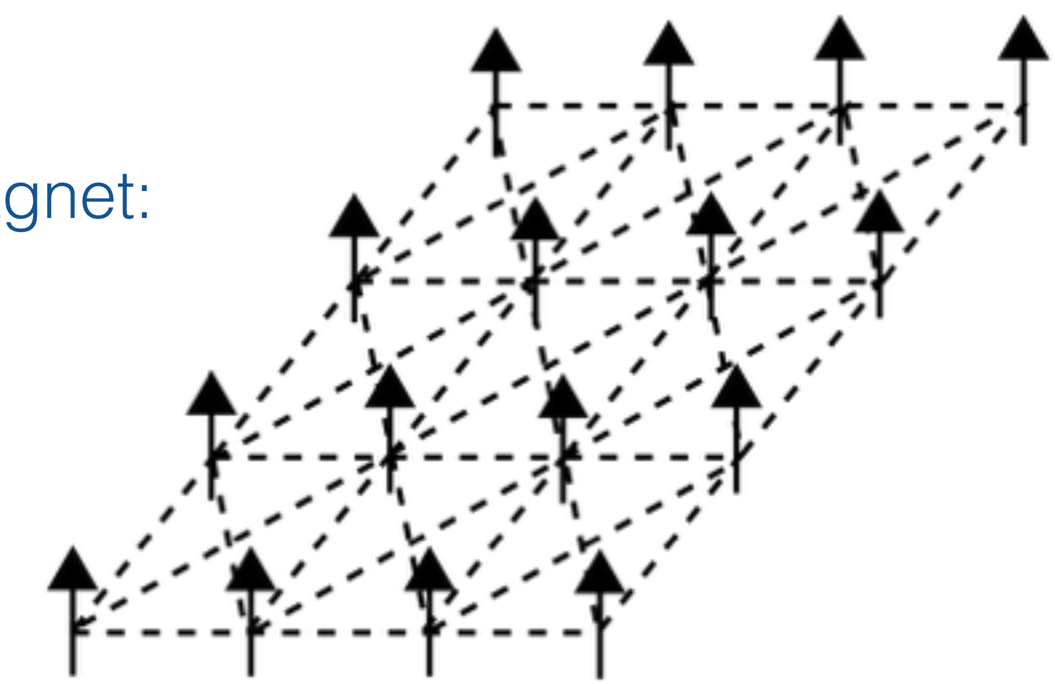
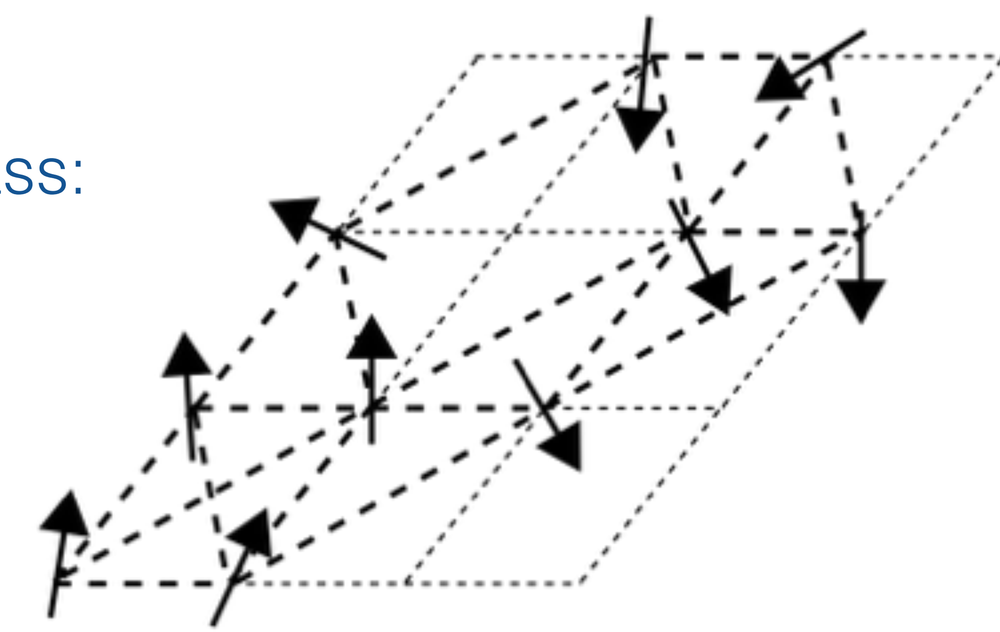
ferromagnet:



spin glass:



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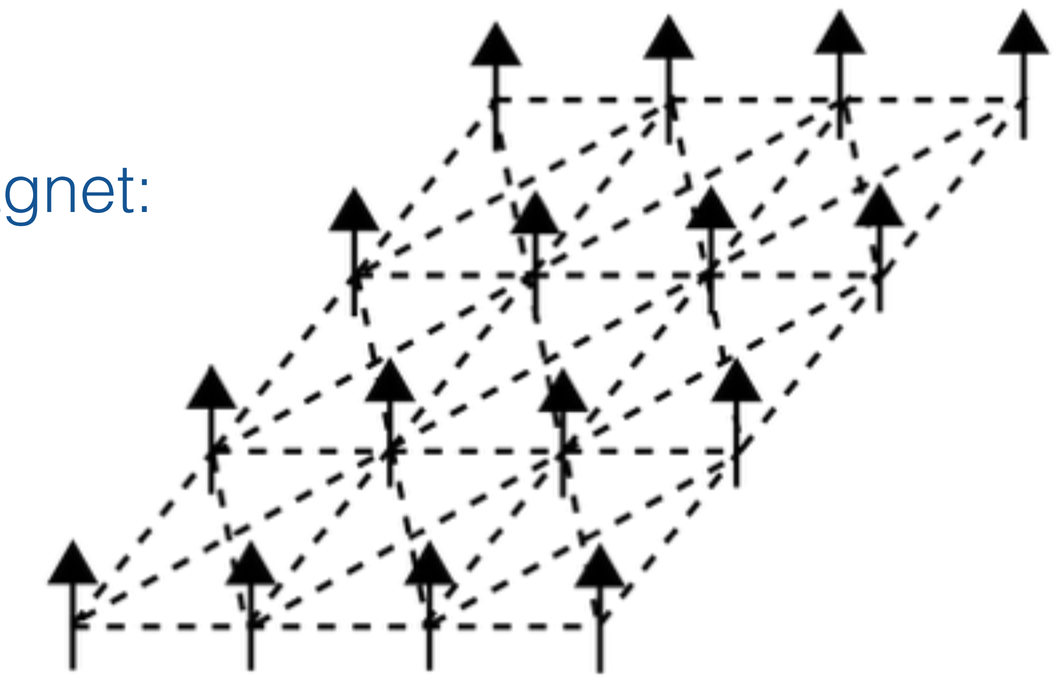

