Learning-augmented Mechanism Design

Jakob Barkalaia

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1. Preliminaries

2. Minimizing egalitarian social cost

3. Minimizing utilitarian social cost

"Learning-Augmented Mechanism Design: Leveraging Predictions for Facility Location", 2022
By Priyank Agrawal, Eric Balkanski , Vasilis Gkatzelis, Tingting Oua , and Xizhi Tan (Columbia University / Drexel University)

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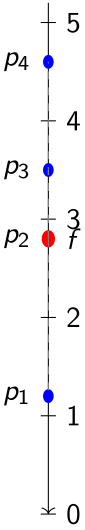
Warmup: Facility Location on the Line

- Given: n preferences $P = (p_1, p_2, \dots, p_n) \subset \mathbb{R}$
- Choose: Facility location $f \in \mathbb{R}$
- Constraints: Agent i incurs cost $|f - p_i|$
- Objective: Minimize a social cost function $C(f, P)$



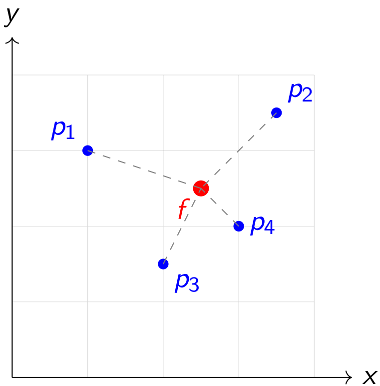
Definition (Egalitarian /Utilitarian cost 1D)

Egalitarian: $C^e := \max_{p \in P} |f - p|$

Utilitarian: $C^u := \frac{1}{n} \sum_{p \in P} |f - p|$

The problem

- Given: n preferences of agents $P = (p_1, p_2, \ldots p_n) \subset \mathbb{R}^2$
- Choose: Facility location $f \in \mathbb{R}^2$
- Constraints: Each agent suffers cost $d(f, p_i)$ (Euclidean distance)
- Objective: Minimize social cost function $C : (f, P) \to \mathbb{R}$



Definition

Egalitarian Cost $C^e := \max_{p \in P} d(f, p)$

Utilitarian Cost $C^u := \sum_{p \in P} \frac{d(f,p)}{n}$

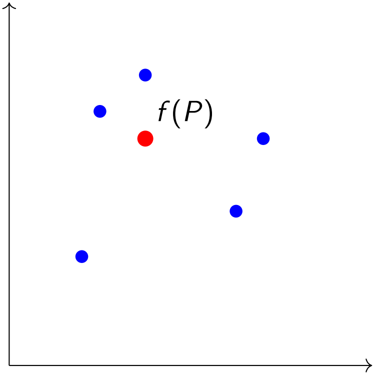
Agents may be strategical

Definition (Strategyproofness)

Mechanism $f : \mathbb{R}^{2n} \to \mathbb{R}^2$ is **strategyproof** iff for all instances $P \in \mathbb{R}^{2n}$, all $i \in [n], p'_i \in \mathbb{R}^2$ it is the case that $d(p_i, f(P)) \leq d(p_i, f(P_{-i}, p'_i))$

In other words: It is the dominant strategy for every player to truthfully report preferences

Coordinatewise Median Mechanism (CM)

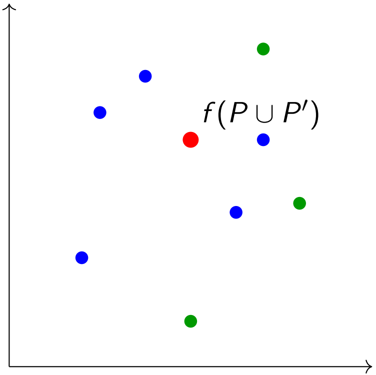


Definition (Coordinatewise Median Mechanism)

Given: Preferences $P = \{(x_1, y_1), \dots, (x_n, y_n)\} \subset \mathbb{R}^2$

Output: $\rightsquigarrow f(P) := \big(\text{Median}(x_1, \dots, x_n),\ \text{Median}(y_1, \dots, y_n)\big)$

Generalized Coordinatewise Median (GCM)



Definition (Generalized CM)

Given: Preferences $P = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ and multiset $P' \subset \mathbb{R}^2$

Output: $\rightsquigarrow f(P) := \mathrm{CM}(P \cup P')$

Median to the rescue

Theorem

The **GCM** *mechanism is deterministic, strategyproof and anonymous (= invariance under permutations of the agents).*

