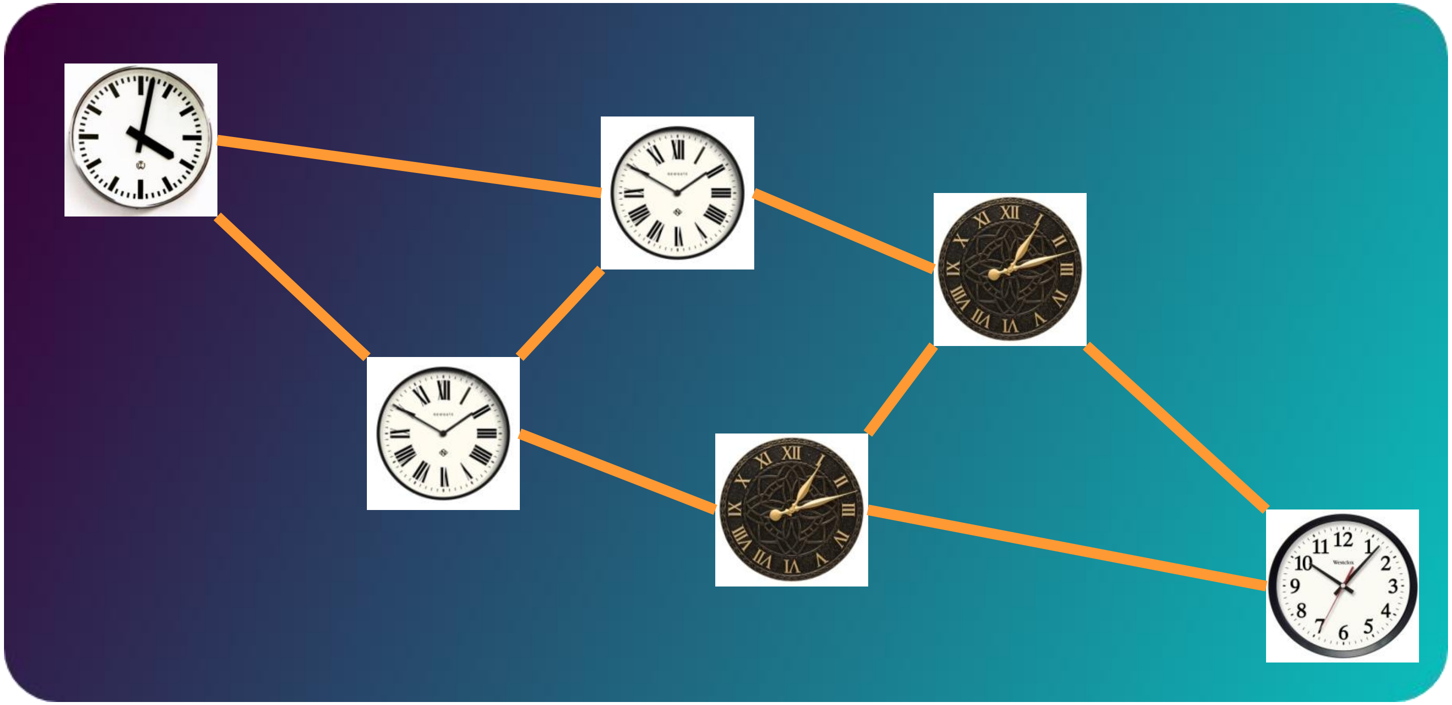
Gradient Clock Synchronization



$$\max_{\{v,w\}\in E} |L_v - L_w| \ll \max_{v,w\in V} |L_v - L_w|$$

Today: GCS Algorithm with log. Skew

Theorem

For any $\mu > \theta$-1, there is an algorithm such that

$$\mathbf{dH/dt \leq dL/dt \leq (1+\mu)dH/dt}$$

and the **local skew** is

$$\mathbf{O((u+\mu d) \log_\sigma D)},$$

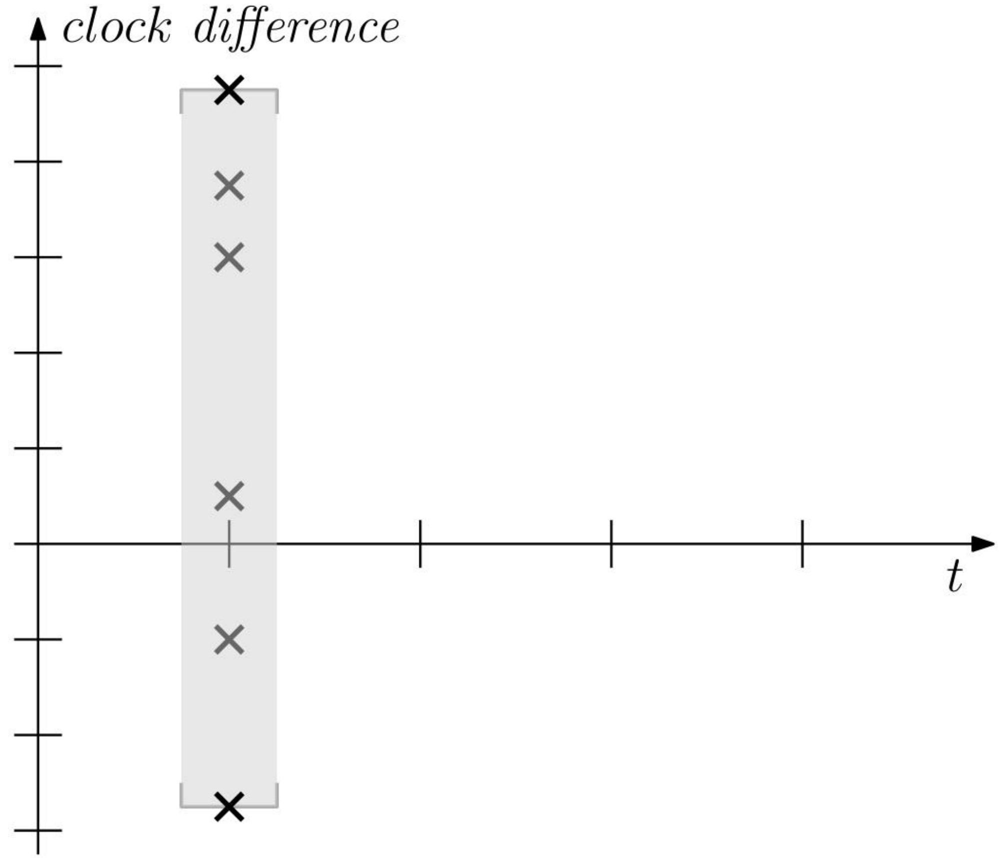
where

$$\sigma = \mu/(\theta-1).$$

GCS: General Approach

repeat:

1. measure skews
   (to neighbors)



GCS: General Approach

repeat:

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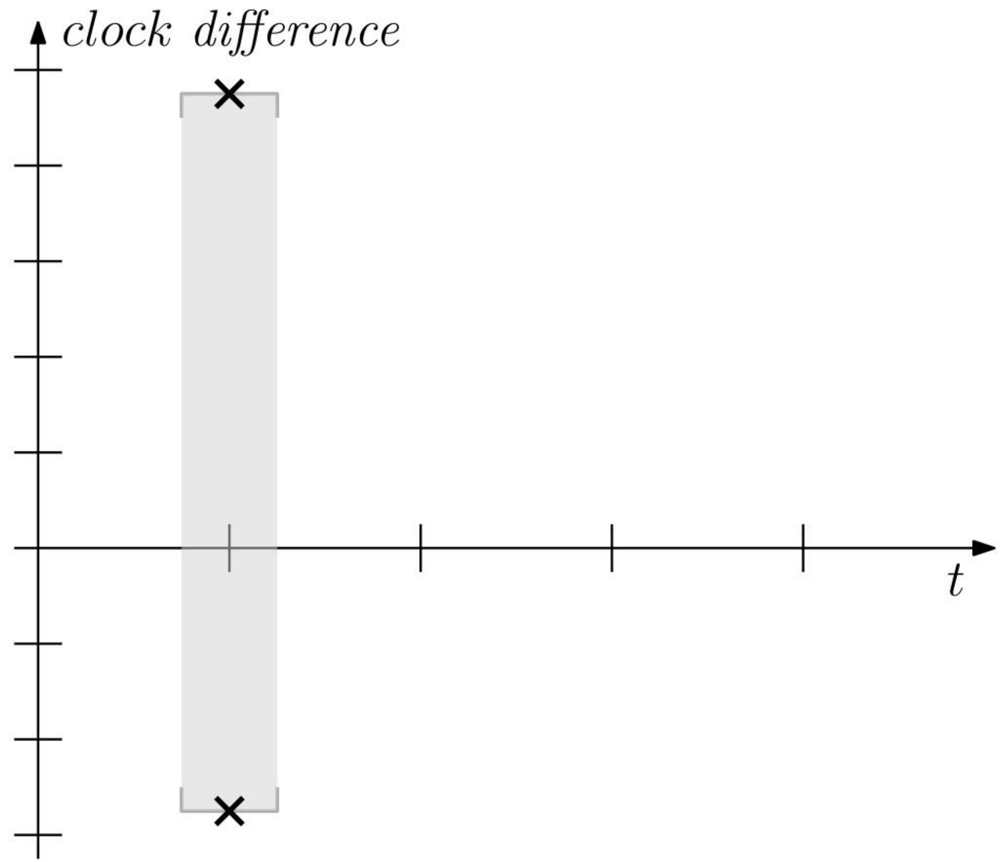

