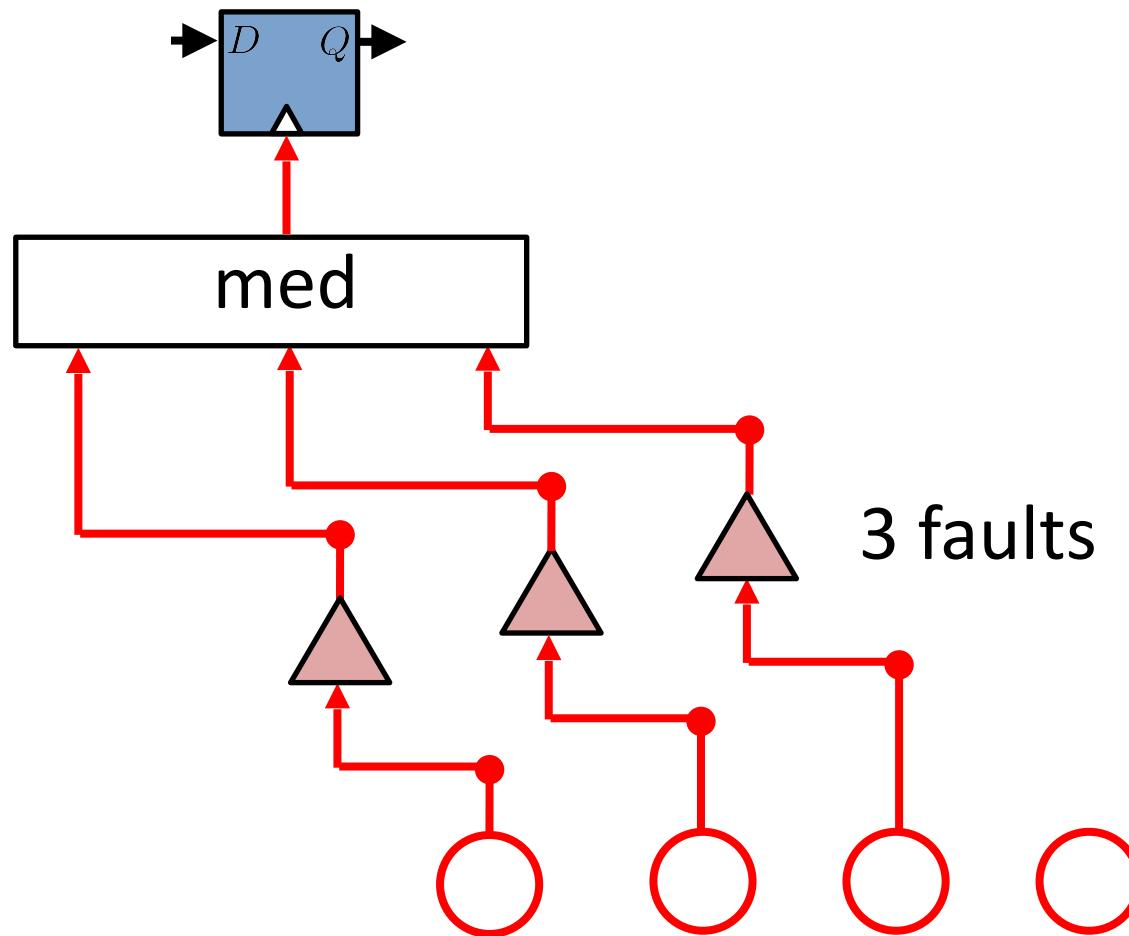


# Cost

3f+1 roots & 2f+1 trees



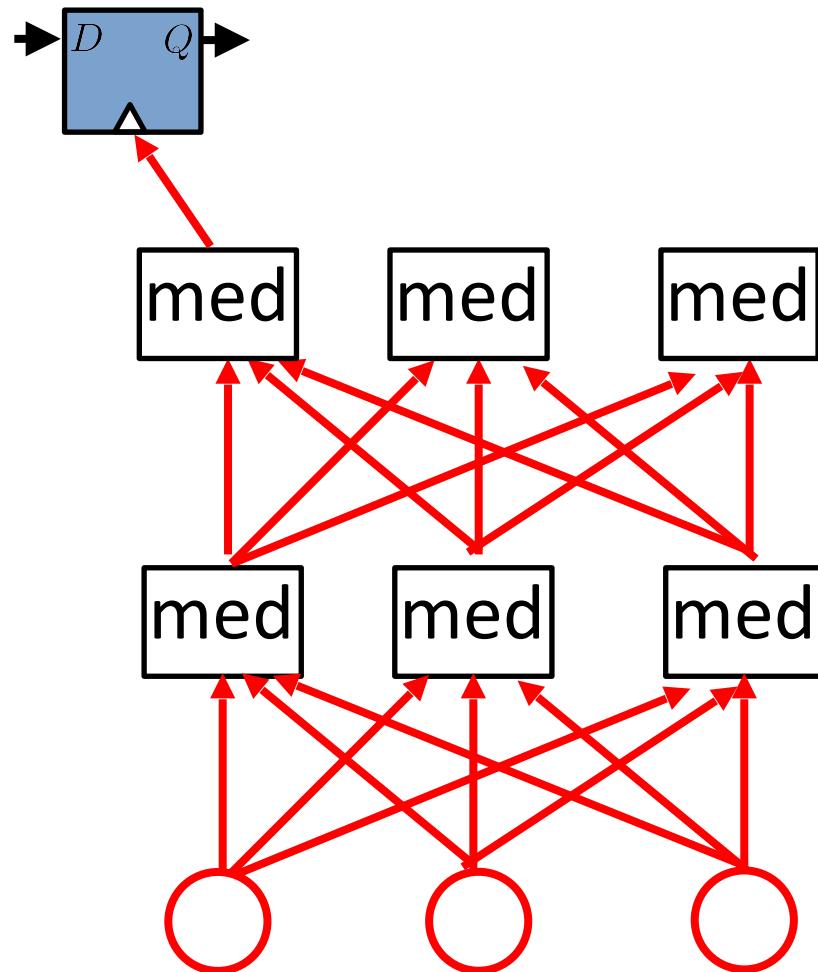
# Depends on the environment



Q: other solutions?