Learning-Augmented Mechanism Design

Now: We are given prediction $\hat{o}$ of optimal facility location $o(P)$

Definition (Consistency and Robustness)

Let C be some social cost function and f some mechanism.
f is α-**consistent**, if an α-approximation is achieved for $\hat{o} = o(P)$:

$$\max_{P} \left\{ \frac{C(f(P, \hat{o} = o(P)), P)}{C(o(P), P)} \right\} \leq \alpha$$

f is β-**robust**, if a β-approximation is achieved for any prediction $\hat{o}$:

$$\max_{P, \hat{o}} \left\{ \frac{C(f(P, \hat{o}), P)}{C(o(P), P)} \right\} \leq \beta$$

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Recap of 1-dimensional case

Definition (MinMaxP mechanism)

Input: $P = (p_1, \ldots, p_n) \in \mathbb{R}^n$, prediction $\hat{o} \in \mathbb{R}$, let
$p_{min} := \min_{p \in P}(p), p_{max} := p_{min} := \max p \in P(p)$

$$\rightsquigarrow f(P, \hat{o}) = \begin{cases} \hat{o} & \text{if } \hat{o} \in [p_{min}, p_{max}] \\ p_{min} & \text{if } \hat{o} < p_{min} \\ p_{max} & \text{if } \hat{o} > p_{max} \end{cases}$$

Equivalently:

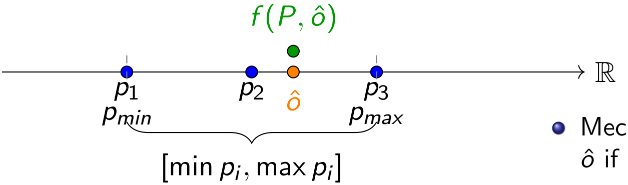
$$f(P, \hat{o}) = CM(P \cup P')$$

where P' contains $n - 1$ copies of $\hat{o}$

The mechanism is strategyproof, 1-consistent and 2-robust as we have seen

Illustration of **MinMaxP** Mechanism (1D)

$$f(P, \hat{o}) = \begin{cases} \hat{o} & \text{if } \hat{o} \in [p_{min}, p_{max}] \\ p_{min} & \text{if } \hat{o} < p_{min} \\ p_{max} & \text{if } \hat{o} > p_{max} \end{cases}$$

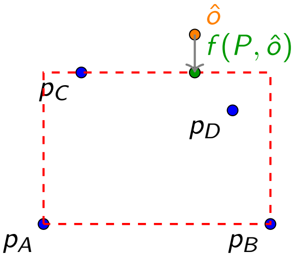


- Mechanism outputs prediction $\hat{o}$ if it's within the agents' range
- Otherwise, clips to nearest endpoint of the interval

2-dimensional case: **Minimum Bounding Box** Mechanism

Definition (Min Bounding Box Mechanism)

$$
\begin{aligned}
\text{Input:}\quad & P = \{(x_1, y_1), \ldots, (x_n, y_n)\} \\
\text{Prediction:}\quad & \hat{o} = (\hat{x}, \hat{y}) \\
\text{Output:}\quad & f(P, \hat{o}) = \text{MinMaxP}(x_1, \ldots, x_n, \hat{x}), \\
& \text{MinMaxP}(y_1, \ldots, y_n, \hat{y})
\end{aligned}
$$



2-dimensional case

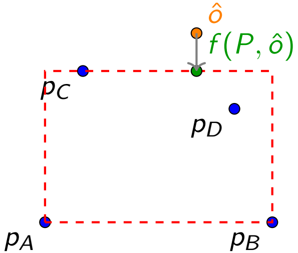
Definition (Minimum Bounding Box mechanism)

Input: $P = ((x_1, y_1), \dots, (x_n, y_n)) \in \mathbb{R}^{2n}$, prediction $\hat{o} = (\hat{x}, \hat{y}) \in \mathbb{R}^2$

$\rightsquigarrow f(P, \hat{o}) = (MinMaxP((x_1, \dots, x_n), \hat{x}), MinMaxP((y_1, \dots, y_n), \hat{y}))$

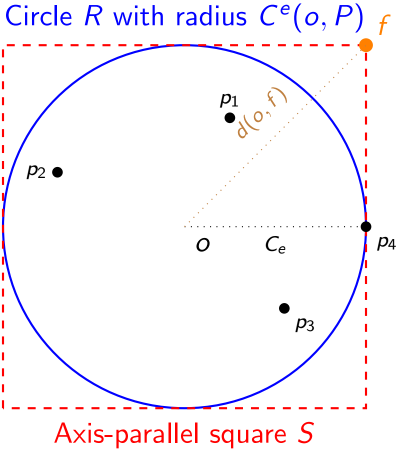
Theorem

The **Minimum Bounding Box** *mechanism is strategyproof,* 1*-consistent and* $1 + \sqrt{2}$ *robust for the egalitarian objective.*