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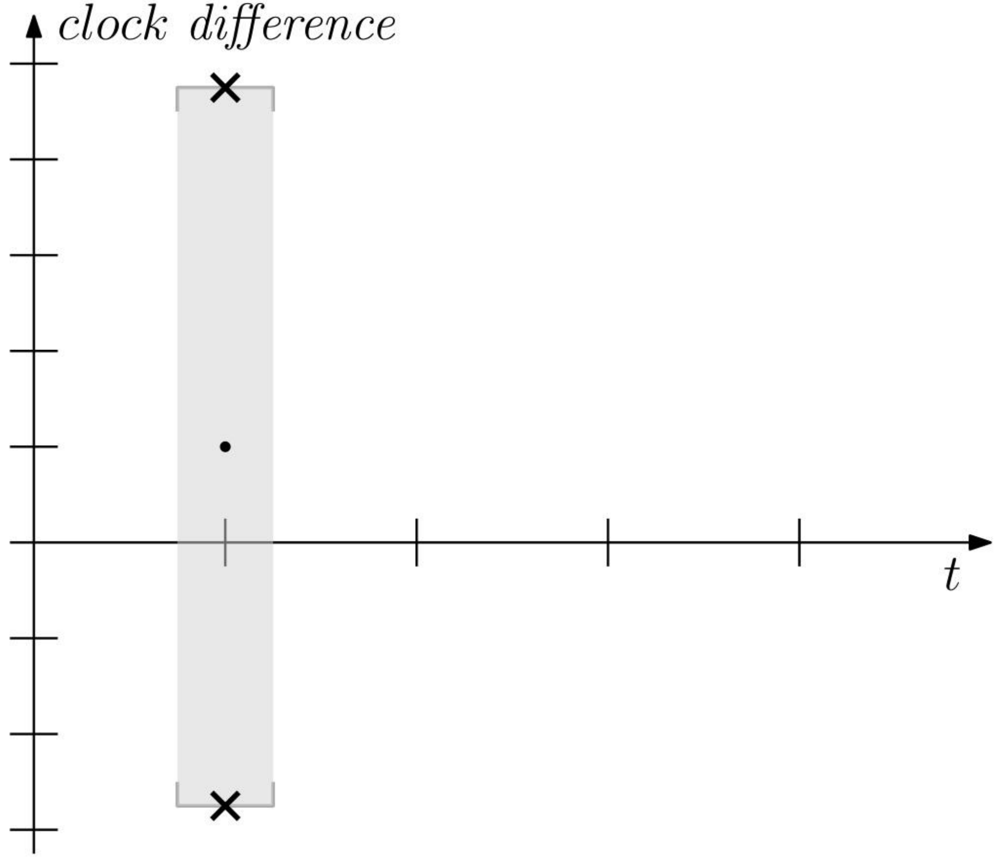

GCS: Naive Averaging Fails

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4. if behind, run faster
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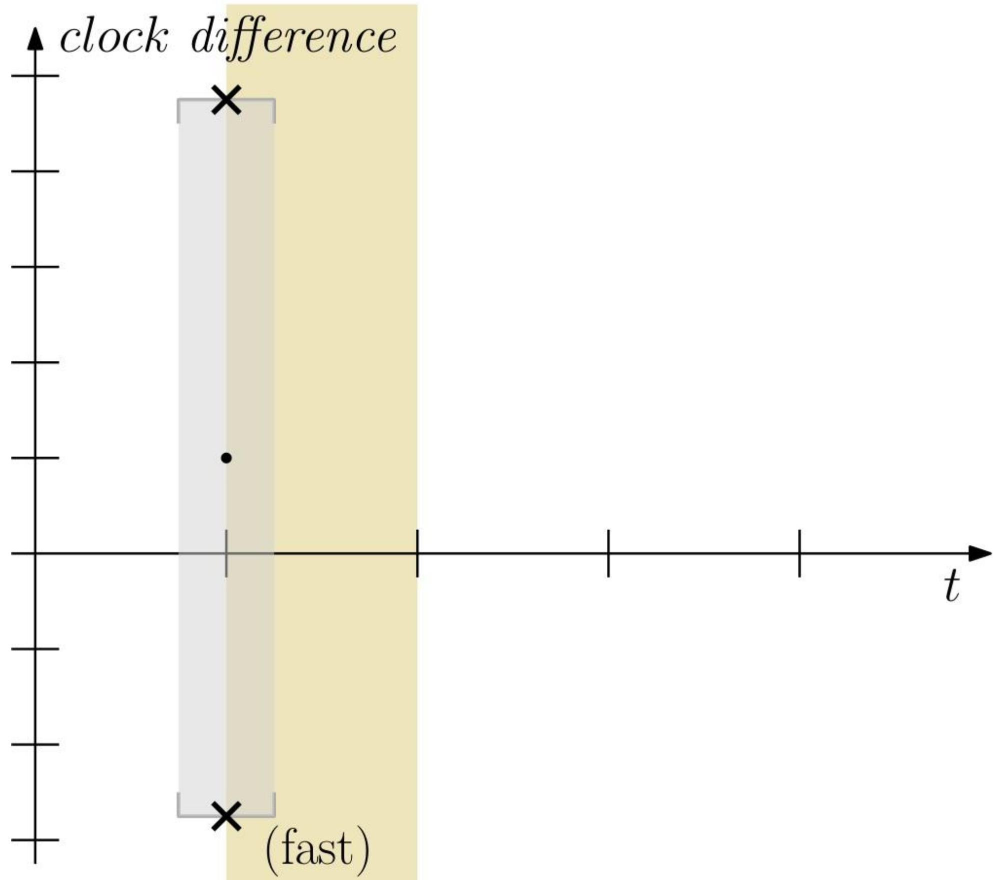

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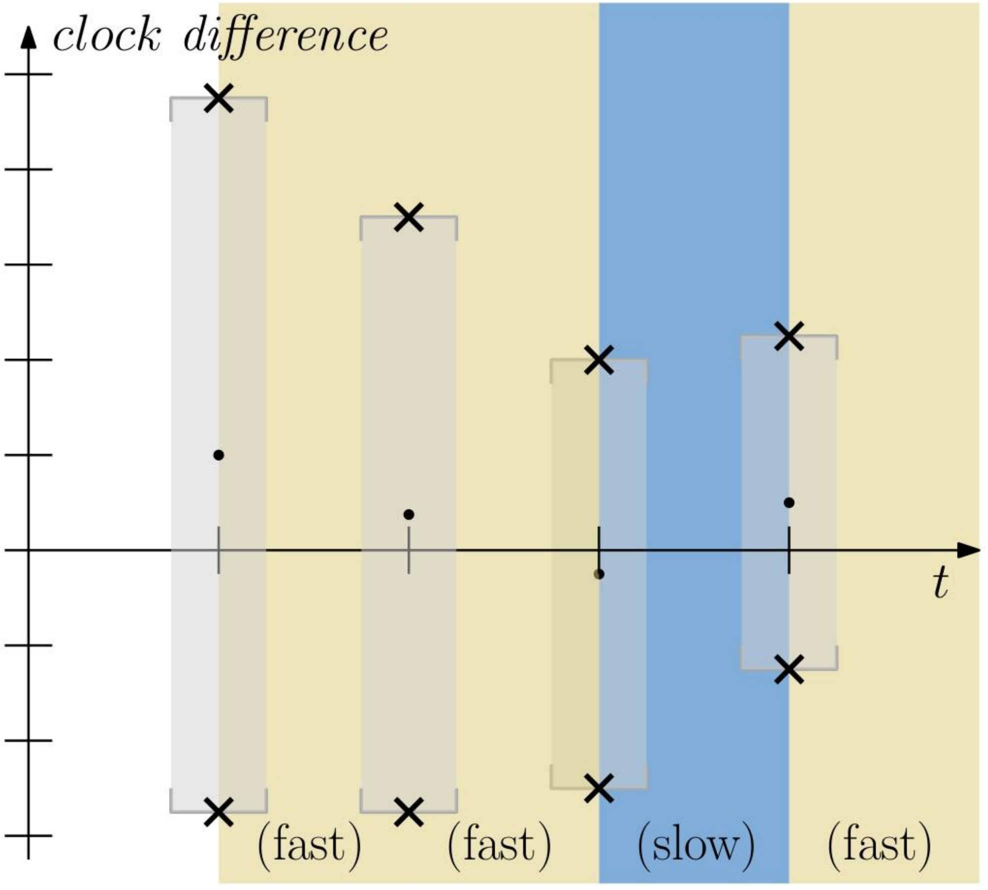

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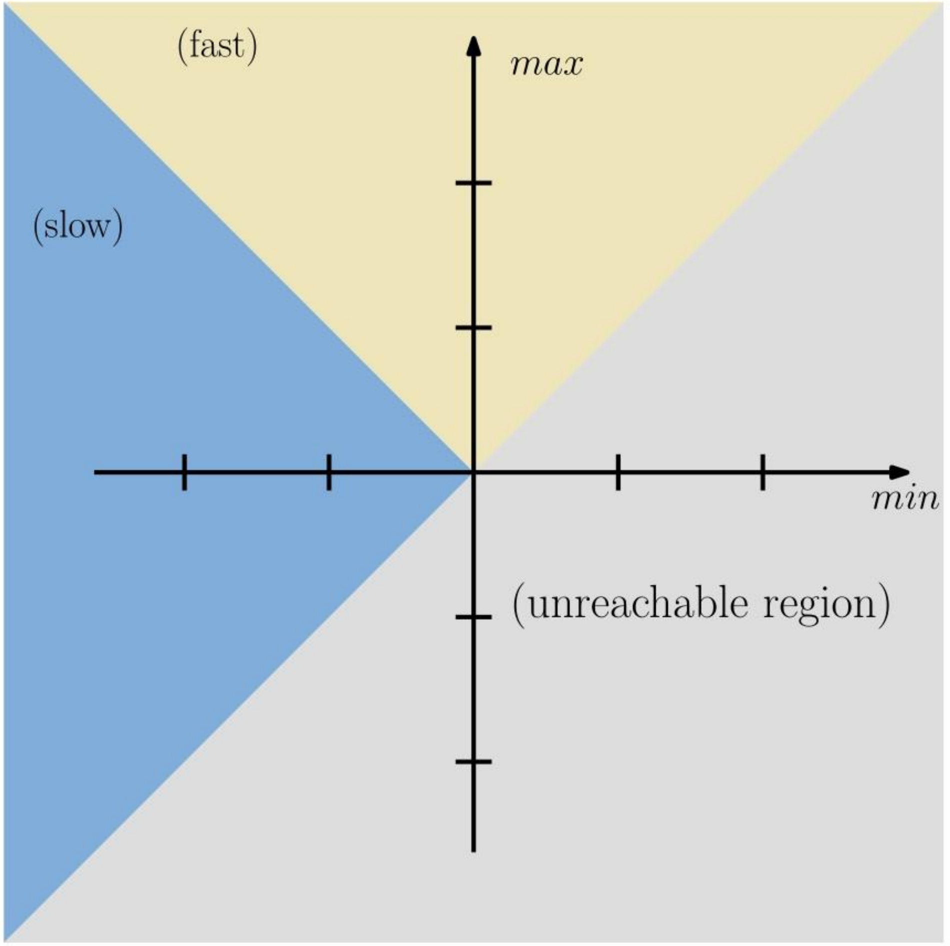

