Problem 8.1 (12 Points) Decide for each of the following statements whether it is true, false or unknown. Each correct answer gives 2 points, no answer gives 1 point and a wrong answer gives 0 points.

a) FPT = W[1] implies P = NP.

b) The problem VERTEXCOVER is contained in W[1].

c) The problem FEEDBACKVERTEXSET is W[1]-hard.

d) The problem k-Path can be solved in time $2^{o(n)}$.

e) There exists an infinite sequence $G_0, G_1, G_2, \ldots$ of graphs such that for every pair $i < j$ the graph $G_i$ is not a minor of $G_j$.

f) The problem VERTEXCOVER can be solved in time $1.5^k n^{O(1)}$.

Problem 8.2 (12 Points) A $d$-uniform hypergraph is a pair of a finite set of vertices $U$ and a set $S$ of subsets of $U$ of size $d$. The $d$-SETPACKING problem asks, given a $d$-uniform hypergraph $(U, S)$ and a positive integer $k$, to decide whether there are $k$ elements of $S$ that are pairwise disjoint. The problem is parameterized by $k$.

Prove that, for every positive integer $d$, the problem $d$-SETPACKING has a kernel with $O(k^d)$ vertices.

Hint: Use the Sunflower Lemma

Problem 8.3 (6+6 Points) For a positive integer $d$, the problem $\#d$-DEGREESUBGRAPH asks, given a graph $G = (V, E)$, to compute the number of sets $S \subseteq E$ such that $(V, S)$ is a subgraph of $G$ in which every vertex has degree exactly $d$.

a) Prove that $\#d$-DEGREESUBGRAPH can be solved in time $(d + 1)^{2\omega(G)} |V|^{O(1)}$.

b) Prove that $\#d$-DEGREESUBGRAPH can be solved in time $(d + 1)^{\omega(G)} |V|^{O(1)}$.

Problem 8.4 (6+6 Points) The problem LONGESTCYCLE asks, given a graph $G = (V, E)$ and a positive integer $k$, to decide whether there exists a cycle in $G$ of length at least $k$.

The problem EXACTCYCLE expects the same input and asks to decide whether there exists a cycle in $G$ of length exactly $k$.

a) Prove that EXACTCYCLE can be solved in time $e^k |V|^{O(1)}$ for some constant $c > 1$.

b) Prove that LONGESTCYCLE can be solved in time $d^k |V|^{O(1)}$ for some constant $d > 1$.

Problem 8.5 (12 Points) The problem MULTICOLOREDBICLIQUE asks, given a positive integer $t$ and a graph $G$ whose vertices are colored with $2t$ different colors, to decide whether there exists a colorful subgraph of $G$ that is isomorphic to the biclique $K_{t,t}$. Here $K_{t,t}$ is the graph with vertices $v_1, \ldots, v_t$ and $u_1, \ldots, u_t$ and edges $\{v_i, u_j\}$ for every pair $1 \leq i, j \leq t$.

Show that MULTICOLOREDBICLIQUE is W[1]-hard when parameterized by $t$. 