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- vertices = particles
- edges = interactions
- each vertex can be in one of q states/spins
  (q depends on the mathematical model)

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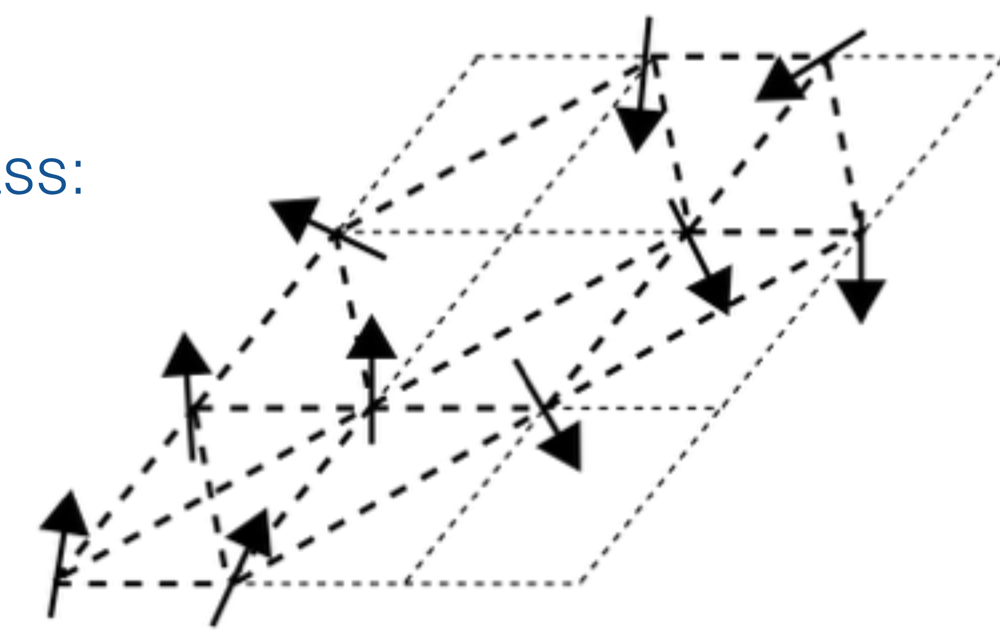

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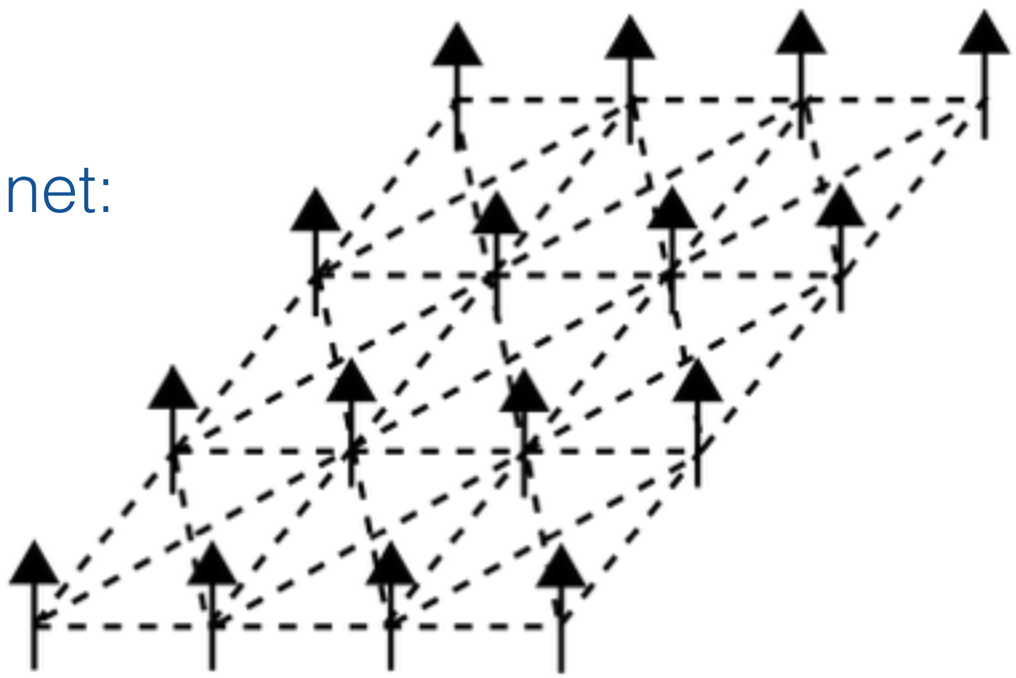