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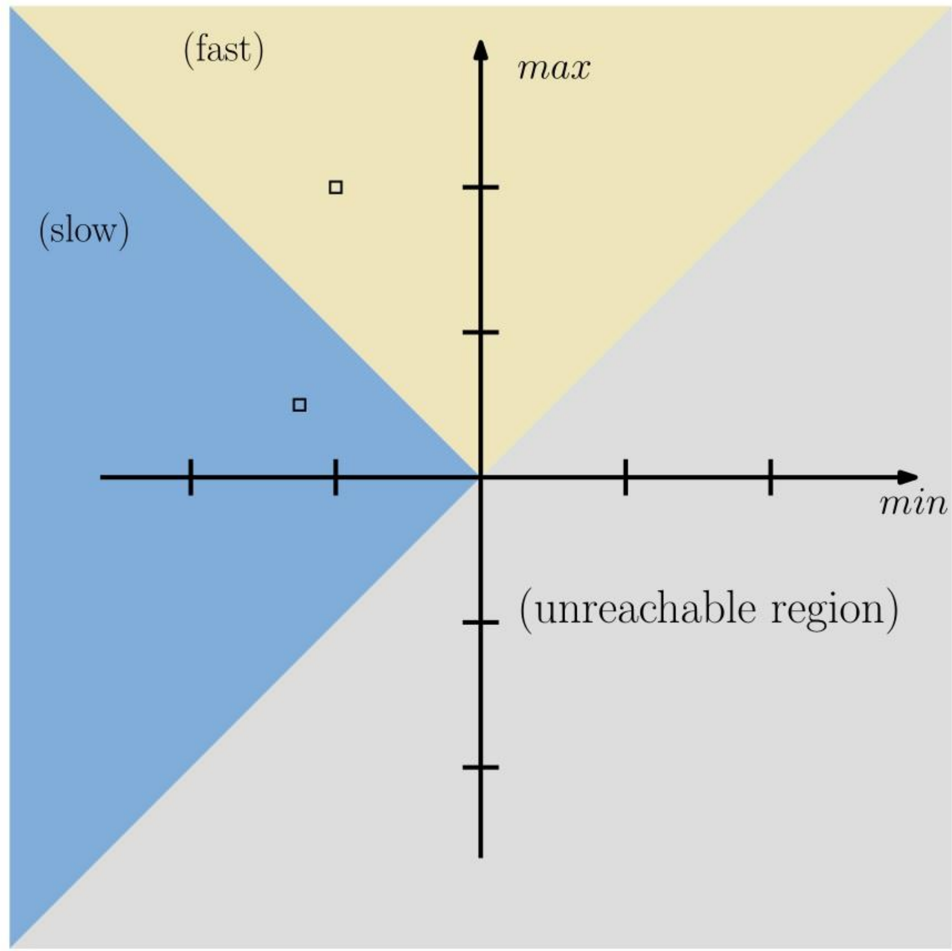

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problem:
Measurements
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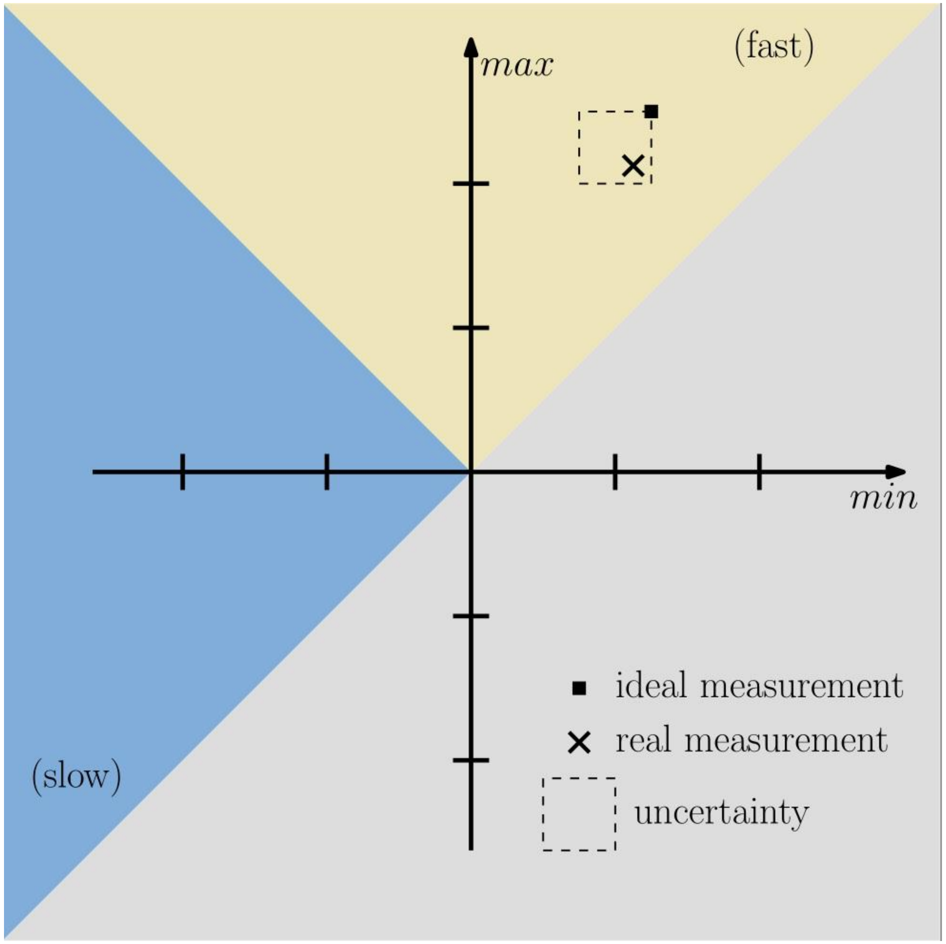

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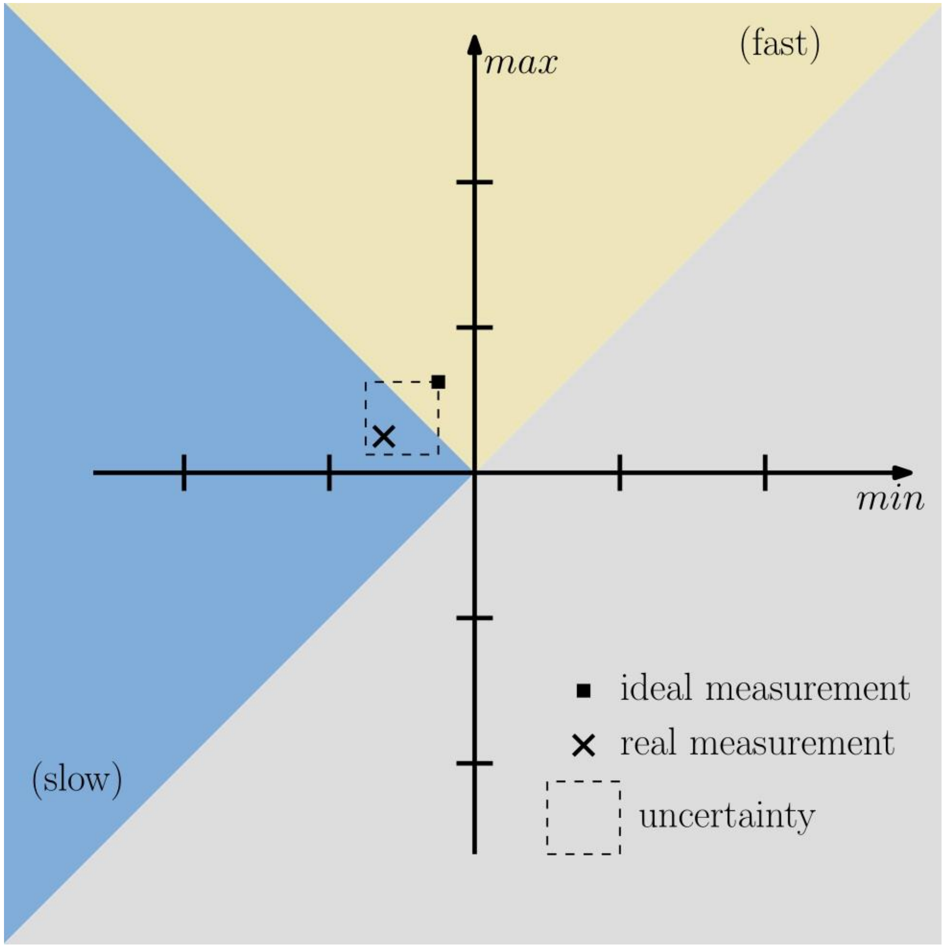

