max planck institut informatik

SAARLAND UNIVERSITY
COMPUTER SCIENCE

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Exercises for Algorithms and Data Structures

http://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter16/algorithms-and-data-structures/

Exercise Sheet 7 Due: **12.12.2016**

The homework must be handed in on Monday before the lecture. You may collaborate with other students on finding the solutions for this problem set, but every student must hand in a writeup in their own words. We also expect you to state your collaborators and sources (books, papers, course notes, web pages, etc.) that you used to arrive at your solutions.

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets.

Whenever you are asked to design an algorithm in this exercise sheet you have to give a proof of its correctness as well as an asymptotic upper bound on its worst case running time.

Exercise 1 (*10 points*)

Use the augmenting path algorithm shown in class to compute a maximum cardinality matching of the bipartite graph in Figure 1. Initialize the algorithm with the matching $M_1 = \{(2, a), (3, c), (4, d), (6, e)\}$. Start by drawing the residual graph depending on M_1. Then for every iteration give the augmenting path you use to extend the current matching and update the residual graph accordingly.

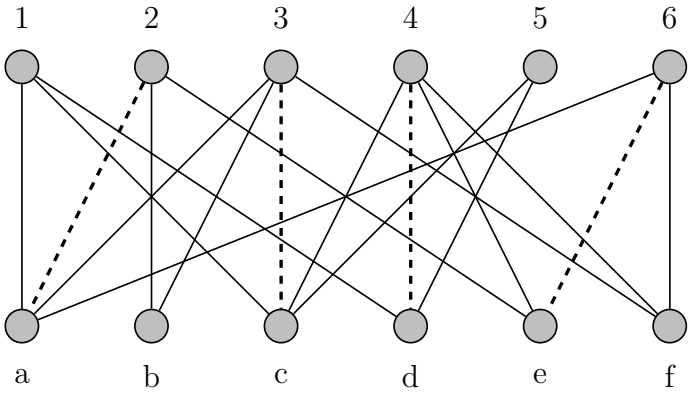

Figure 1: Input graph G for Exercise 1. The edges in M_1 are drawn with dashed lines.

Exercise 2 (*10 points*)

The famous Saarbrücken school of music currently employs n piano teachers $t_1, \ldots, t_n$ and there are m piano students $s_1, \ldots, s_m$ enrolled. Every teacher can only take one student per semester. To make the lessons most beneficial for the students, all lessons are private lessons. Not every student wants to learn from every teacher as they all specialize in different genres of music. Therefore every student is asked to provide a list of the teachers they would like to take lessons from.

Only students that are assigned to a teacher will have to pay tuition. Give an algorithm that assigns the maximum number of students to teachers in the next semester and therefore maximizes the profit for the school.