Dániel Marx and Roohani Sharma Summer 2023

# Parameterized Algorithms, Exercise Sheet 1

cms.cispa.saarland/paramalg_23

Total Points: 50 Due: Tuesday, **April 25**, 2023

*You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, **using your own words**. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Barış (*baris-can.esmer@cispa.de*).*

## Exercise 1 — **10** points

Given an instance $(x, k)$, a branching algorithm produces three instances $(x_1, k-2)$, $(x_2, k-2)$, and $(x_3, k-2)$ in polynomial time and recursively solves them. What bound can we give on the running time of the algorithm?

## Exercise 2 — **5 + 5** points

In the 5-Bounded-Degree Deletion problem, we are given an undirected graph $G$ and a positive integer $k$, and the task is to find at most $k$ vertices whose removal decreases the maximum vertex degree of the graph to at most 5.

a) Use the method of bounded depth search trees to show that the problem is FPT parameterized by $k$.

b) Show that 5-Bounded-Degree Deletion admits a polynomial kernel.

## Exercise 3 — **5 + 5** points

For a graph $G$, the *diameter* $\mathrm{diam}(G)$ is defined as the maximum distance between any two vertices in the graph; that is, $\mathrm{diam}(G) := \max_{x,y \in V(G)} \mathrm{dist}(x, y)$. Are the following graph properties closed under taking induced subgraphs? Prove or give counterexamples.

a) $G \in \mathcal{P}_a :\Longleftrightarrow$ The diameter of $G$ is at *most* 3, that is, $\mathrm{diam}(G) \leq 3$.

b) $G \in \mathcal{P}_b :\Longleftrightarrow$ The diameter of $G$ is at *least* 3, that is, $\mathrm{diam}(G) \geq 3$.

## Exercise 4 — **5 + 5** points

Consider a parameterized problem $P$ where the input is a graph $G$ together with two integers $k$ and $d$, and we know that $d$ is always *at most* $k^2$. Which of the following statements are true? Justify your answer.

a) If $P$ is FPT parameterized by $k$, then $P$ is also FPT parameterized by $d$.

b) If $P$ is FPT parameterized by $d$, then $P$ is also FPT parameterized by $k$.

## Exercise 5 — **5+5** points

Consider the Closest String problem, as defined on Slide 5 of Lecture 2.

a) Show that the problem is FPT with combined parameters $|\Sigma|$ and $L$.

b) Show that the problem is polynomial-time solvable for $k = 2$.