GCS: Naive Averaging Fails

problem:
Measurements
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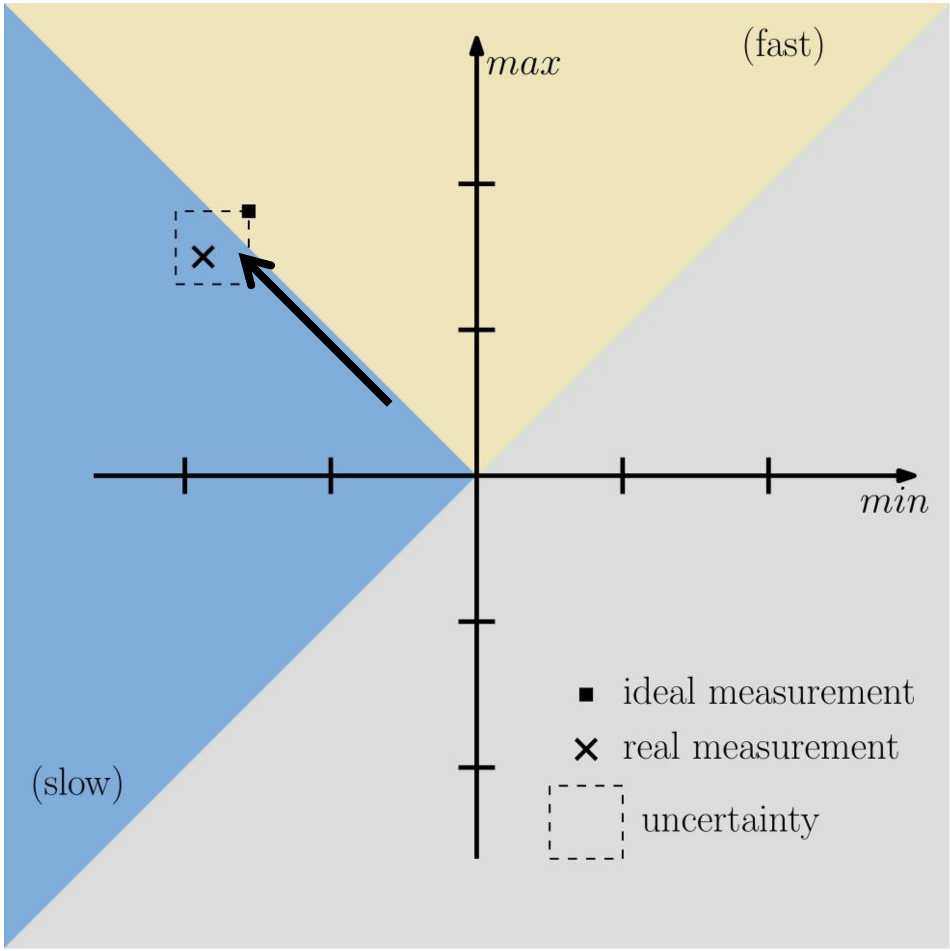
This blurs the line
between fast and slow.

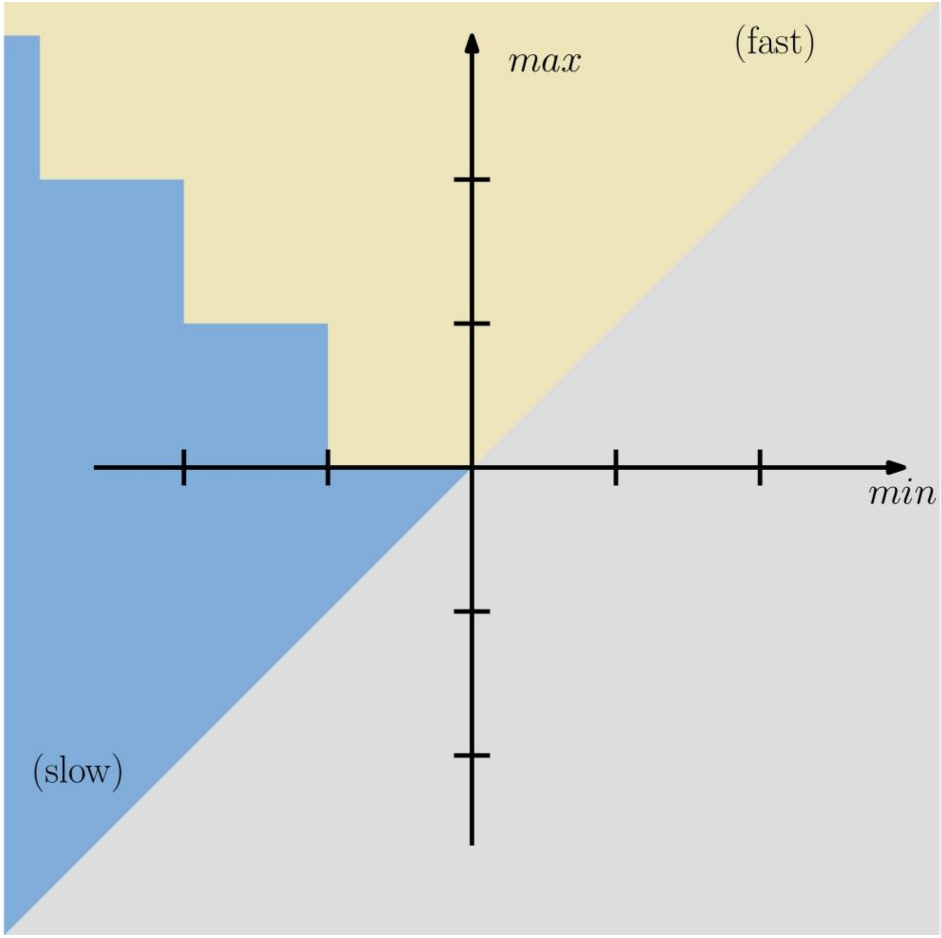
=> system might not
respond to build-up
of skew!



GCS Algorithm

idea:
discretize skews and
round conservatively

=> local & limited
response to skews



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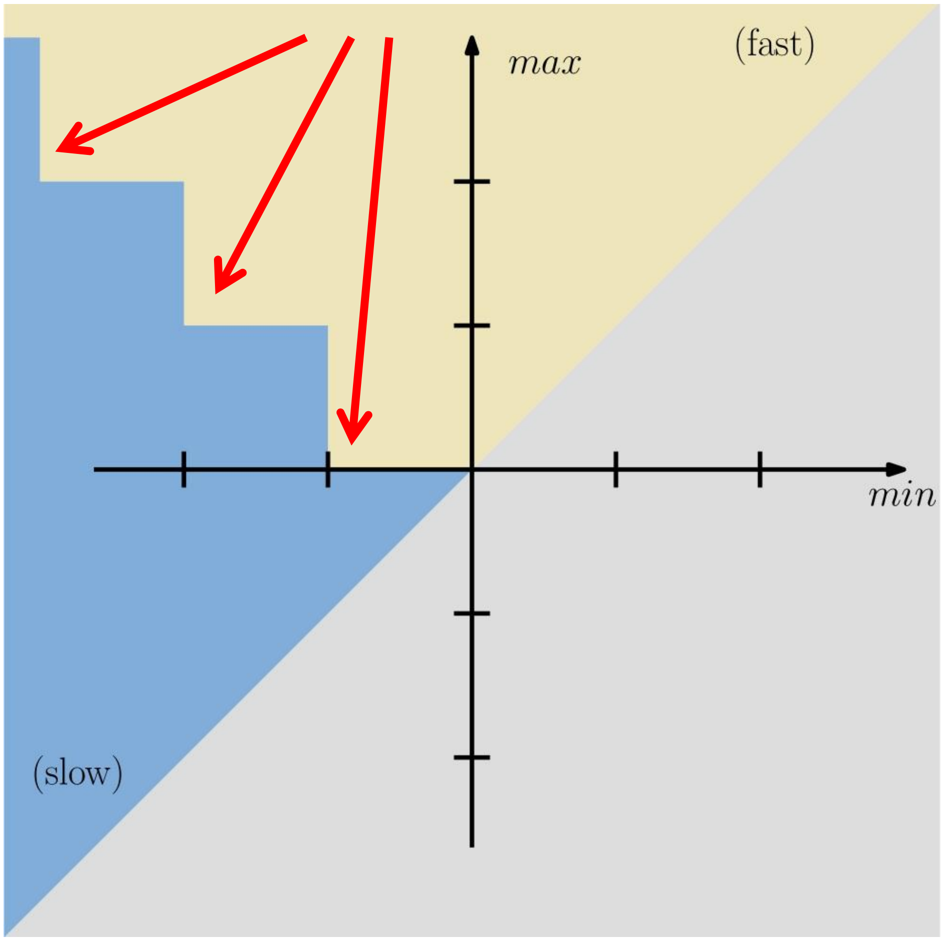

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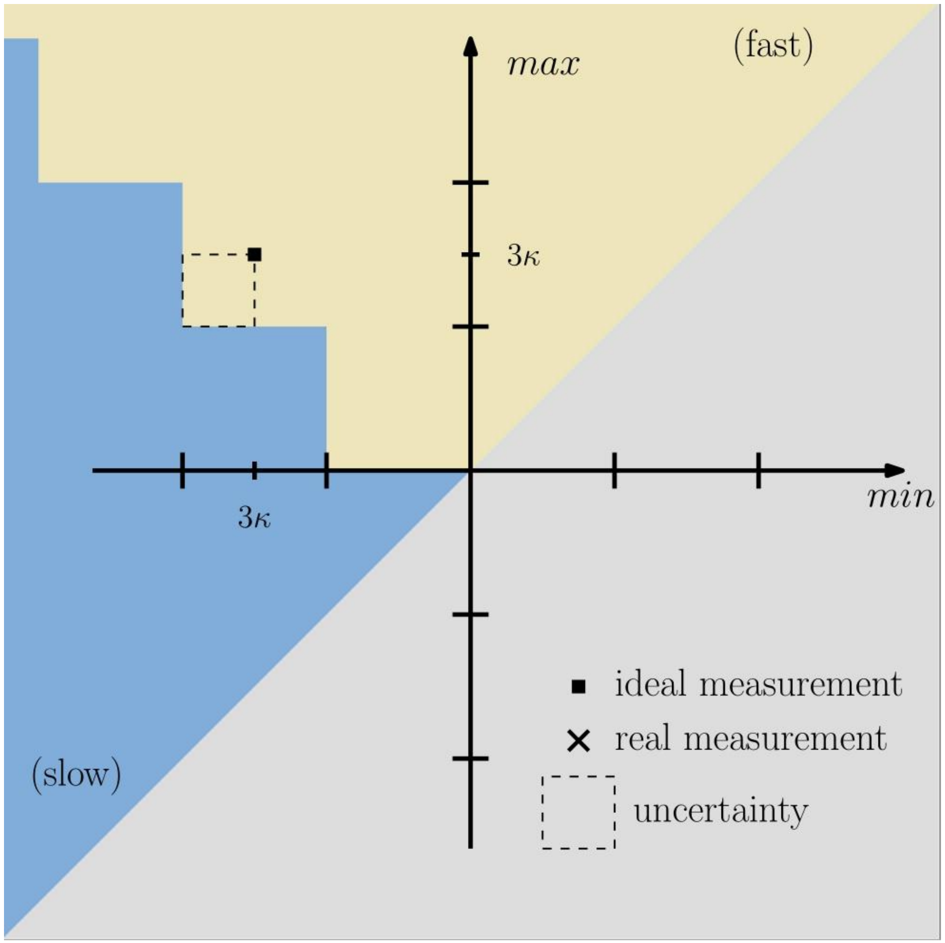

