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Exercises for Randomized and Approximation Algorithms

www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter18/rand-apx-algo/

Exercise Sheet 12: Hardness of Approximation

To be handed in by **January 29nd, 2019** via e-mail to André Nusser
(CC to Antonios Antoniadis and Marvin Künnemann)

Exercise 1 *(6 Points)* Consider the Traveling Salesperson Problem (TSP): Given a complete (undirected) graph $G = (V, E)$ with positive edge weights $w : E \to \mathbb{N}$, find a Hamiltonian tour of minimum weight.

Show, by means of a gap-introducing reduction from a suitable decision problem, that there is no polynomial-time $2^{n^{1-\varepsilon}}$-approximation for TSP (for any constant $\varepsilon > 0$) unless $\mathsf{P} = \mathsf{NP}$.

Exercise 2 *(7 Points)* Find a reasonable notion of reduction $\leq_{\text{approx}}$ such that the following holds. If $\Pi \leq_{\text{approx}} \Pi'$ for some NPO problems Π, Π', then Π' admitting a PTAS implies that Π admits a PTAS. What other properties should such a notion fulfill? Does your notion fulfill them?

Exercise 3 *(7 Points)* Prove that if there is a (constant-)gap-introducing reduction from SAT to Max3SAT, then $\mathsf{NP} \subseteq \mathsf{PCP}(O(\log n), O(1))$.

Exercise 4 *(10 Points)* Recall the (unweighted) VertexCover problem and consider the related IndSet problem:

IndSet: Given an undirected graph $G = (V, E)$, determine the largest subset of vertices $S \subset V$ that is *independent* (i.e., no edge of G connects a vertex from S to another vertex in S).

a) *(5 Points)* Show that solving VertexCover and IndSet exactly is polynomial-time equivalent.

b) *(5 Points)* Recall that we have shown (1) a 2-approximation for VertexCover and (2) the inapproximability result that for some constant $\gamma > 0$, VertexCover has no $(1 + \gamma)$-approximation unless $\mathsf{P} = \mathsf{NP}$. What are the best approximability and inapproximability results that you can achieve for IndSet, using the reductions from a)? Justify you answer!

Exercise 5 *(10 Points)* Show, by means of gap-preserving reduction from VertexCover, that there is some $\gamma > 0$ such that the following problem has no polynomial-time $(1+\gamma)$-approximation unless $\mathsf{P} = \mathsf{NP}$.

Given an undirected graph $G = (V, E)$ with positive edge weights $E : V \to \mathbb{N}$, where $V = R \cup S$ is partitioned into the *required* vertices R and the *Steiner* vertices S find the minimum-cost tree in G that contains all required vertices and any subset of Steiner vertices.