Mayank Goswami and Andreas Wiese **Winter 2015/2016**

Exercises for Approximation Algorithms

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter15/approx

Tutorials: Andreas Schmid

Exercise Sheet 1 Due: **11.11.2015**

NOTE: *The tutorial has been moved to a different time slot! Starting from 11th of November we meet every other Wednesday from 2:15-3:45 pm in room 024 in the ground floor of the MPI-INF building E1.4.*

Your homework must be handed in on Wednesday at the beginning of the tutorial.

You need to collect at least 50% of all points over all exercise sheets. You are allowed to work on these exercises in groups but every student has to hand in his/her own write-up.

Exercise 1 (*10 points*)

Given a graph $G = (V, E)$, we say that a set of vertices $C \subseteq V$, covers E if any edge in E is incident to at least one vertex in C. In the Vertex-Cover Problem we want to find the minimum size vertex cover for a given graph G.

Show that if G is a tree, one can find an optimal vertex-cover in polynomial time.

Exercise 2 (*10 points*)

Give a 2-approximation algorithm for the Vertex Cover Problem in general graphs. Can you also give an instance on which your algorithm gives a solution that is not better than a 2-approximation? (i.e., the algorithm gives no α-approximation for any $\alpha < 2$)

Exercise 3 (*10 points*)

Give examples showing that

a) FirstFitDecreasing for Bin Packing is not an α-approximation for any constant $\alpha < \frac{3}{2}$.

b) Greedy II (the 2-approximation algorithm shown in the lecture) for the Knapsack Problem is not an α-approximation for any constant $\alpha < 2$.

Exercise 4 (*10 + 5(BONUS) points*)

You are given a knapsack of size B and a set of items $I = \{1, ...n\}$ where each item i has a size s_i and a cost c_i. All sizes and costs are positive integers. A Knapsack-Cover is a set of items $J \subseteq I$ such that $\sum_{i \in J} s_i > B$. In the Knapsack-Cover Problem we want to find the minimum Knapsack-Cover, i.e. the Knapsack-Cover with the lowest sum of costs.

a) Assume that there is no single item with a cost greater than the cost of the optimal solution. Give a 2-approximation algorithm for the Knapsack Cover Problem under this assumption.

BONUS b) Can you give a 2-approximation without the assumption of part a)?