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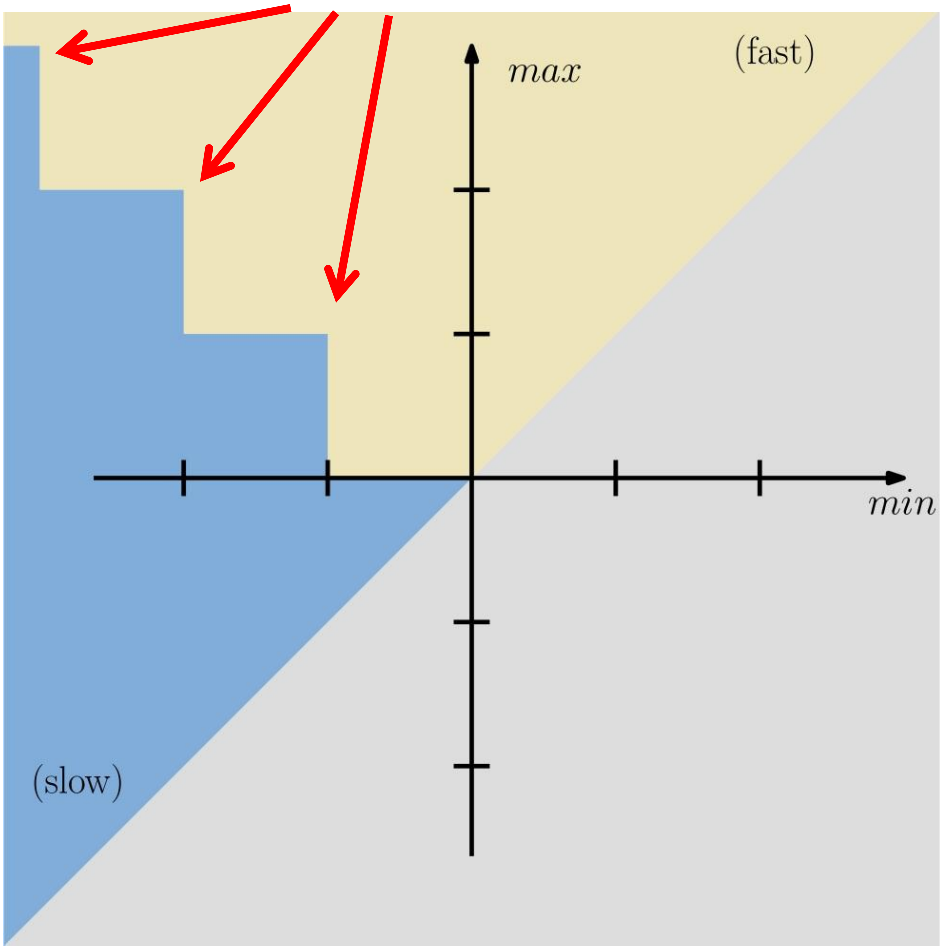

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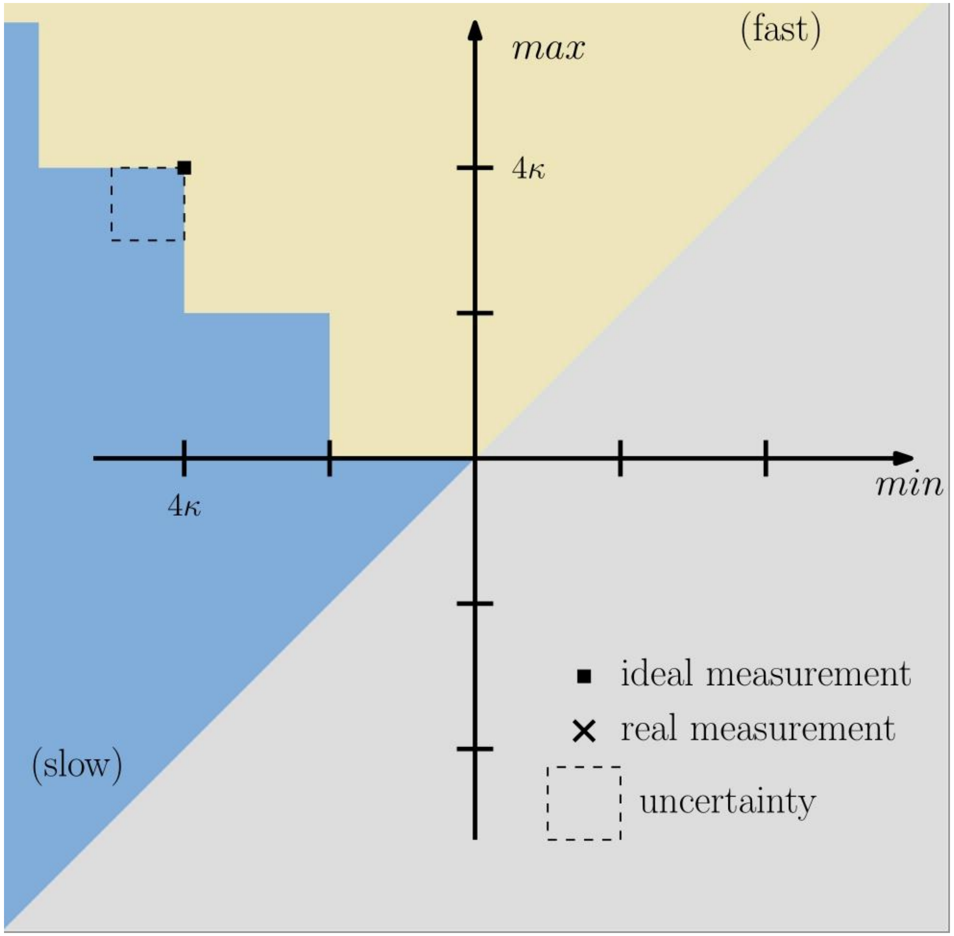


GCS Algorithm

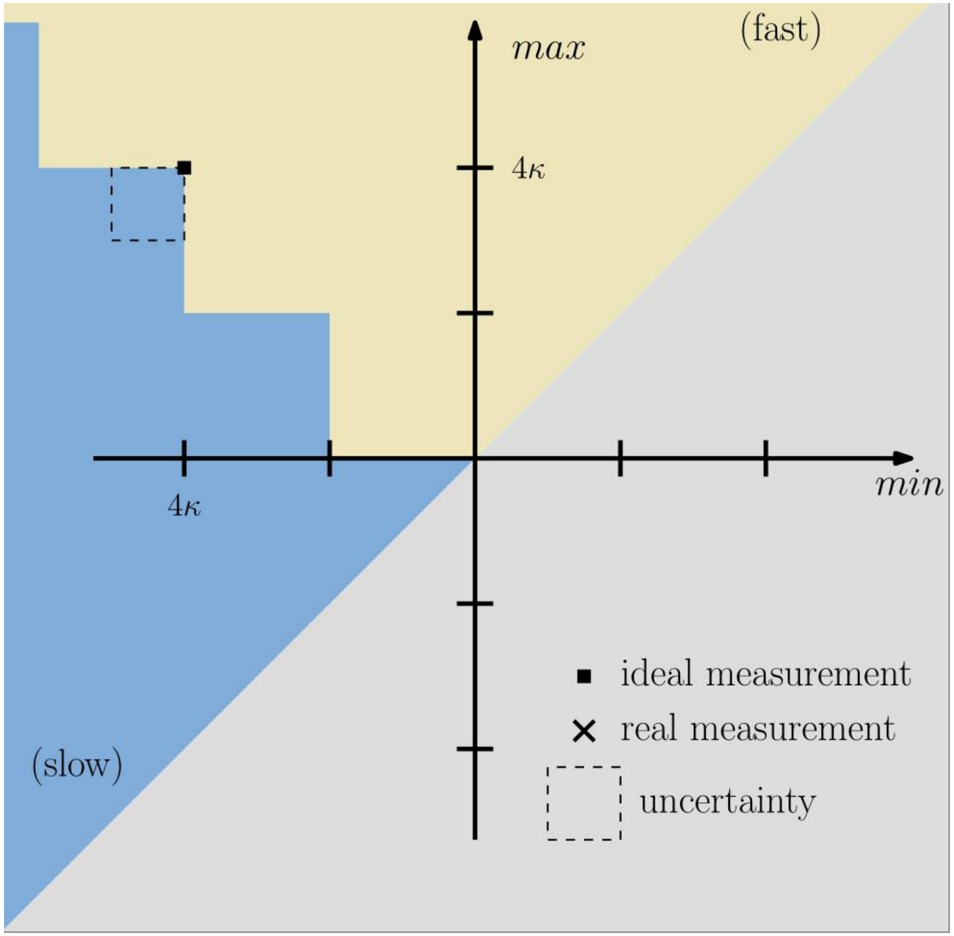
idea:

discretize skews and round conservatively

=> local & limited response to skews

2κ = “height of stairs”

δ = side of square



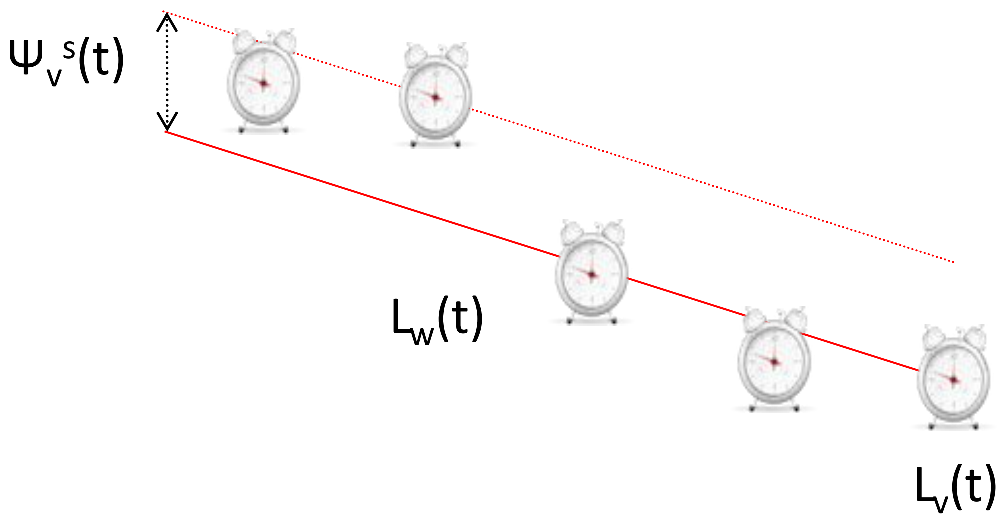
Computing Clock Estimates

breakout session:

What‘s δ (asymptotically)?