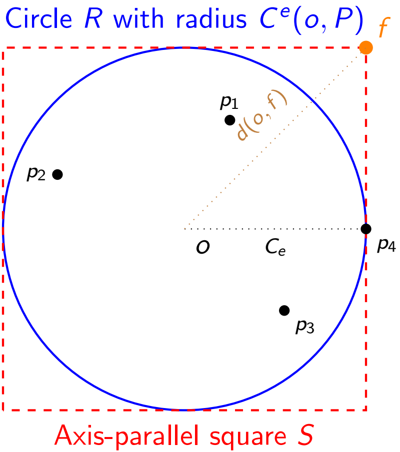
Proof $(1+\sqrt{2})$-Robustness

- R contains all points from P $\implies$ S contains all points from P



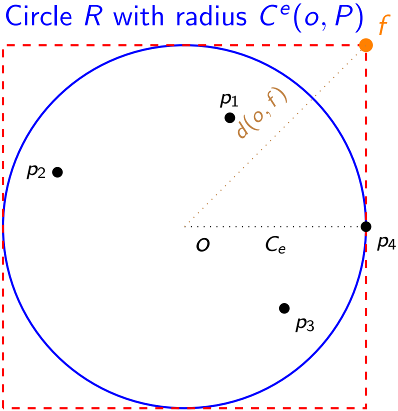
Proof $(1+\sqrt{2})$-Robustness

- R contains all points from P $\implies$ S contains all points from P
- $\implies$ All bounds for S also hold for the minimum axis-parallel bounding box



Proof $(1+\sqrt{2})$-Robustness

- R contains all points from P $\implies$ S contains all points from P
- $\implies$ All bounds for S also hold for the minimum axis-parallel bounding box
- $d(o, f) \leq \sqrt{2}C^e(o, p)$



Theorem

There is no deterministic, strategyproof, and anonymous mechanism that is $(2-\varepsilon)$*-consistent and* $(1+\sqrt{2}-\varepsilon)$ *robust with respect to the egalitarian objective for any* $\varepsilon > 0$.

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Lemma (Peters et al. (2013))

Any deterministic, strategyproof, anonymous, unanimous mechanism solving the problem is equivalent to **GCM** *with $n-1$ phantom points.*

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Lemma (Peters et al. (2013))

Any deterministic, strategyproof, anonymous, unanimous mechanism solving the problem is equivalent to **GCM** *with $n-1$ phantom points.*

- 1. For consistency better than 2, we have to place all $n-1$ phantom points on $\hat{o}$
- 2. If we place all $n-1$ phantom points on $\hat{o}$, robustness is at least $1+\sqrt{2}$

We have to place all phantoms on the prediction

- Assume atleast one of the $n-1$ phantoms q' is not on $\hat{o}$. Wlog $y_{q'} < y_{\hat{o}}$

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- Choose $\bar{y} = \max_{q \in P : y_q < y_{\hat{o}}} y_q$ and $\varepsilon = y_{q'} - \bar{y}$

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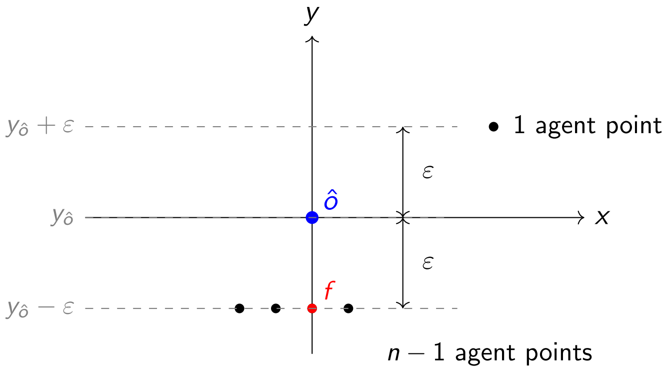
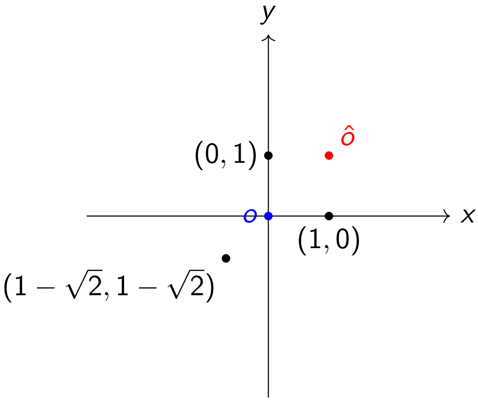
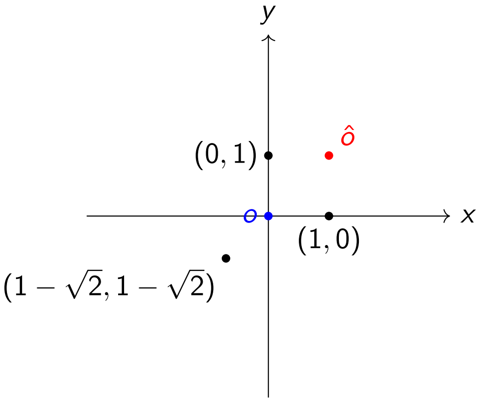