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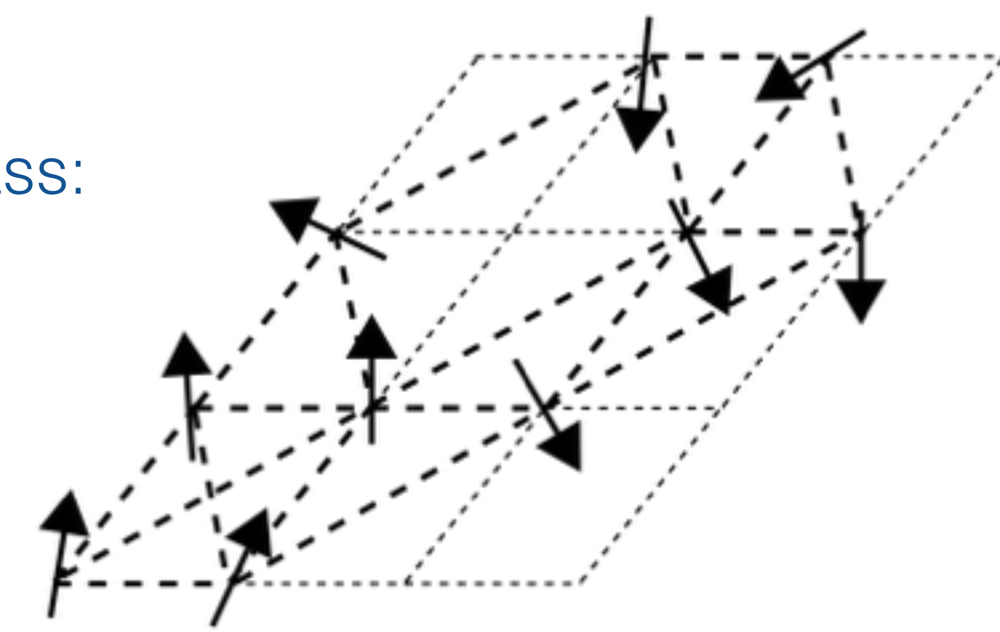

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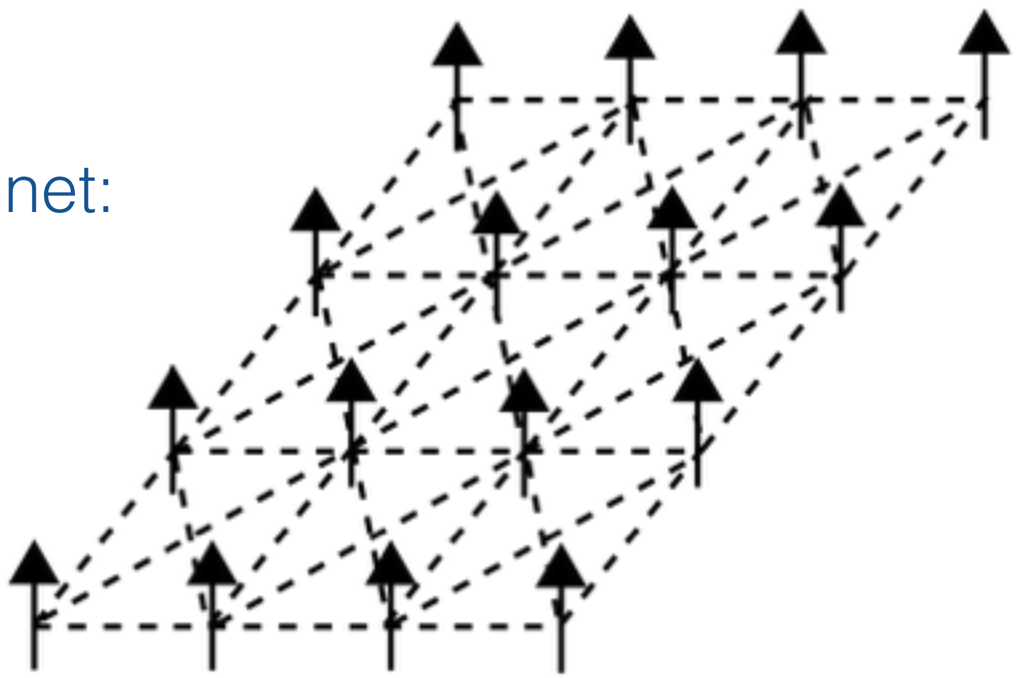

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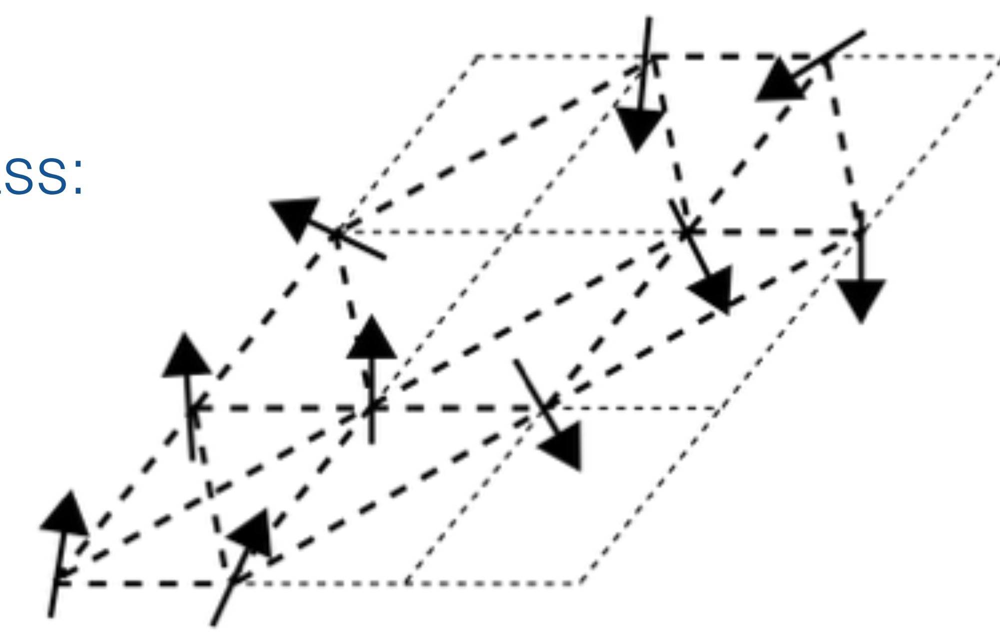

