Homework 3: Directed Treewidth

Algorithms on Directed Graphs, Winter 2018/9

Due: 16.11.2018 by 16:00

Exercise 1 (Cops and Rober Game: Undirected Treewidth game)**.** In the lecture you learned about cops and robber game, here we dig a bit more into it.

1. A $k \times k$ grid is a planar graph consisting of k disjoint vertical paths $P_1, \ldots, P_k$ and k horizental paths $Q_1, \ldots, Q_k$ each of length $k-1$, so that P_i interesects P_j for all $i, j \in [k]$. Provide a proof sketch to show that k cops are not sufficient to win visible cops and robber game on $k \times k$ grid and conclude that treewidth of a grid is k. (hint: first show $k-1$ cops cannot catch the robber).

2. *Pathwidth* cops and robber game is defined similarly to treewidth game, except that robber is invisible and cops should sweep the graph so that the robber cannot elude.

   Show that there is a class of graphs of treewidth $O(1)$ however the number of cops needed to win pathwidth game is unbounded (hint: there is a class of graphs with path width in $\Omega(\log n)$, what this number means to you?).

Exercise 2 (DAGs)**.** Many problems are easy on DAGs but they are hard on general graphs, in this exercise you will let us know what are your favorite problems of this kind.

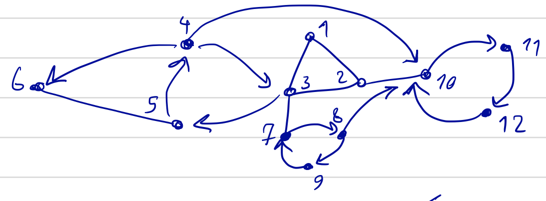
1. Name five problems that are polynomial time solvable on DAGs while they are hard in general digraphs.

2. Provide a sketch of polynomial time algorithms for two of the above problems.

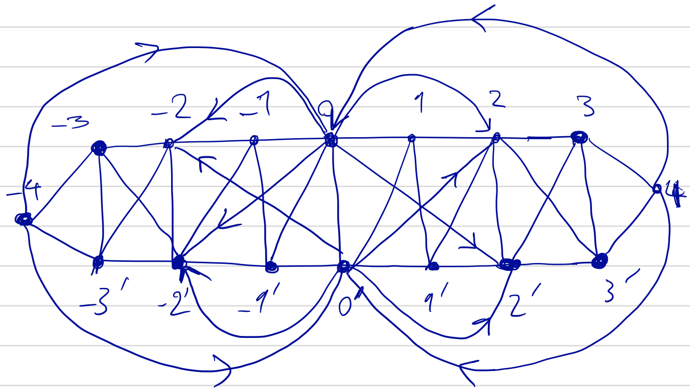
Exercise 3 (Directed Treewidth Examples)**.** In this exercise we expand our understanding of directed treewidth and cops and robber game. Consider the following graphs.

1. Calculate directed tree width and directed tree-decomposition of minimum width of each of the following graphs.

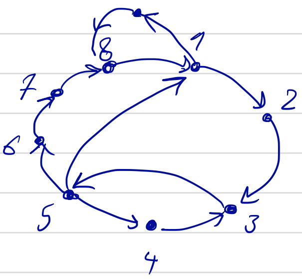
1.



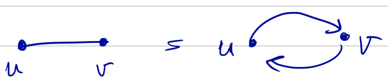
2.



3.



undirected edge = bidirected edge i.e.



2. Recall that cops and robber game is robber monotone if robber's search space only shrinks. In the example 2, if we play a normal game, 4 cops can win the game, however, if we play robber monotone we need at least 5 cops. Can you explain why we cannot win with 4 cops in a robber monotone game? (hint: this example is taken from paper of Isolde Adler: Directed Treewidth Examples, if you cannot prove it, try to use step by step hints she provides in that paper).

Exercise 4 (k-linkedness)**.** A set $W \subseteq V(G)$ is k-linked if it does not contain a balanced (strong) separator of order k. A digraph G is k-linked if there is a set $W \subseteq V(G)$ so that W is k-linked.

Prove if directed treewidth of a graph G is bounded above by k then G is not $k+1$-linked.