With n-1 phantom points on prediction , robustness at least $1+\sqrt{2}$



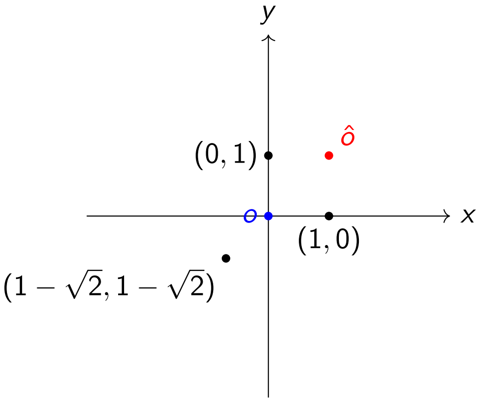
- optimum at o with cost 1

With n-1 phantom points on prediction , robustness at least $1+\sqrt{2}$



- optimum at o with cost 1
- We have 2 phantom points at $\hat{o}$, hence 3 points with $x=1$, 3 point with $y=1$

With n-1 phantom points on prediction , robustness at least $1 + \sqrt{2}$



- optimum at o with cost 1
- We have 2 phantom points at $\hat{o}$, hence 3 points with $x = 1$, 3 point with $y = 1$
- f is placed at $(1, 1)$ and cost is $1 + \sqrt{2}$

2-dimensional case

Definition (Prediction Error)

Let P be an instance, $\hat{o}$ a prediction and $o(P)$ the optimal facility location. Define the prediction error

$$\eta(\hat{o}, P) := \frac{d(\hat{o}, o(P))}{C(o(P), P)}$$

as the distance to the optimal location $o(P)$ normalized by the optimal social cost

Smoothness

Lemma

Let P be an instance of the problem. For any two predictions $\hat{o}$ and $\tilde{o}$, the returned facility locations $f(P, \hat{o})$ and $f(P, \tilde{o})$ by the **Minimum Bounding Box** *mechanism satisfy*

$$d(f(P, \hat{o}), f(P, \tilde{o})) \leq d(\hat{o}, \tilde{o})$$

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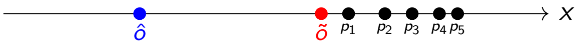
Case 1:



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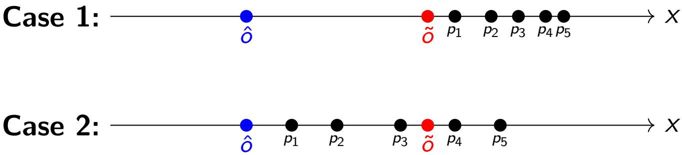

Theorem

The **Minimum Bounding Box** *mechanism achieves a* $\min\{1+\eta, 1+\sqrt{2}\}$ *approximation*

Proof.

- By definition $d(\hat{o}, o) = \eta C^e(o, P)$
- By previous lemma $d(f(P, \hat{o}), f(P, o)) \leq d(\hat{o}, o), \eta C^e(o, P)$

$\square$

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$$C^e(f(P, \hat{o}), P) = \max_{i \in [n]} d(p_i, f(P, \hat{o}))$$

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$$
\begin{aligned}
C^e(f(P,\hat{o}), P) &= \max_{i \in [n]} d(p_i, f(P,\hat{o})) \\
&\leq \max_{i \in [n]} [d(p_i, f(P,o)) + d(f(P,o), f(P,\hat{o}))]
\end{aligned}
$$