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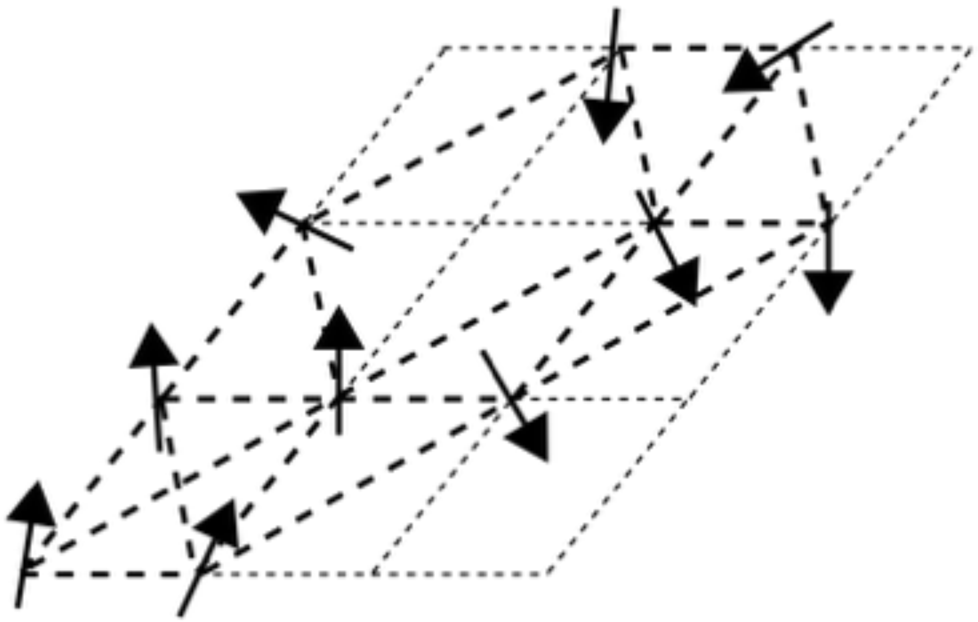

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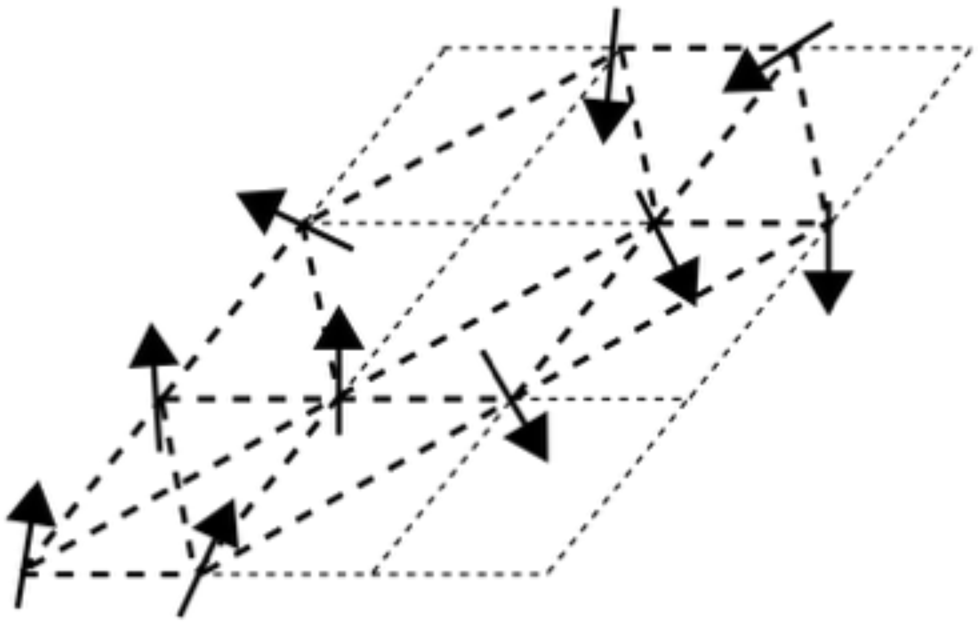

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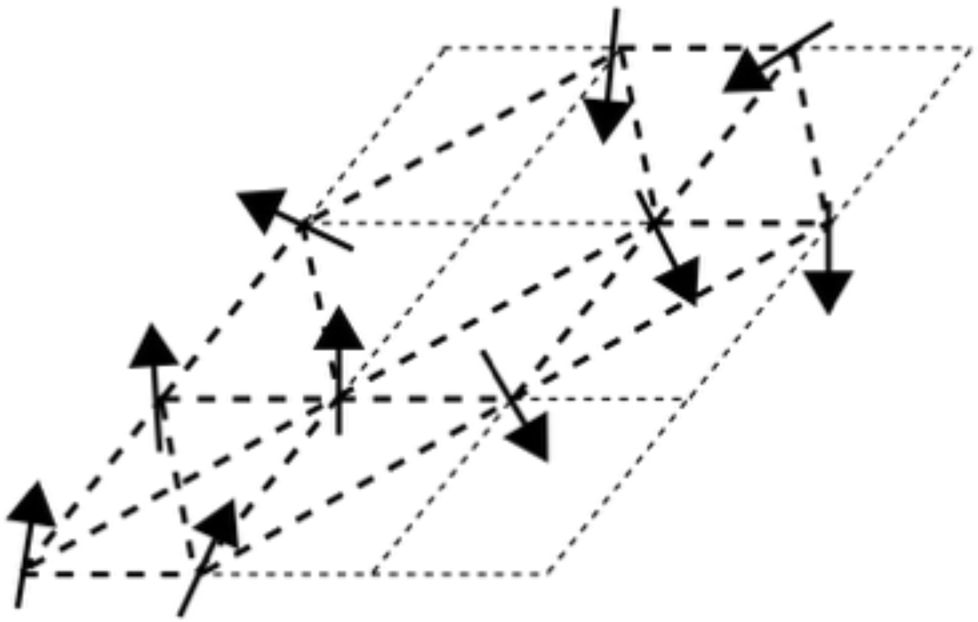

