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Exercises for Algorithms and Data Structures

http://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter16/algorithms-and-data-structures/

Exercise Sheet 1 Due: **31.10.2016**

The homework must be handed in on Monday before the lecture. You may collaborate with other students on finding the solutions for this problem set, but every student must hand in a writeup in their own words. We also expect you to state your collaborators and sources (books, papers, course notes, web pages, etc.) that you used to arrive at your solutions.

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets.

Exercise 0 (*-10 points if incomplete*)

First, sort the following three tutorial slots by your preference. Start with your most favorable one.

- Thursday 14:00-16:00
- Thursday 16:00-18:00
- Friday 14:00-16:00

Next, flip a coin and write down H (for heads) or T (for tails) depending on the outcome of your experiment.

Finally, go to the course website and sign up for the mailing list. Once we assigned every student to a tutorial group we will publish the result there.

Exercise 1 (*10 points*)

The underlying model for this exercise is an integer RAM with unit cost operations $+, -, *, \text{div}$. We usually assume that in our RAM only integers bounded by a polynomial in the size of the input can be stored in a single cell. If the input has size n such numbers can be represented by $O(\log n)$ bits.

a) How many computation steps does it take to generate the number $2^n - 1$.

b) Assume now that you can store integers of arbitrary size in a cell. How many computation steps does it take to generate the number $2^{2^n} - 1$ (a number with 2^n bits).

c) Another computational task that becomes easy if we drop the size restriction is factorizing large integers into their prime components. Can you think of a real world application that depends on this task being difficult?

Exercise 2 (*10 points*)

Order the following functions by their asymptotic growth rate. How would you prove that your ordering is indeed correct?

$$(3/2)^n \quad n \log n \quad n^{3/2} \quad n \cdot 2^{\sqrt{\log n}} \quad n^{1+\epsilon}$$

Here all logarithms are with respect to base 2, and ϵ stands for an arbitrary positive real number less than 1/2.