GCS Analysis: Potential Function

$\Psi_v^s(t) = \max_{w \in V} \{L_w(t) - L_v(t) - (2s - 1)\kappa \, \mathrm{dist}(v,w)\}$



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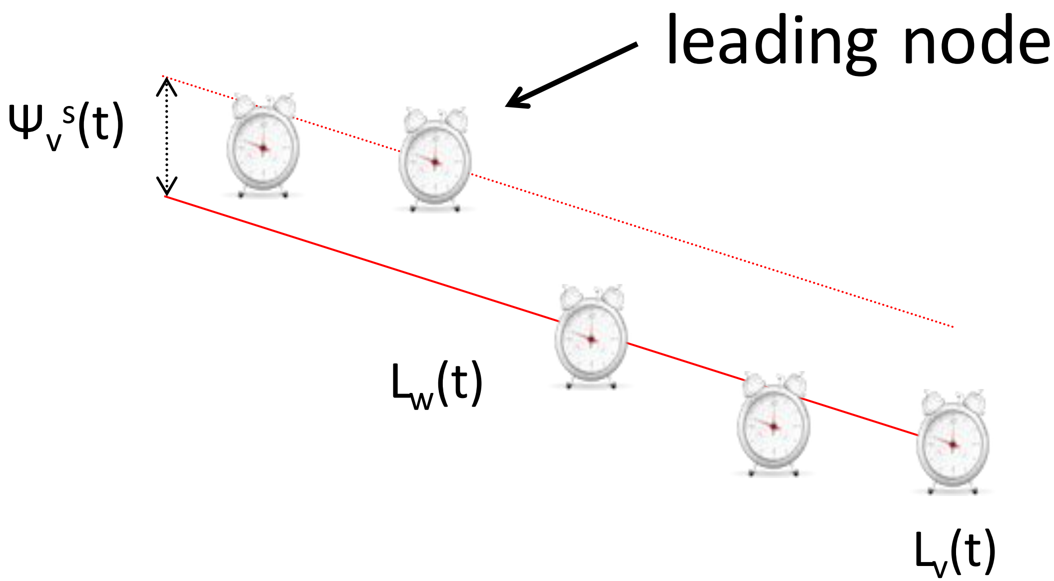

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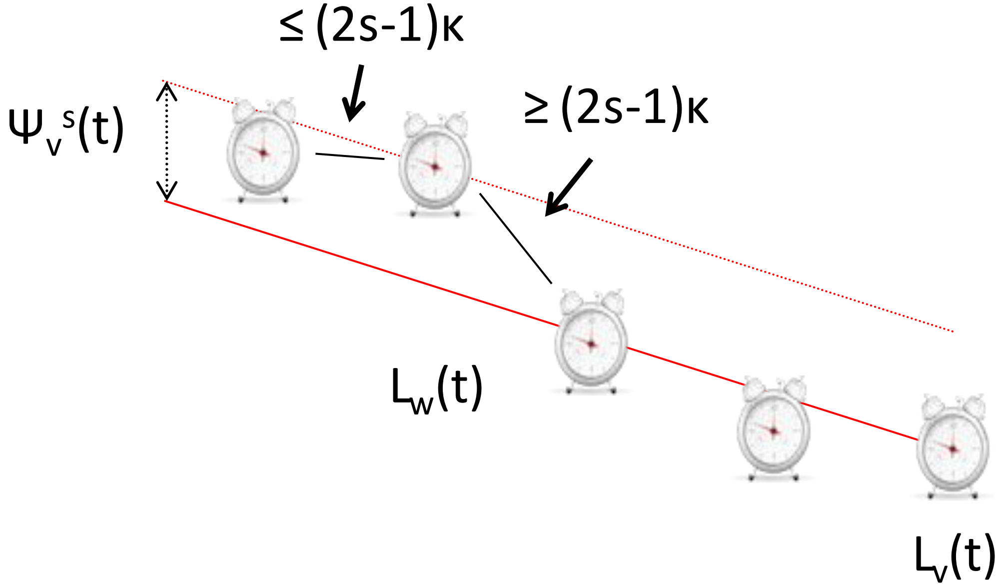

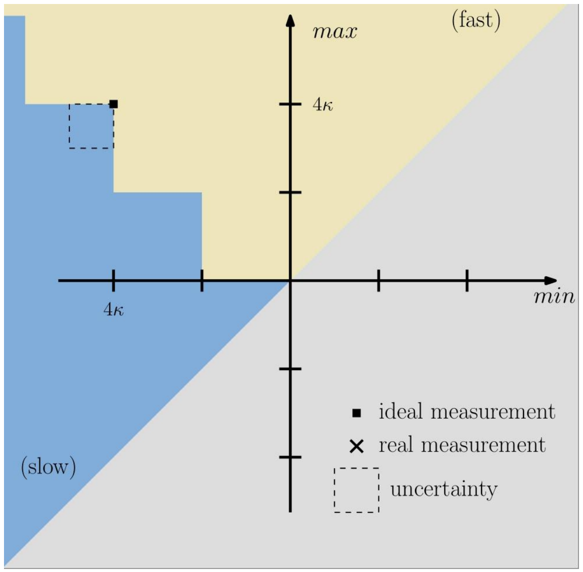

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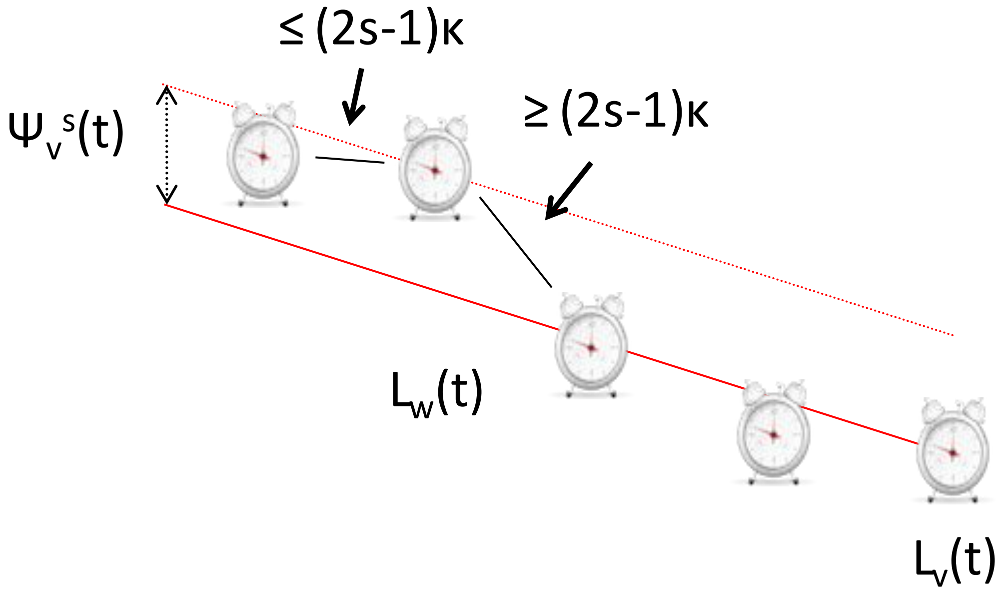

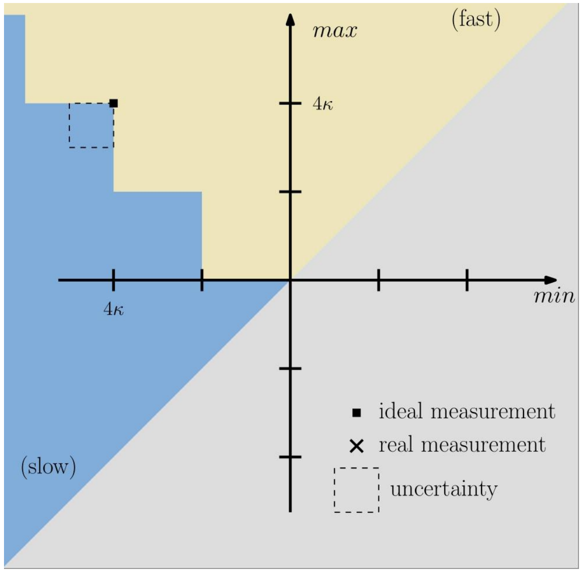

- estimates are ≤ actual clock values

=> slow mode trigger holds for all leading nodes

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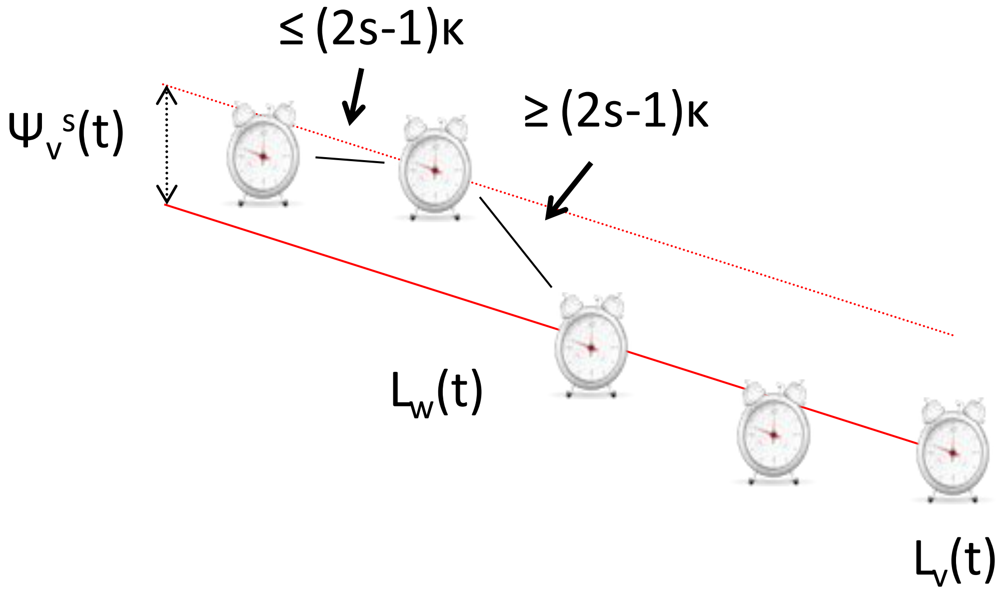