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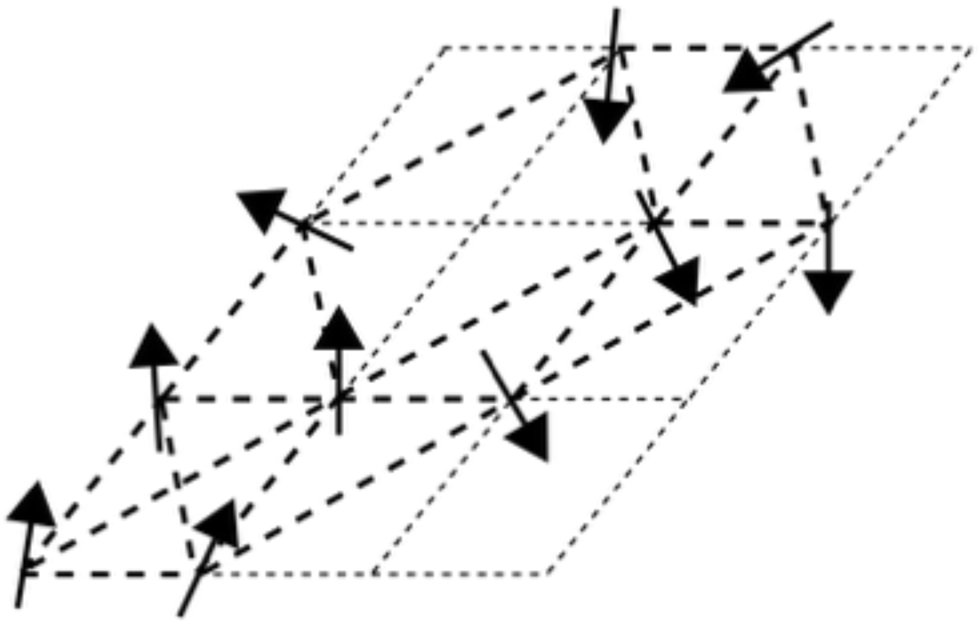
Probability that spin system is in configuration σ

$$p(\sigma) \propto e^{-H(\sigma)/cT}$$

(Gibbs distribution)



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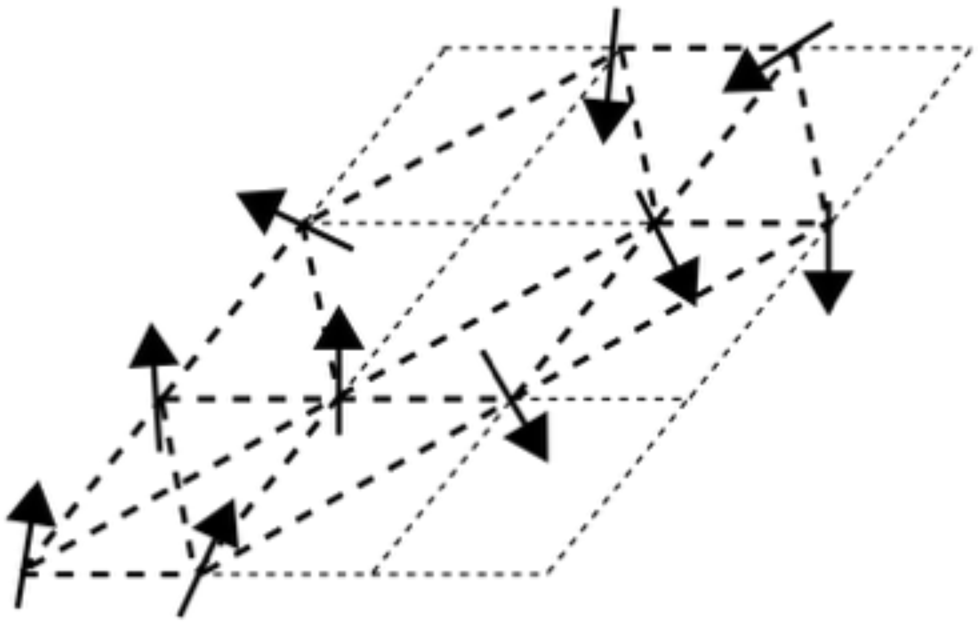

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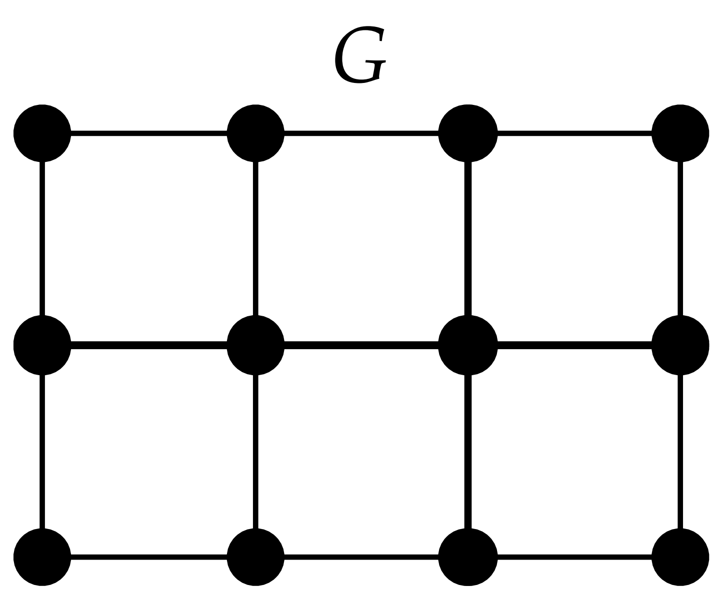
Example:
hard-core gas model



