Multivariate Algorithmics:
Problem set 8 Trial Exam

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Due: Before the lecture on February 05, 2019 Tutorial: February 07, 2019

Problem 8.1 (12 Points) Decide for each of the following statements whether it is *true*, *false* or *unknown*. Each correct answer gives 2 points, no answer gives 1 point and a wrong answer gives 0 points.

a) $\mathsf{FPT} = \mathsf{W}[1]$ implies $\mathsf{P} = \mathsf{NP}$.

b) The problem VertexCover is contained in $\mathsf{W}[1]$.

c) The problem FeedbackVertexSet is $\mathsf{W}[1]$-hard.

d) The problem k-Path can be solved in time $2^{o(k)}n^{O(1)}$.

e) There exists an infinite sequence $G_0, G_1, G_2, \ldots$ of graphs such that for every pair $i < j$ the graph G_i is not a minor of G_j.

f) The problem VertexCover can be solved in time $1.5^k n^{O(1)}$.

Problem 8.2 (12 Points) A *d-uniform hypergraph* is a pair of a finite set of vertices $\mathcal{U}$ and a set $\mathcal{S}$ of subsets of $\mathcal{U}$ of size d. The d-SetPacking problem asks, given a d-uniform hypergraph $(\mathcal{U}, \mathcal{S})$ and a positive integer k, to decide whether there are k elements of $\mathcal{S}$ that are pairwise disjoint. The problem is parameterized by k.

Prove that, for every positive integer d, the problem d-SetPacking has a kernel with $O(k^d)$ vertices.
Hint: Use the Sunflower Lemma

Problem 8.3 (6+6 Points) For a positive integer d, the problem $\#d$-DegreeSubgraph asks, given a graph $G = (V, E)$, to compute *the number* of sets $S \subseteq E$ such that (V, S) is a subgraph of G in which every vertex has degree exactly d.

a) Prove that $\#d$-DegreeSubgraph can be solved in time $(d+1)^{2\mathsf{tw}(G)}|V|^{O(1)}$.

b) Prove that $\#d$-DegreeSubgraph can be solved in time $(d+1)^{\mathsf{tw}(G)}|V|^{O(1)}$.

Problem 8.4 (6+6 Points) The problem LongestCycle asks, given a graph $G = (V, E)$ and a positive integer k, to decide whether there exists a cycle in G of length at least k.
The problem ExactCycle expects the same input and asks to decide whether there exists a cycle in G of length *exactly* k.

a) Prove that ExactCycle can be solved in time $c^k|V|^{O(1)}$ for some constant $c > 1$.

b) Prove that LongestCycle can be solved in time $d^k|V|^{O(1)}$ for some constant $d > 1$.

Problem 8.5 (12 Points) The problem MulticoloredBiclique asks, given a positive integer t and a graph G whose vertices are colored with $2t$ different colors, to decide whether there exists a colorful subgraph of G that is isomorphic to the biclique $K_{t,t}$. Here $K_{t,t}$ is the graph with vertices $v_1, \ldots, v_t$ and $u_1, \ldots, u_t$ and edges $\{v_i, u_j\}$ for every pair $1 \le i, j \le t$.

Show that MulticoloredBiclique is $\mathsf{W}[1]$-hard when parameterized by t.