Final Exam Announcement
Algorithms on Digraphs
Winter 2018–9

Updated: February 4, 2019

Exam Date & Format

Oral examinations will be held on Monday, 25. February, 2019. Students will meet individually with the instructors for approximately 50 minutes. During the exam, students will be asked to discuss two topics appearing on the list below. The first topic will be chosen by the student, while the second topic will be chosen randomly. Students will be interviewed about the two topics, and may be asked to provide the following:

- An explanation of the main problem(s) addressed by the topic.
- A description of the important definitions used to address the main problem(s).
- A statement of the main or interesting results related to the topic.
- An overview of the techniques used to obtain the interesting results.

In addition to the items listed above, students may contribute anything they found interesting or valuable related to their chosen/assigned topic, even if it was not explicitly covered in class.

In evaluating student performance on the exam, we will emphasize conceptual understanding, rather than technical details. In particular, students will not be expected to recall precise calculations, definitions, or results. Rather, students should focus on explaining how the conceptual aspects of a topic fit together to give the results discussed in class.

List of Topics

1. Color coding and its application to long directed cycles. Understand what the color coding technique is, and how a long “colored” path is easy to detect in a graph. Understand the high-level ideas of Zehavi’s algorithm for finding a long directed cycle.
   - Parameterized Algorithms by Cygan et al., Section 5.2.
   - Zehavi2016

2. Disjoint paths. Know the difference between having $k$ vertex disjoint paths from $A$ to $B$ and a $k$-linkage from $A$ to $B$. Understand why the former problem is is “easy” (solvable in polynomial time) and $k$-linkage is hard (in general). Understand why $k$-linkage is solvable in polynomial time in graphs of bounded directed treewidth.
   - Classes of Directed Graphs, Section 9.5.1.
   - Definitions/techniques in Homework 4
3. Static and dynamic packet routing. Understand the “buffer” model of a directed network, and understand the connection between schedules and buffer loads. Understand the randomized strategy employed by the “preliminary” algorithm (Section 3.1) in Leighton1994. Understand how the Lovasz Local Lemma applies to the analysis of the preliminary algorithm in Leighton1994. Understand the online packet routing problem of Patt-Shamir2017. Know the greedy, centralized, and Odd-Even Downhill forwarding rules (see the slides linked below), as well as their worst-case bounds.

- Leighton1994 (Up to and including Section 3.1.)
- Patt-Shamir2017
- Forwarding on a path slides (Up to, not including, “More General Lower Bounds”)

4. Treewidth and grid theorem:

- Undirected treewidth: We expect a student to be able to explain definitions and a generic dynamic programming technique on graphs of bounded treewidth. For this, you may read chapter 7 of the parametrized algorithm book, sections 7.1, 7.2, 7.3.1, 7.3.2 (either of the latter two is enough).
- Directed treewidth: We expect a student to be able to define directed treewidth, reformulate it as a cops and robber game, be able to understand its relation to well-linked set and strong separators. DirectedTreewidth to the end of section 2, for simpler and more clear definitions one may use the more clear version in the Erdős-Pósa property for Digraphs, for the well-linked set, balanced separators, etc, one may take a look at chapter 9, sections 1 and 3. For cops and robber game, we expect a student have an idea what is a difference between monotone and non-monotone game (consequently the role of guards), this can be found in the section 2 of Directed Tree-Width Examples.

5. The Erdős-Pósa property: We expect to hear basic definitions and the ideas on how to prove or disprove a certain graph admits Erdős-Pósa property. For the corresponding materials check the followings.

- Undirected graphs: Graph minor V section 8,
- Directed graphs: Erdős-Pósa property for Digraphs up to the end of section 4.

6. Important cuts, iterative compression: We expect an student to be able to: provide us related definitions, explain the relation between cuts and submodular functions and the consequences of this relation, how we can employ important cuts to solve problems: in particular for feedback arc set problem, how iterative compression helps us to solve feedback arc set problem both in general digraphs and tournaments. For materials on this topic see chapter 8 of parametrized algorithm book except for sections: 8.4, 8.7. From the digraph book, take a look at 2.9.3 and 2.9.4.

7. Plane-sweep approach: We expect a student to be able to describe the two plane-sweep algorithms we discussed in class: an algorithm for computing the intersection points of line segments and an algorithm for computing the Voronoi diagram. Students should describe algorithms and analyze the time complexity of each algorithm. The students can refer to the following papers.

- Line segment intersection: [BO 79]
- Voronoi diagrams: Section 2.1 of [Fortune 87]