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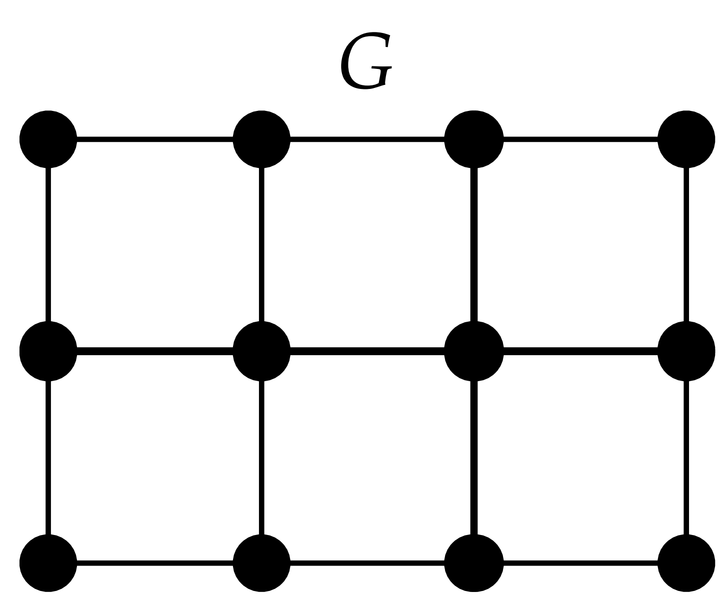

2 states:
-unoccupied
-occupied

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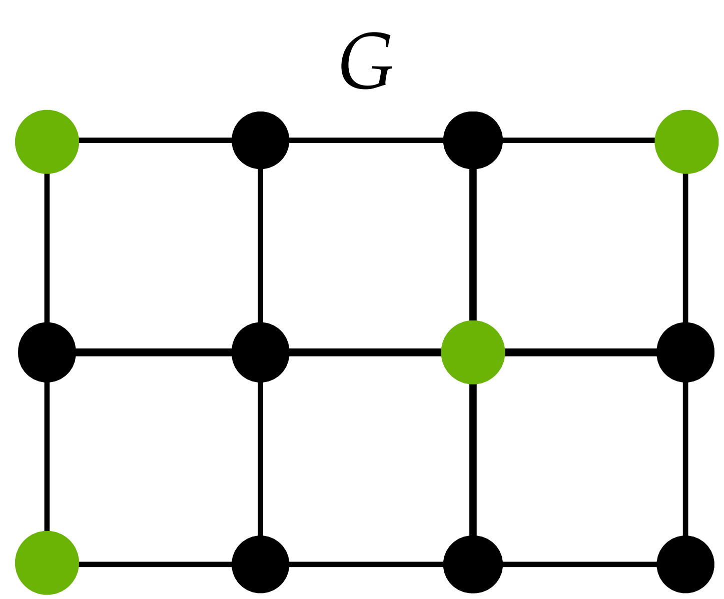

no adjacent occupied sites!

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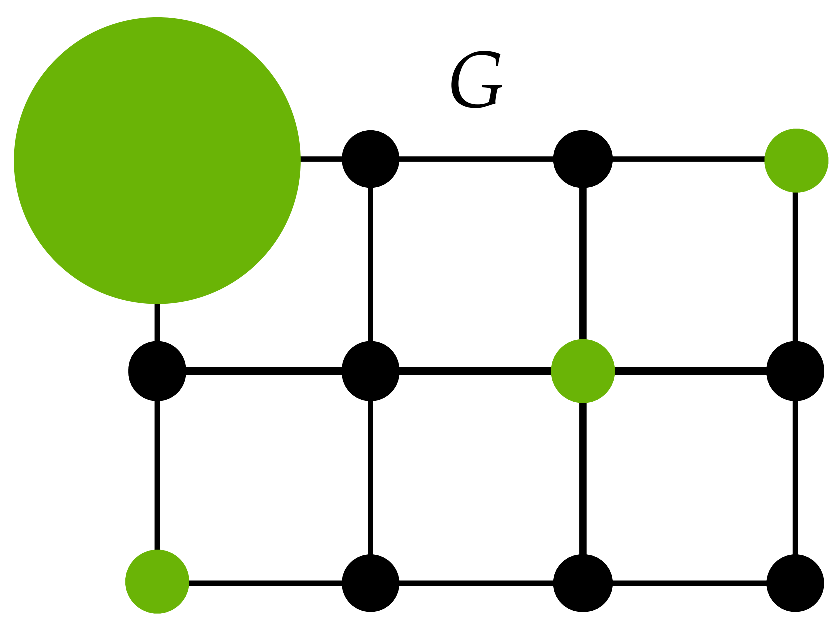