□

Theorem

The **Minimum Bounding Box** *mechanism achieves a* $\min\{1+\eta, 1+\sqrt{2}\}$ *approximation*

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$$\begin{aligned} C^e(f(P,\hat{o}), P) &= \max_{i\in[n]} d(p_i, f(P,\hat{o})) \\ &\leq \max_{i\in[n]} [d(p_i, f(P,o)) + d(f(P,o), f(P,\hat{o}))] \\ &\leq \max_{i\in[n]} [d(p_i, o) + \eta \cdot C^e(f(P,o), P)] \end{aligned}$$

$\square$

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C^e(f(P,\hat{o}), P) &= \max_{i\in[n]} d(p_i, f(P,\hat{o})) \\
&\leq \max_{i\in[n]} [d(p_i, f(P,o)) + d(f(P,o), f(P,\hat{o}))] \\
&\leq \max_{i\in[n]} [d(p_i, o) + \eta \cdot C^e(f(P,o), P)] \\
&\leq (1+\eta) \cdot C^e(f(P,o), P)
\end{aligned}$$

$\square$

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The plan

- 1-Dimensional case: Median is optimal and strategyproof
- 2-Dimensional case: CM mechanism is a $\sqrt{2}$-approximation and no deterministic, strategyproof and anonymous algorithm can provide a better approximation [Meir 2019]

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- 1-Dimensional case: Median is optimal and strategyproof
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What can we achieve if we add predictions into the mix?

The plan

- 1-Dimensional case: Median is optimal and strategyproof
- 2-Dimensional case: CM mechanism is a $\sqrt{2}$-approximation and no deterministic, strategyproof and anonymous algorithm can provide a better approximation [Meir 2019]

What can we achieve if we add predictions into the mix?

$$\implies \sqrt{2c^2+2}/(1+c)\text{-consistency and } \sqrt{2c^2+2}/(1-c)\text{-robustness}$$

where $c \in [0,1)$

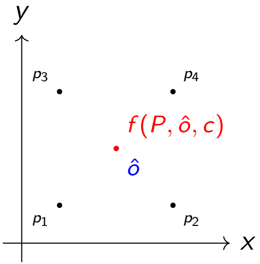
The Mechanism

Definition (Coordinate Median with Predictions (CMP) mechanism)

Input: Locations $P \in \mathbb{R}^{2n}$, prediction $\hat{o} \in \mathbb{R}^2$, confidence value $c \in [0, 1)$

$$\rightsquigarrow f(P, \hat{o}, c) = CM(P \cup P')$$

where P' contains cn copies of $\hat{o}$



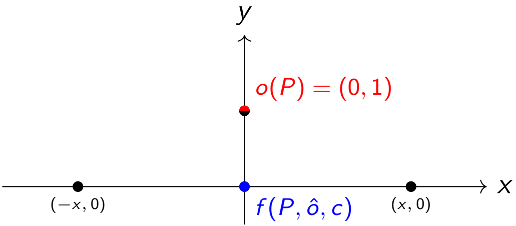
The Mechanism

Definition (Clusters-and-Opt-on-Axes Instances)

Let $c \in [0,1)$. Define $\mathcal{P}$ to be the class of all instances $(P, \hat{o})$ such that

- $f(P, \hat{o}, c) = (0,0)$
- $o(P) = (0,1)$
- $\forall p \in P : p \in \{(0,1), (x,0), (-x,0)\}$ for some $x \in \mathbb{R}_{\geq 0}$

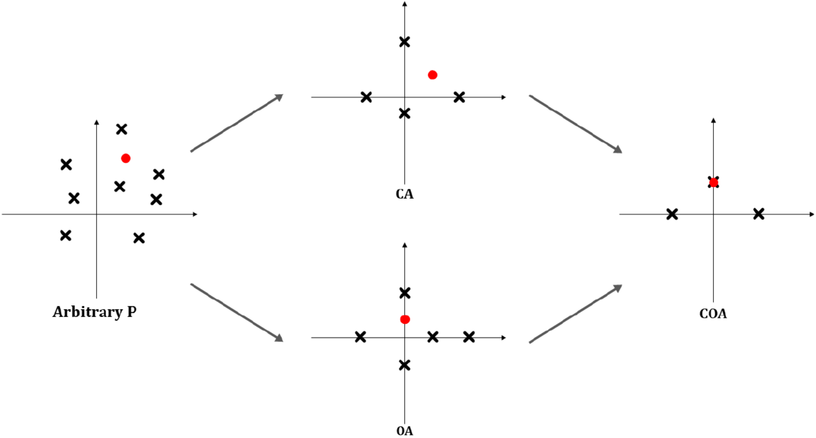
Let $\mathcal{P}^C_{coa}(c) \subset \mathcal{P}$ where $\hat{o} = o(P)$ and $\mathcal{P}^R_{coa} \subset \mathcal{P}$ where $\hat{o} = (0,0)$



