Exercise 1 (10 points)

Use the augmenting path algorithm shown in class to compute a maximum cardinality matching of the bipartite graph in Figure 1. Initialize the algorithm with the matching $M_1 = \{(2, a), (3, c), (4, d), (6, e)\}$. Start by drawing the residual graph depending on $M_1$. Then for every iteration give the augmenting path you use to extend the current matching and update the residual graph accordingly.

Figure 1: Input graph $G$ for Exercise 1. The edges in $M_1$ are drawn with dashed lines.
Exercise 2 (10 points)

The famous Saarbrücken school of music currently employs $n$ piano teachers $t_1, \ldots, t_n$ and there are $m$ piano students $s_1, \ldots, s_m$ enrolled. Every teacher can only take one student per semester. To make the lessons most beneficial for the students, all lessons are private lessons. Not every student wants to learn from every teacher as they all specialize in different genres of music. Therefore every student is asked to provide a list of the teachers they would like to take lessons from.

Only students that are assigned to a teacher will have to pay tuition. Give an algorithm that assigns the maximum number of students to teachers in the next semester and therefore maximizes the profit for the school.