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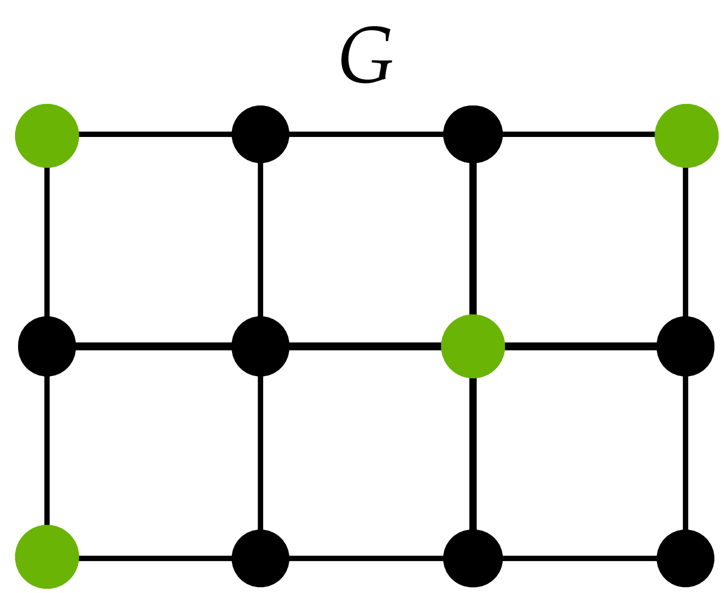
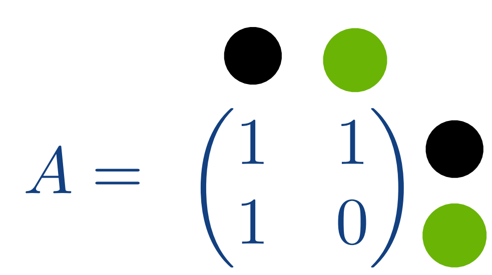


$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$



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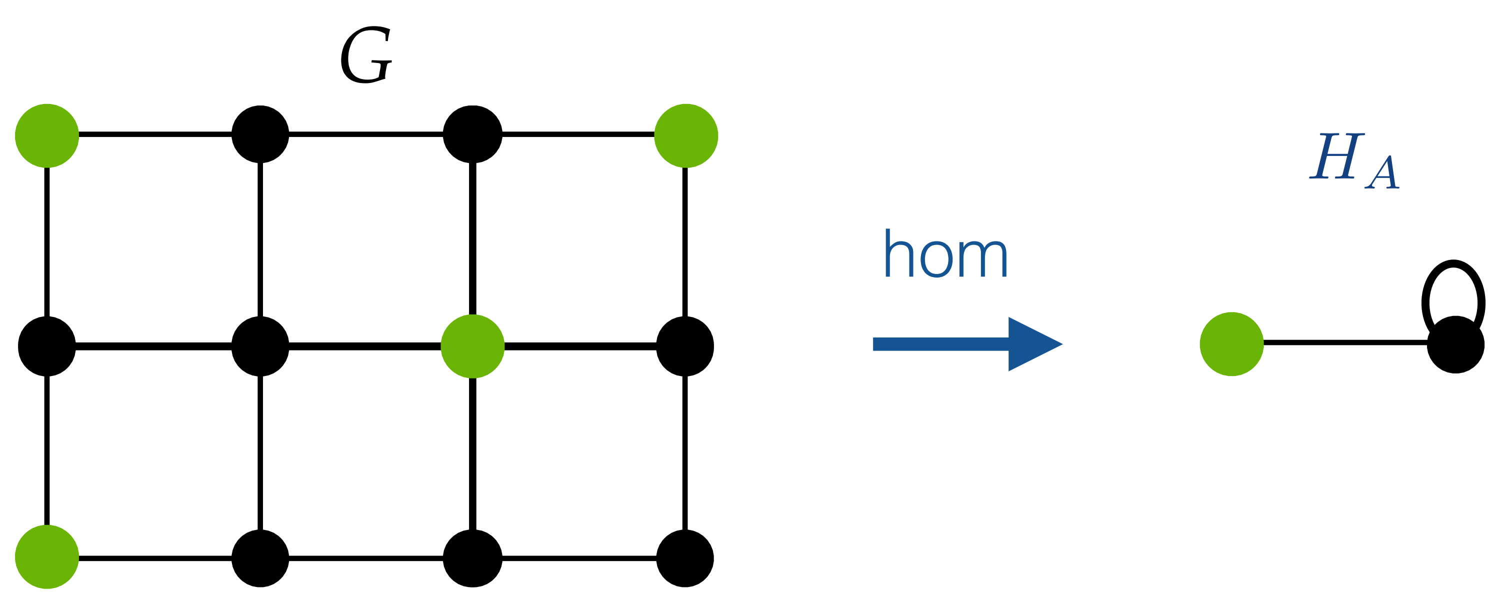

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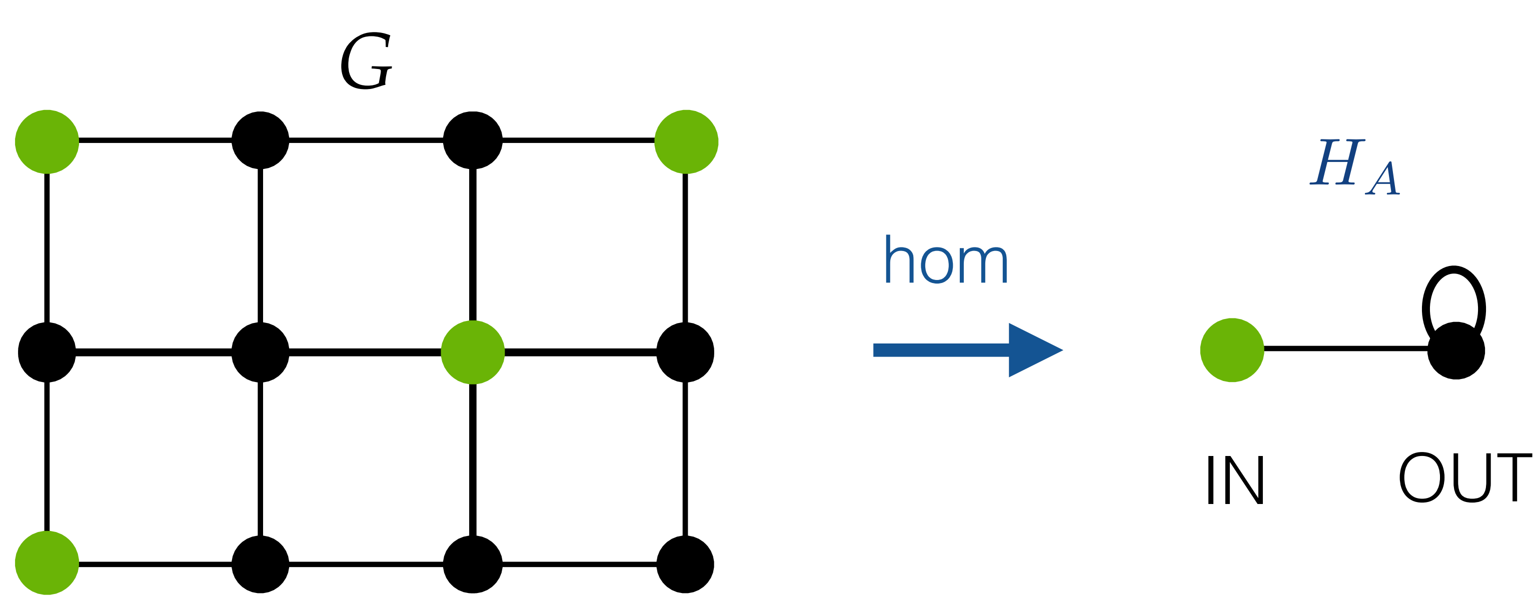