Lemma (COA instances always contain a worst-case instance)

Let $r(P, \hat{o}, c)$ be the achieved approx-ratio. For any $c \in [0, 1)$, the CMP-mechanism is α-consistent and β-robust where

$\alpha = \max_{P \in \mathcal{P}^C_{coa}} r(P, \hat{o} = o(P), c)$ *and* $\beta = \max_{P \in \mathcal{P}^R_{coa}} r(P, \hat{o} = (0, 0), c)$

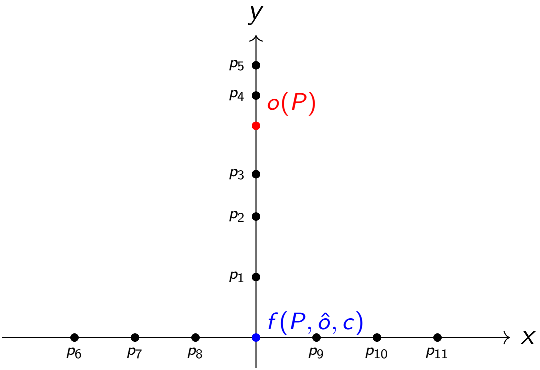


Definition (Optimal-on-Axes family)

For some c, $\hat{o}$, define $\mathcal{P}_{oa}$ be the family of multisets P such that

- $f(P, \hat{o}, c) = (0, 0)$
- $x_o(P) = 0$, $y_o(P) > 0$
- $\forall p \in P : p \in A_x \cup A_y$

Let $\mathcal{P}_{oa}^C(c)$ be the family when $\hat{o} = o(P)$ and $\mathcal{P}_{oa}^R(c)$ when $\hat{o} = (0, 0)$

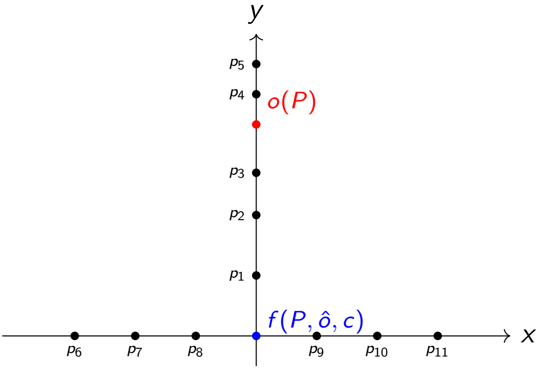


Lemma (Convert OA instance to COA or make it strictly worse)

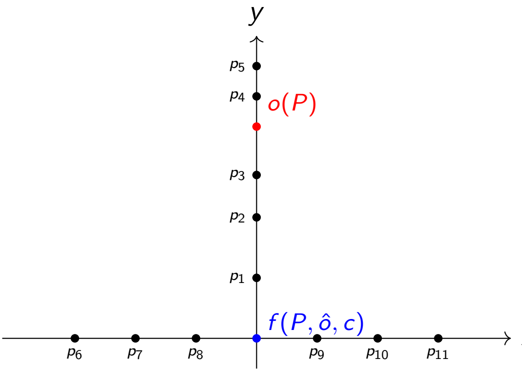
For any $c \in [0, 1)$, $P \in \mathcal{P}_{oa}^{C}$

- *There is some Q such that $r(Q, o(Q), c) > r(P, o(P), c)$*
- *OR there is some $Q \in \mathcal{P}_{coa}^{C}$ such that $r(Q, o(Q), c) \geq r(P, o(P), c)$*

(This holds analogously for the robustness case)