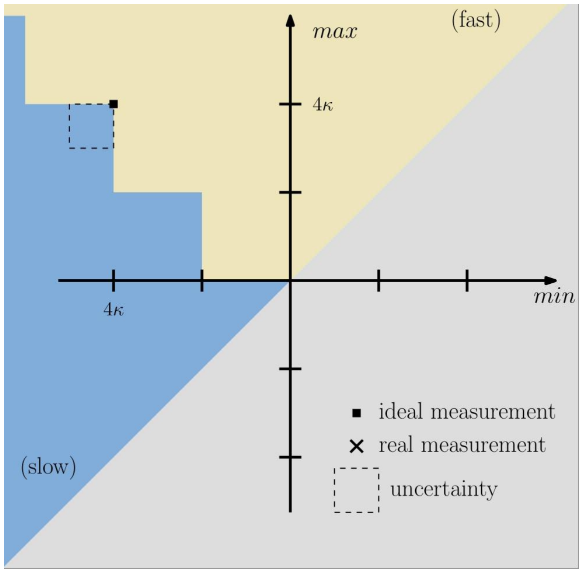

- estimates are ≤ actual clock values

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=> $\Psi_v^s(t') \le \Psi_v^s(t) + \theta(t' - t) - (L_v(t') - L_v(t))$

GCS Analysis: Catching Up

$\Psi_v^s(t) = \max_{w \in V} \{L_w(t) - L_v(t) - (2s - 1)\kappa\, \mathrm{dist}(v,w)\}$

$\Psi^s(t) = \max_{v \in V} \{\Psi_v^s(t)\}$

Lemma:

For times

$$t' \geq t + \Psi^{s-1}(t)/\mu$$

we have

$$L_v(t') - L_v(t) \geq t' - t + \Psi_v^s(t).$$

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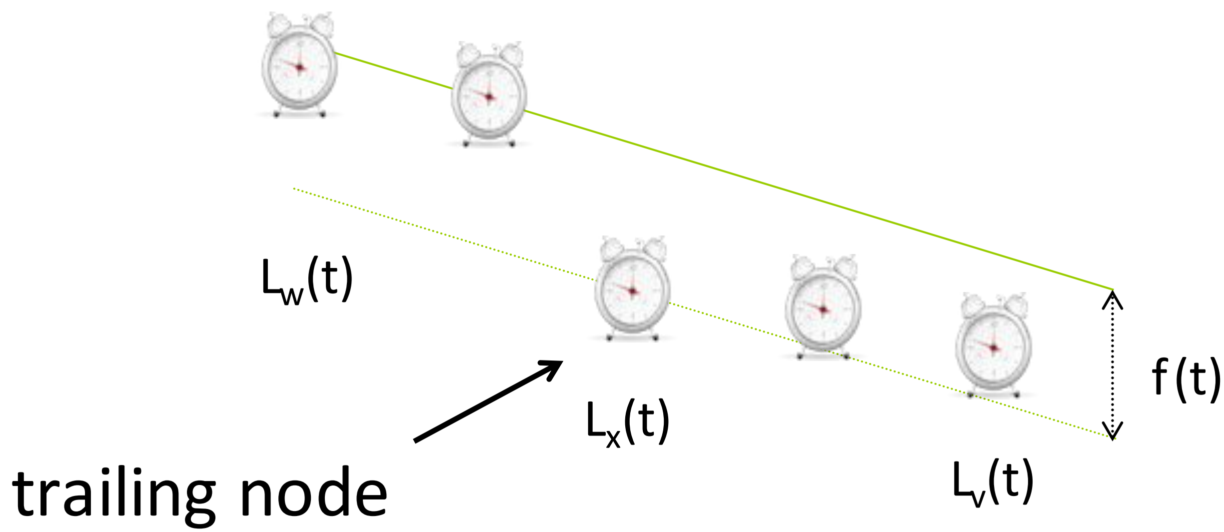
fix t and w that maximizes $\Psi_v^s(t)$, and set

$f_x(t') = L_w(t) + t' - t - L_x(t') - (2s - 2)\kappa\,\mathrm{dist}(v,w)$

$f(t') = \max_{x \in V}\{f_x(t')\}$



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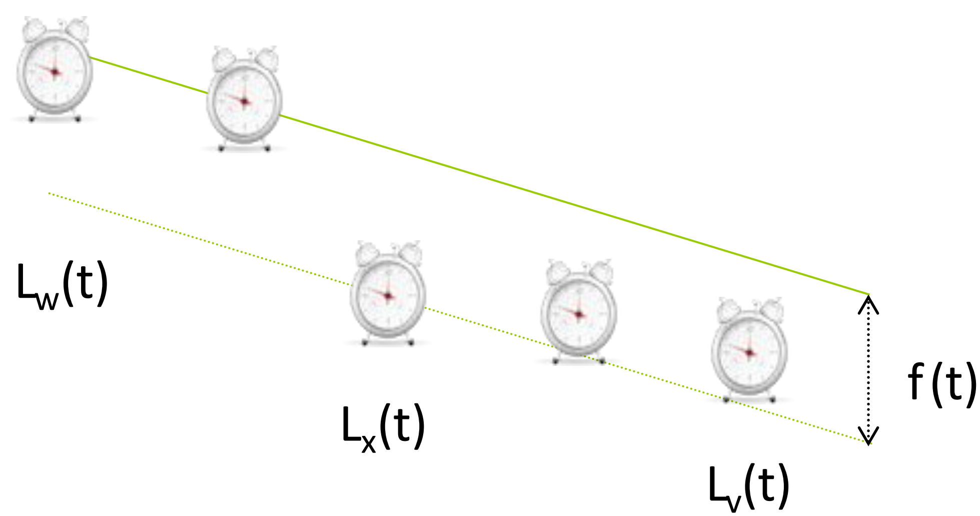

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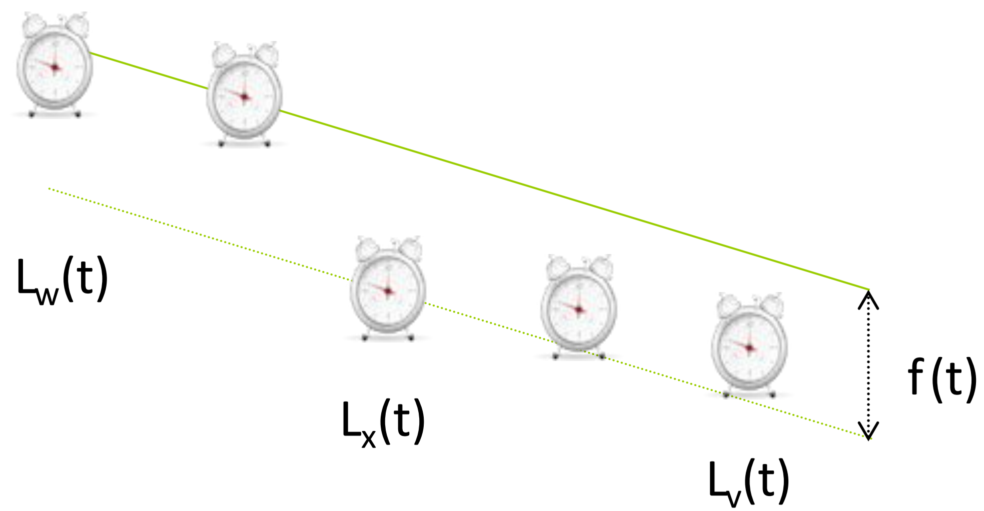

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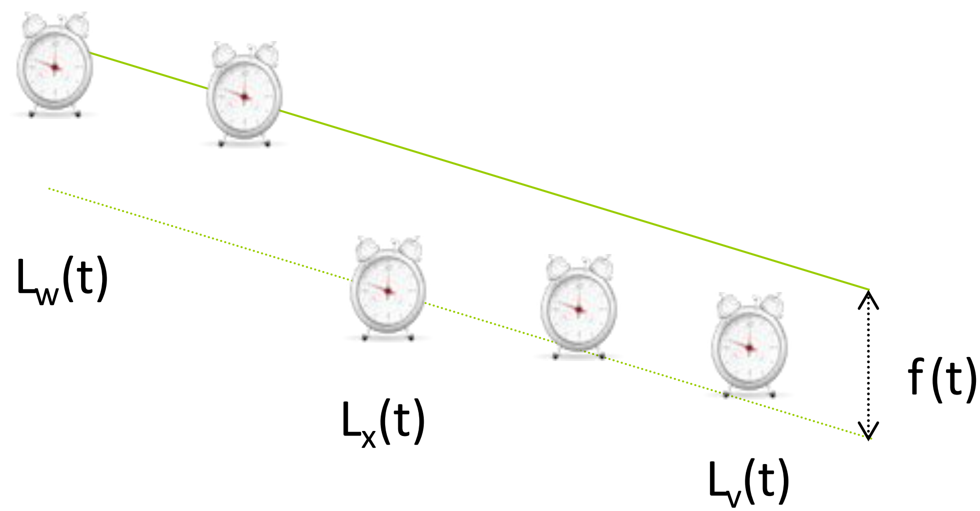
$\Psi_v^s(t) \leq f_v(t)$