Let H be a fixed graph.

#Hom(H)

Input: Graph G without loops.
Output: Number of Homomorphisms from G to H.

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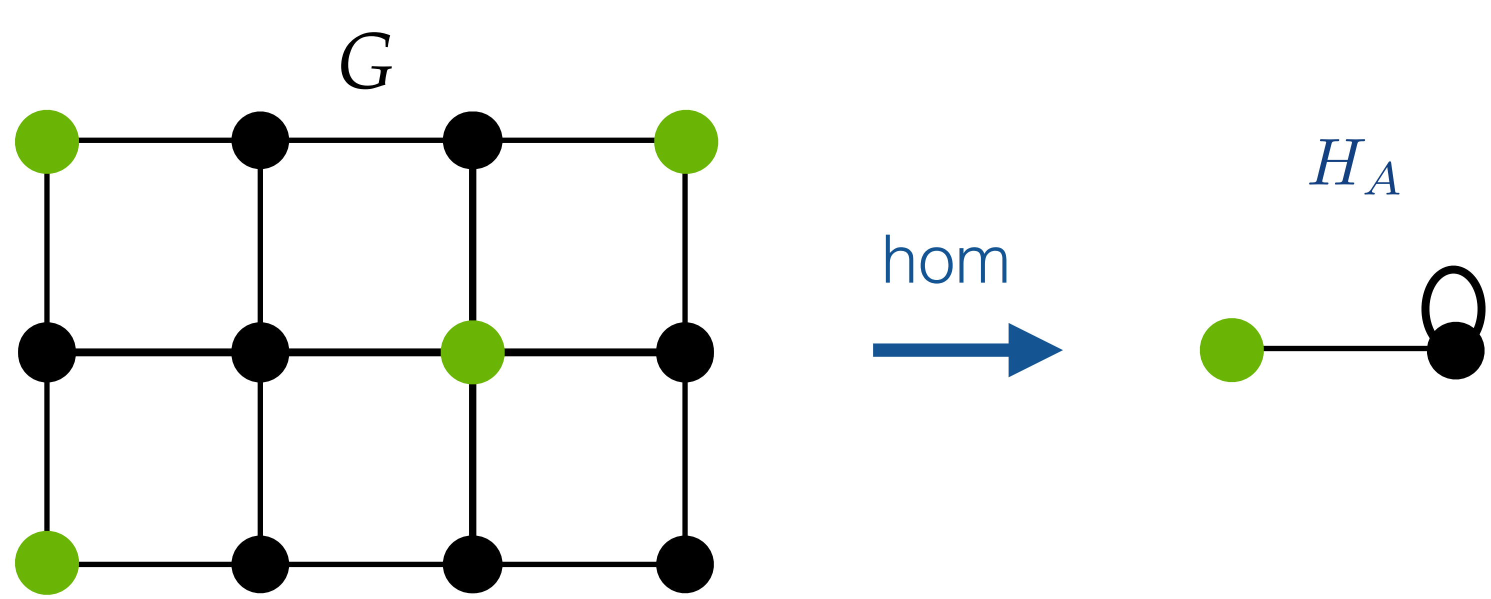

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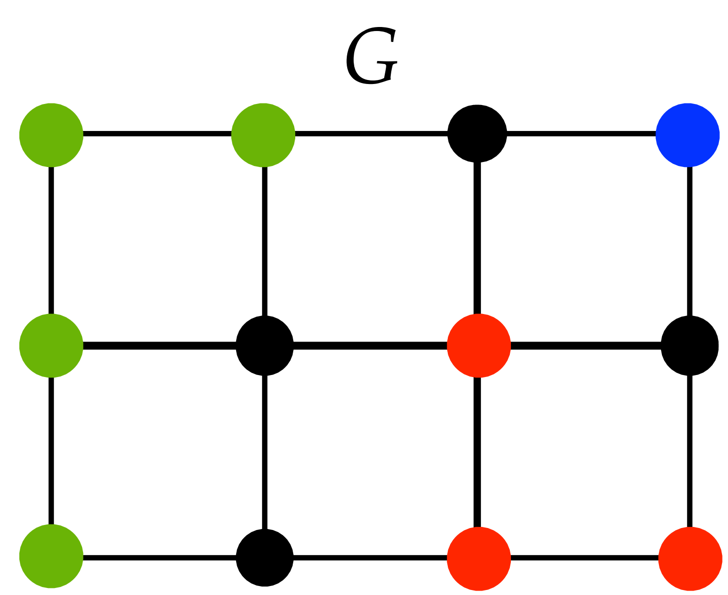
3-particle Widom-Rowlinson gas model



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