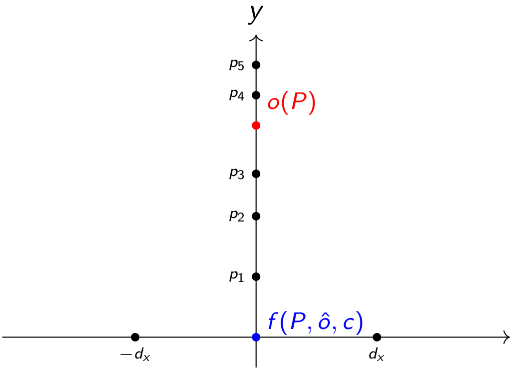


OA to COA



OA to COA

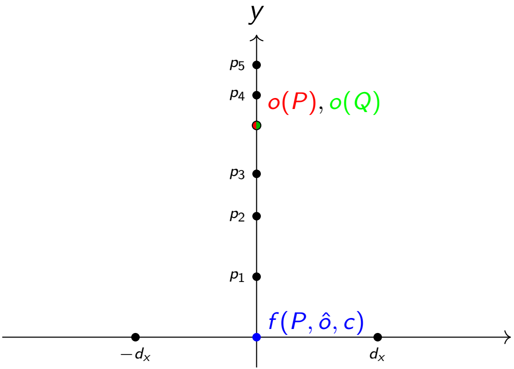
Step 1: Move x-Points to $d_x, -d_x$ where $d_x :=$ average distance to f.



OA to COA

Step 1: Move x-Points to $d_x, -d_x$ where $d_x :=$ average distance to f.

Step 2: In the new instance Q we have $o(P) = o(Q)$.



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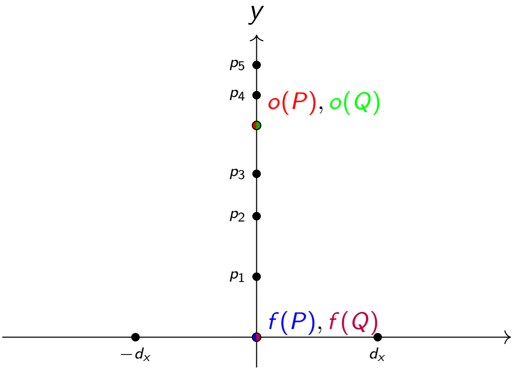
Step 3: In the new instance Q we have $f(P) = f(Q)$ hence social cost doesnt change, optimal cost weakly improves $\implies r(Q) \geq r(P)$.



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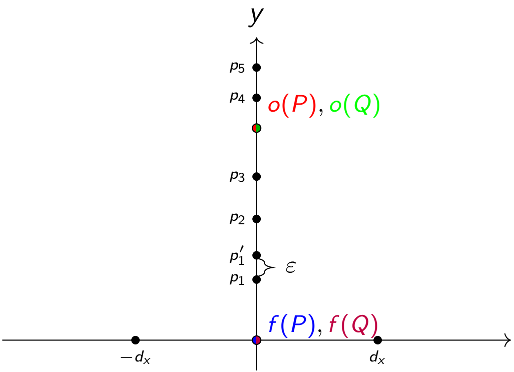
Step 3: In the new instance Q we have $f(P) = f(Q)$ hence social cost doesnt change, optimal cost weakly improves $\implies r(Q) \geq r(P)$.

Step 4: If not all y-points are on $o(Q)$, move one towards it to obtain a strictly worse instance



Lemma

Let

$$\alpha := \max_{P \in \mathcal{P}^C_{oa} \cup \mathcal{P}^C_{ca}} r(P, o(P), c)$$

and

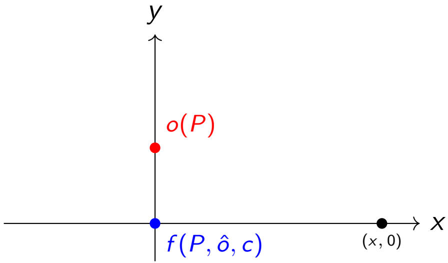
$$\beta := \max_{P \in \mathcal{P}^R_{oa} \cup \mathcal{P}^R_{ca}} r(P, (0,0), c)$$

then the **CMP** *mechanism is* α*-consistent and* β*-robust*

Consistency and Robustness

Lemma

The **CMP** *mechanism is with parameter* $c \in [0, 1)$ *is* $\frac{\sqrt{2c^2+2}}{1+c}$*-consistent and* $\frac{\sqrt{2c^2+2}}{1-c}$*-robust*



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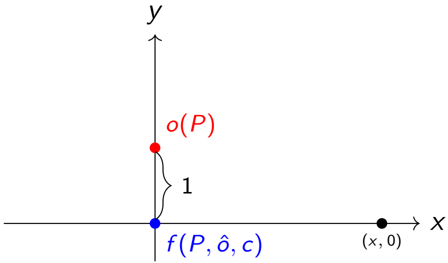

