Mayank Goswami and Andreas Wiese **Winter 2015/2016**

Exercises for Approximation Algorithms

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter15/approx

Tutorials: Andreas Schmid

Exercise Sheet 3 Due: **9.12.2015**

Your homework must be handed in on Wednesday at the beginning of the tutorial.

You need to collect at least 50% of all points over all exercise sheets. You are allowed to work on these exercises in groups but every student has to hand in his/her own write-up.

Exercise 1 (*10 points*) [Shmoys-Williamson Exercise 10.2]

We say a graph $G = (V, E)$ is k-colorable if we can assign each vertex one of k colors such that for any edge $(i, j) \in E$, i and j are assigned different colors. Suppose that G has a tree decomposition T of treewidth t , for constant t. Prove that for any k, one can decide in polynomial time whether the graph is k -colorable

Exercise 2 (*10 points*)

In Exercise Sheet 1 we studied the vertex cover problem. In case you forgot here is the problem definition again. Given a undirected graph $G = (V, E)$, we say that a set of vertices $C \subseteq V$, covers E if any edge in E is incident to at least one vertex in C. This time every vertex $v \in V$ also has a nonnegative weight w_v. We now want to find a minimum-weight vertex cover.

In the lecture we have seen a $PTAS$ for the maximum independent set problem in planar graphs. Show that this can be adapted to give a $PTAS$ for the vertex cover problem in planar graphs.

Exercise 3 (*10 points*) [k-packing problem]

Given a set of points $P = \{p_1, \cdots, p_n\}$ in the plane, for any set C of k centers $\{c_1, \cdots, c_k\}$, (where $c_i = p_j$ for some j), define the *packing distance* of C as $r_C(P) = \min_{1 \le i,j \le k} d(c_i, c_j)$.

Consider the following k-packing problem: The input is a point set P and an integer $k > 0$. The output is a set of k-centers OPT such that $r_{OPT}(P)$ is the maximum among all possible choices of k centers; i.e., $r_{OPT}(P) \ge r_C(P)$ for any other set $C \ne OPT$ of k centers. In other words, we are trying to choose cluster centers so that the centers are “as far apart as possible”.

Show that the furthest point insertion algorithm we discussed in class, that gave a 2-approximation for the k-center problem, also gives a 2-approximation for the k-packing problem. In other words, the minimum distance between the set of centers it produces is at least $\frac{1}{2}r_{OPT}(P)$.

Exercise 4 (*10 points*)

Let H be a tree such that the degree of all its vertices is at most 3. Show that there is a polygon P and a triangulation T of P such that the dual of $GT(P)$ is H. Moreover, show that there is a polygon P such that P has a unique triangulation T whose dual is H.