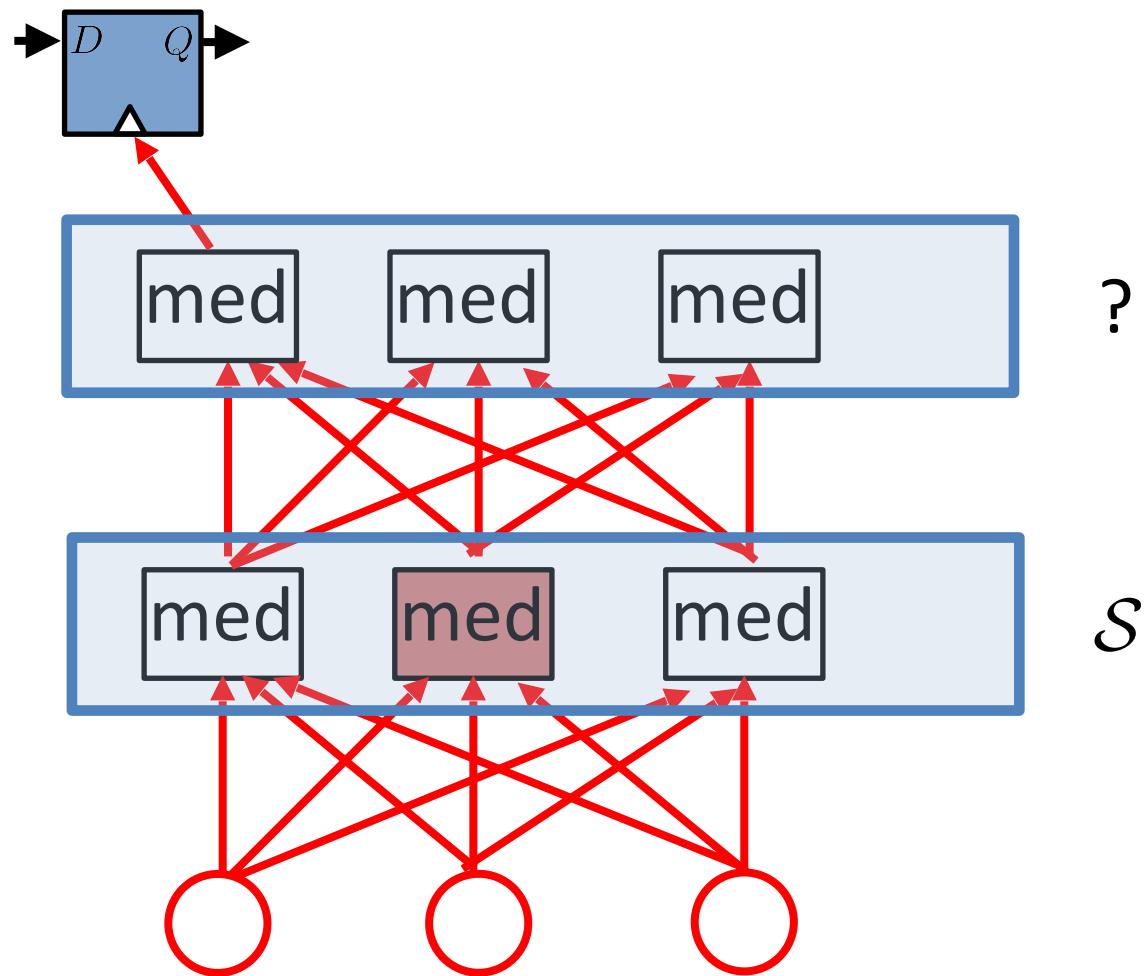
# Idea 3: interlinked trees

median at each stage of the tree



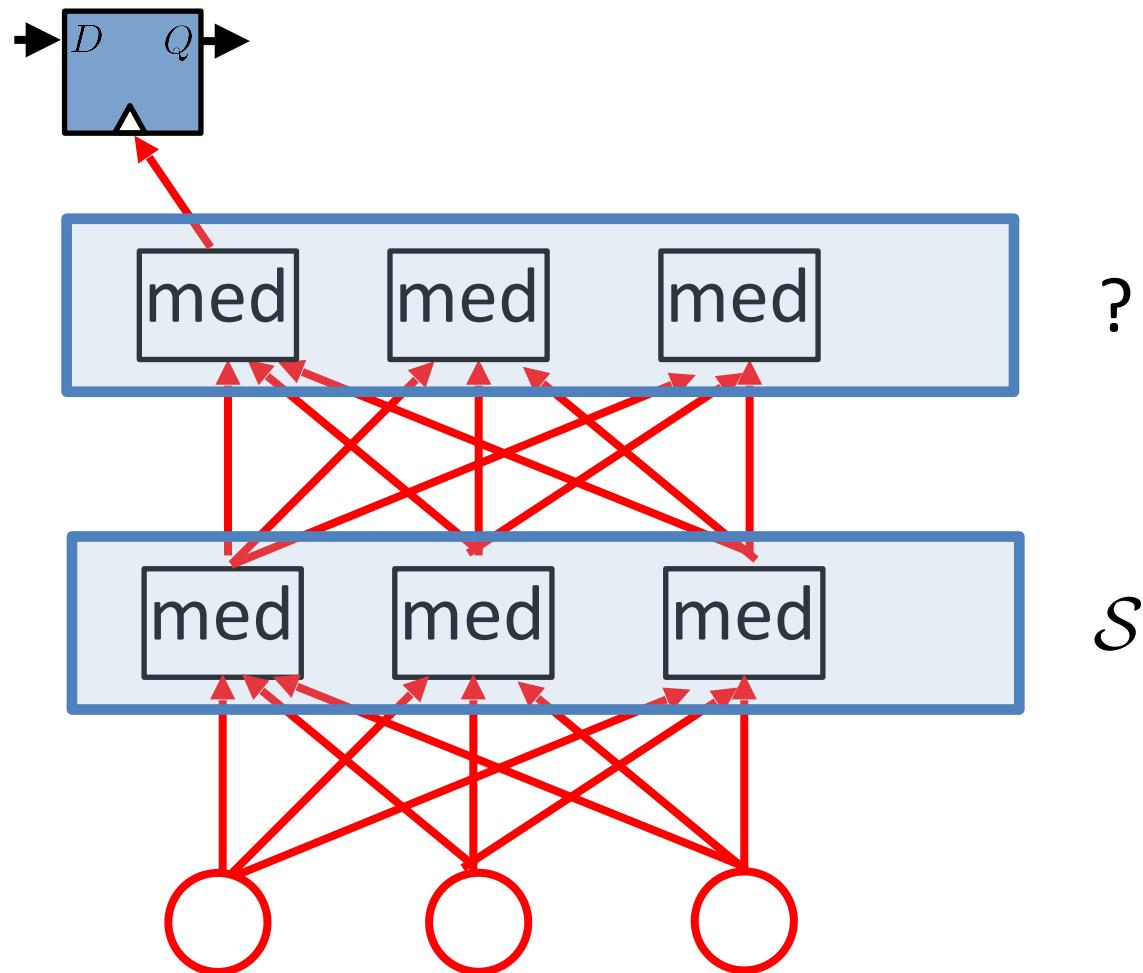
# Skew

median at each stage of the tree



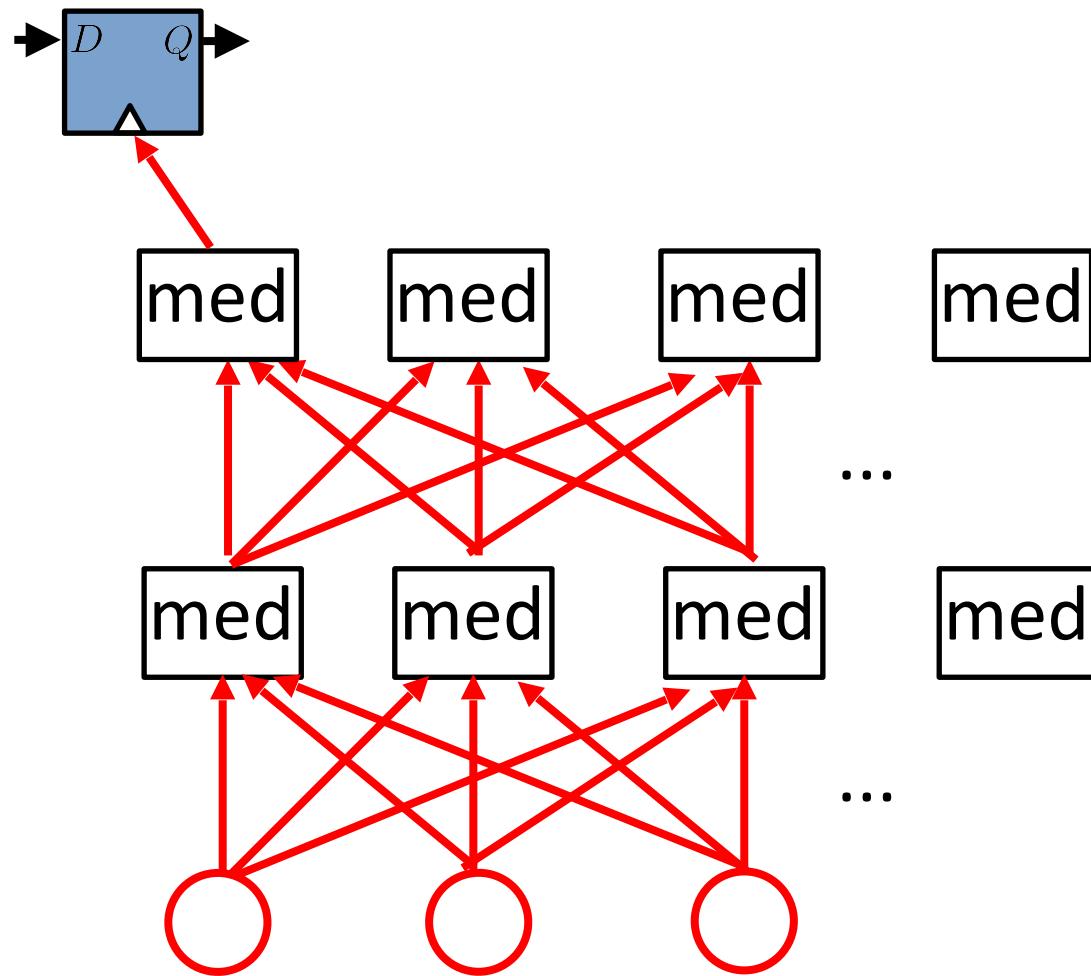
# Skew

median at each stage of the tree



# Idea 4: interlinked trees v2

median at each stage of the tree





## Ex 11.1

$$P(\geq 2 \text{ faults}) = 1 - P(\leq 1 \text{ faults}) \approx 0.80$$

$$1 - \sum_{i=0}^1 \binom{300}{i} p^i (1-p)^{300-i} \text{ where } p = 0.01$$

$$P(\geq 100 \text{ faults}) = 1 - P(\leq 99 \text{ faults}) \approx 6.9 \cdot 10^{-15}$$

$$1 - \sum_{i=0}^{99} \binom{300}{i} p^i (1-p)^{300-i} \text{ where } p = 0.01$$

# 100 faults

global constraint (idea 1, LW):

$$P(\geq 100 \text{ faults}) = 1 - P(\leq 99 \text{ faults}) \approx 6.9 \cdot 10^{-15}$$

local constraints (idea 2, redundant trees):

$$P(\text{correct}) = P(\leq 1 \text{ faulty tree})$$

$$\{p = 0.01, c = (1 - p)^{100}, c^3 + 3c(1 - c)^2\}$$

$$c(4c^2 - 6c + 3) \approx 0.490383$$

local constraints (idea 3, interlinked trees):

$$P(\text{correct}) = P(\leq 1 \text{ fault per triple})$$

$$\{p = 0.01, c = (1 - p)^3 + 3(1 - p)^2 p, c^{100}\}$$

$$c^{100} \approx 0.970635$$