$f(t') \leq 0 \Rightarrow f_v(t') \leq 0$

$\Rightarrow L_v(t') - L_v(t) =$

$\quad t' - t + f_v(t) \geq$

$\quad t' - t + \Psi_v^s(t)$



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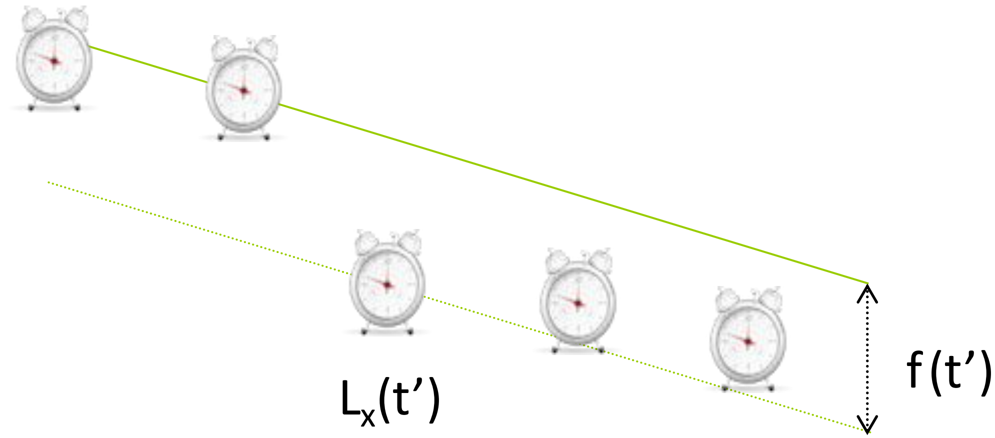

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$t' \geq t + \Psi^{s-1}(t)/\mu$

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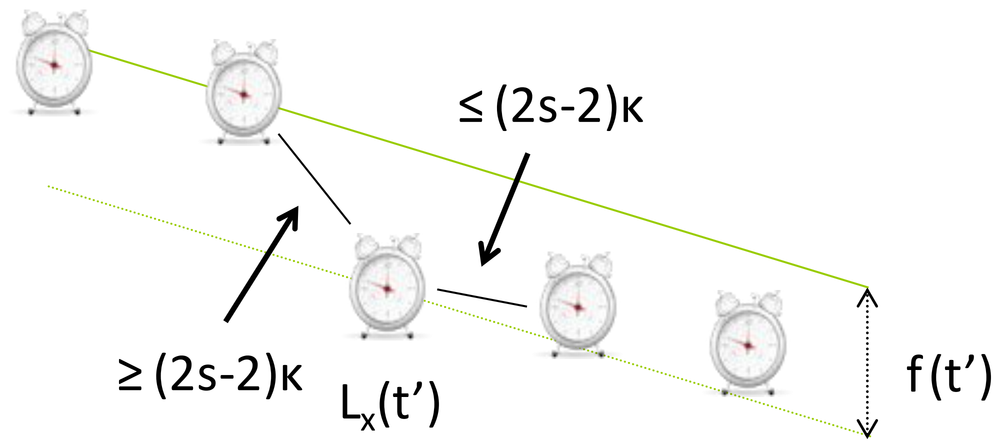

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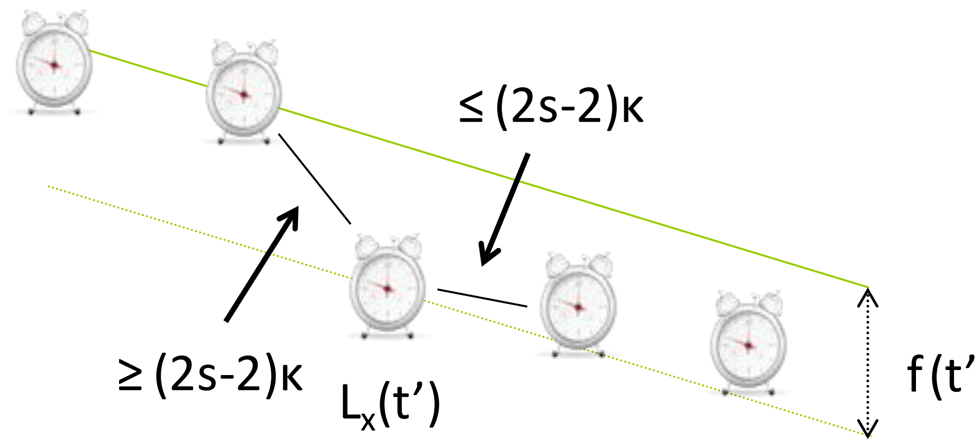

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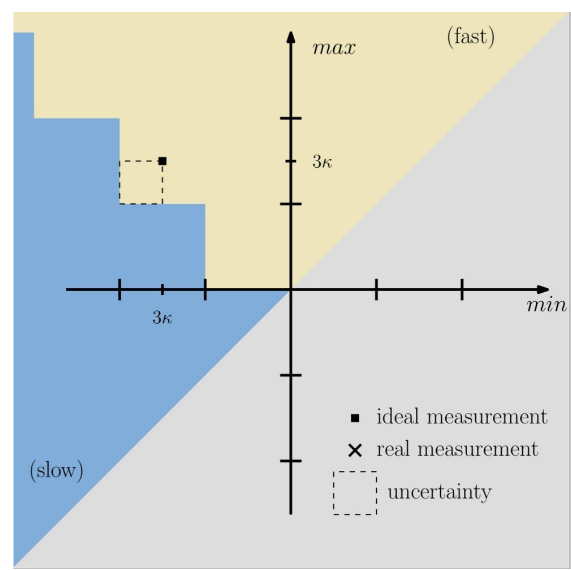

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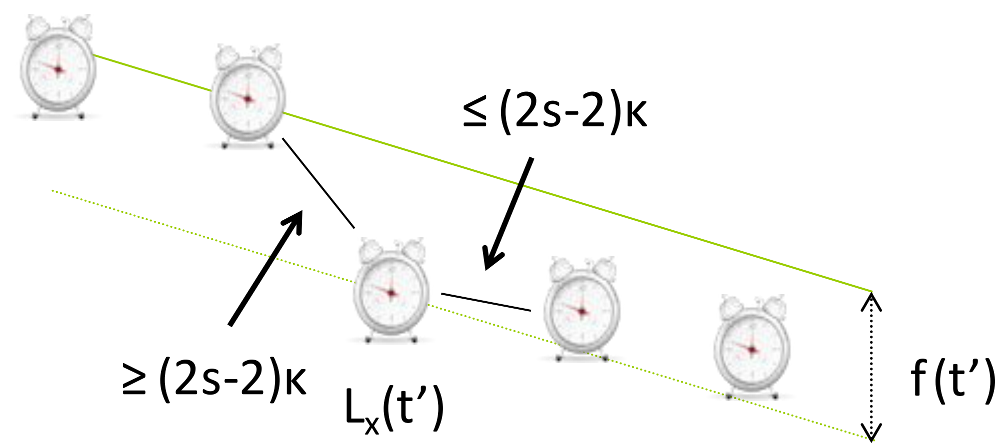

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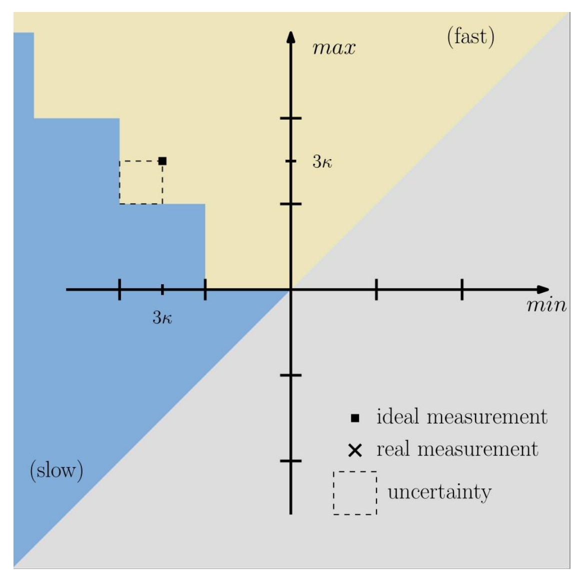
$\Rightarrow f(t') \leq 0$

$f(t) = \max_{x \in V}\{f_x(t)\}$

$\leq \max_{x \in V}\{L_w(t) - L_x(t)$

$- (2s - 3)\kappa\,\mathrm{dist}(x,w)\}$

$\leq \Psi^{s-1}(t)$



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Theorem:

Let G upper bound the global skew and $\Psi^1(0) = 0$. Then $\Psi^s(t) \leq G/\sigma^{s-1}$.

GCS Analysis: Proof of Local Skew Bound

$\Psi^1(t) \leq G(t) \leq G = G/\sigma^0$

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for $t' \leq G/(\mu\sigma^{s-2})$:

$\Psi_v^s(t') \leq \Psi^s(0) + \theta t' - (L_v(t') - L_v(0)) \leq (\theta - 1)t' \leq G/\sigma^{s-1}$

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for $t' > G/(\mu\sigma^{s-2})$ set $t = t' - G/(\mu\sigma^{s-2}) \leq t' - \Psi^{s-1}(t)/\mu$; thus

$$\begin{aligned}\Psi_v^s(t') &\leq \Psi_v^s(t) + \theta(t' - t) - (L_v(t') - L_v(t)) \\ &\leq \Psi_v^s(t) + \theta(t' - t) - (t' - t + \Psi_v^s(t)) \\ &= (\theta - 1)(t' - t) = G/\sigma^{s-1}\end{aligned}$$

GCS Analysis: Skew Bounds

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Theorem:

If $\Psi^1(0) = 0$, then $G \leq \sigma\kappa D/(\sigma - 1) = (1 + 1/(\sigma - 1))\kappa D$.

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=> for $\{v,w\} \in E$: $|L_v(t) - L_w(t)| \leq 2\kappa(\log_\sigma(G/\kappa) + 1)$

Theorem:

If $\Psi^1(0) = 0$, then $G \leq \sigma\kappa D/(\sigma - 1) = (1 + 1/(\sigma - 1))\kappa D$.

for t' just after skew G has been exceeded:

$\Psi_v^1(t') \leq (\theta - 1)(t' - \max\{0, t' - G/\mu\}) \leq G/\sigma \leq \kappa D/(\sigma - 1)$

GCS Analysis: Skew Bounds

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