Dániel Marx and Roohani Sharma Summer 2023

Parameterized Algorithms, Exercise Sheet 3

cms.cispa.saarland/paramalg_23

Total Points: 50 Due: Tuesday, **May 23**, 2023

*You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, **using your own words**. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Barış (*baris-can.esmer@cispa.de*).*

Exercise 1 — **10** points

Let Π be a parameterized problem that admits an FPT algorithm with running time $8^k \cdot n^{O(1)}$, where k is the parameter and n is the size of the input instance. Show that Π admits a kernel of size 2^k.

Exercise 2 — **5+5** points

a) In the MAX 3-SAT problem, the input is a 3-CNF formula ψ and a positive integer k. The goal is to determine if there exists a (boolean) assignment that satisfies at least k clauses of ψ. Show that MAX 3-SAT admits a linear kernel when the parameter is k.

b) A graph is *planar* if it can be drawn on a plane such that the edges do not cross each other. The *Four Coloring Theorem* says that every planar graph is 4-colorable, that is one can color the vertices of any planar graph with colors from the set $\{1, 2, 3, 4\}$ such that no two adjacent vertices get the same color.

In the PLANAR INDEPENDENT SET problem the input is a planar graph G and a positive integer k. The goal is to determine if G has an independent set of size at least k. Using the Four Coloring Theorem, show that PLANAR INDEPENDENT SET admits a linear vertex kernel when the parameter is k.

Exercise 3 — **10** points

In the POINT LINE COVER problem, we are given n points on the plane, and the goal is to cover these points with at most k lines. A line covers a point if the point lies on the line. Design a polynomial kernel for the POINT LINE COVER problem parameterized by k.

Make sure to show that all the reduction rules in your proposed kernelization algorithm are safe and can be applied in polynomial time.

Exercise 4 — **10** points

In the DELETION TO DEGREE-d problem, the input is an undirected graph G and an integer k. The goal is to determine if there exists a set of vertices, say S, of G such that $|S| \leq k$ and the maximum degree of $G - S$ is at most d (that is each vertex in $V(G) \setminus S$ has at most d neighbours in $G - S$). Note that when $d = 0$, the problem is the same as VERTEX COVER.

Design a kernel for DELETION TO DEGREE-d whose size is polynomial in k and d.

Exercise 5 — **5+5** points

In the k-PATH problem *parameterized by vertex cover*, we are given as input an undirected graph G, a vertex cover C of G and a positive integer k. The goal is to determine if G has a path of length k. The parameter is $|C|$.

a) Design a kernel for the above problem with $|C|^{O(1)}$ vertices without using the Expansion Lemma.

b) Improve the size of the above kernel to $O(|C|^2)$ vertices using the Expansion Lemma.