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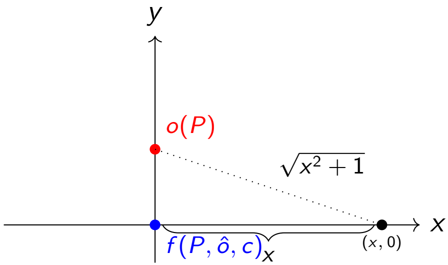

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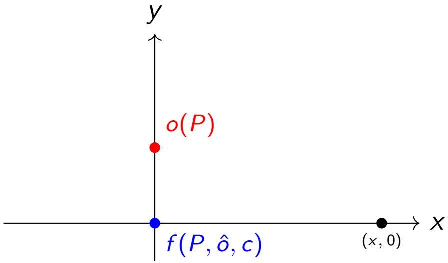
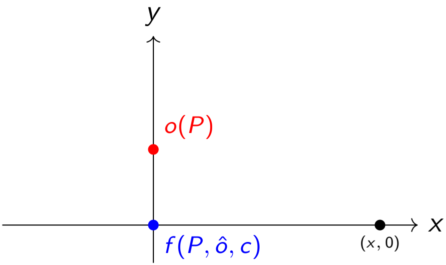
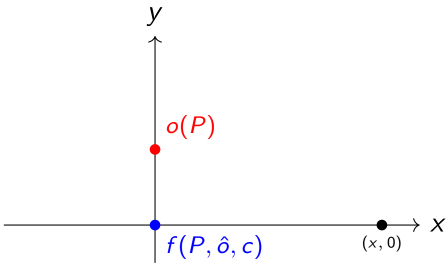


Consistency proof



- z $\leq \frac{1-c}{2}n$ where z is number of agent points with $y = 1$

Consistency proof



- $z \leq \frac{1-c}{2}n$ where z is number of agent points with $y = 1$
- $\alpha = \frac{C^u(f(P, \hat{o}=o(P), c), P)}{C^u(o(P), P)} = \frac{\frac{1+c}{2}nx + \frac{1-c}{2}n}{\frac{1+c}{2}n\sqrt{x^2+1}}$

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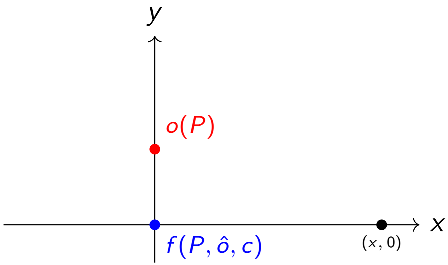

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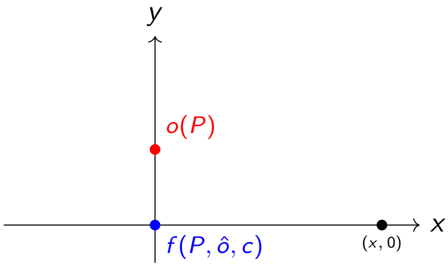

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- $\implies \alpha = \frac{\sqrt{2c^2+2}}{1+c}$

Optimality

Lemma

Any deterministic, strategyproof and anonymous anonymous mechanism with $\frac{\sqrt{2c^2+2}}{1+c}$-consistency has a robustness no better than $\frac{\sqrt{2c^2+2}}{1+c}$

Approximation in relation to the prediction error

Lemma

Let P be an instance of the problem. For any two prediction $\hat{o}$ and $\tilde{o}$ the returned facility locations $f(P, \hat{o}, c)$ and $f(P, \tilde{o}, c)$ by the **CMP** *mechanism satisfy*

$$d(f(P, \hat{o}, c), f(P, \tilde{o}, c)) \leq d(\hat{o}, \tilde{o})$$

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Theorem

The **CMP** *mechanism achieves a* $\min\{\frac{\sqrt{2c^2+2}}{1+c} + \eta, \frac{\sqrt{2c^2+2}}{1-c}\}$ *approximation*

Agrawal, Priyank et al. (2022). "Learning-Augmented Mechanism Design: Leveraging Predictions for Facility Location". In: *Proceedings of the 23rd ACM Conference on Economics and Computation*. EC '22. Boulder, CO, USA: Association for Computing Machinery, pp. 497–528. ISBN: 9781450391504. DOI: 10.1145/3490486.3538306. URL: https://doi.org/10.1145/3490486.3538306.

Meir, Reshef (2019). "Strategyproof Facility Location for Three Agents on a Circle". In: *Algorithmic Game Theory: 12th International Symposium, SAGT 2019, Athens, Greece, September 30 – October 3, 2019, Proceedings*. Athens, Greece: Springer-Verlag, pp. 18–33. ISBN: 978-3-030-30472-0.