max planck institut informatik

SAARLAND UNIVERSITY

COMPUTER SCIENCE

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Exercises for Approximation Algorithms

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter15/approx

Tutorials: Andreas Schmid

Exercise Sheet 6 Due: **11:59pm, 5.2.2016**

You need to collect at least 50% of all points over all exercise sheets. You are allowed to work on these exercises in groups but every student has to hand in his/her own write-up.

Exercise 1 (*10 points*)[De Berg et.al. Exercise 8.7]

Let R be a set of n red points in the plane, and let B be a set of n blue points in the plane. We call a line ℓ a separator for R and B if ℓ has all points of R to one side and all points of B to the other side. Give a randomized algorithm that can decide in $O(n)$ expected time whether R and B have a separator.

Exercise 2 (*10 points*) [De Berg et.al. Exercise 9.11]

A *Euclidean minimum spanning tree* ($EMST$) of a set P of points in the plane is a tree of minimum total edge length connecting all the points. $EMST$s are interesting in applications where we want to connect sites in a planar environment by communication lines (local area networks),roads, railroads, or the like.

a) Prove that the set of edges of a Delaunay triangulation of P contains an $EMST$ for P.

b) Use this result to give an $O(n \log n)$ algorithm to compute an $EMST$ for P.

Exercise 3 (*10 points*) [De Berg et.al. Exercise 9.13]

The Gabriel graph of a set P of points in the plane is defined as follows: Two points p and q are connected by an edge of the Gabriel graph if and only if the disc with diameter pq does not contain any other point of P.

a) Prove that the Delaunay graph $DG(P)$ of P contains the Gabriel graph of P.

b) Prove that p and q are adjacent in the Gabriel graph of P if and only if the Delaunay edge between p and q intersects its dual Voronoi edge.

c) Give an $O(n \log n)$ time algorithm to compute the Gabriel graph of a set of n points.

Exercise 4 (*10 points*) [De Berg et.al. Exercise 9.14]

The relative neighborhood graph of a set P of points in the plane is defined as follows: Two points p and q are connected by an edge of the relative neighborhood graph if and only if

$$d(p,q) \leq \min_{r \in P, r \neq p,q} \max(d(p,r), d(q,r))$$

a) Given two points p and q, let $lune(p,q)$ be the moon-shaped region formed as the intersection of the two circles around p and q whose radius is $d(p,q)$. Prove that p and q are connected in the relative neighborhood graph if and only if $lune(p,q)$ does not contain any point of P in its interior.

b) Prove that the Delaunay graph $DG(P)$ of P contains the relative neighborhood graph of P.

c) Design an algorithm to compute the relative neighborhood graph of a given point set.

Exercise 5 (***BONUS*** *10 points*) [De Berg et.al. Exercise 9.15]

Prove the following relationship between the edge sets of an $EMST$, of the relative neighborhood graph RNG, the Gabriel graph GG, and the Delaunay graph DG of a point set P.

$$EMST \subseteq RNG \subseteq GG \subseteq DG$$