Homework 10: Separators and Cuts Part I

Algorithms on Directed Graphs, Winter 2018/9

Due: 19.01.2019 by 16:00

Materials of this lecture are from chapter 8 of the parametrized algorithm book. For definitions please take a look at the book.

Before going into exercises I have to fix one of my mistakes in the lecture: In the proof of Lemma 8.10 I said $d(R_{max} \cup R) > d(R)$ and we arrive at a contradiction, but as one of you complained during the lecture this is wrong. The correct argument is that: if $d(R_{max} \cup R) \leq d(R)$ with the fact that $R_{max} \not\subseteq R$ we have that R is a proper subset of $R_{max} \cup R$. Given that $d(R_{max} \cup R)$ is small we get a contradiction with the assumption that R is an important cut.

Exercise 1 (submodular functions)**.** Prove the followings.

1. Prove the function d_G is submodular for every undirected graph G.
2. Let define $\Delta_G(R)$ for digraph G to be the set of edges so that their tail is in R and their head is in $G \setminus R$. Define $d_G(R) = |\Delta_G(R)|$. Is d_G a submodular function?
3. Solve the exercise 8.2 of the book.

Exercise 2 ($R_{\min}, R_{\max}$)**.** Solve the exercises 8.6 and 8.7.

Exercise 3 (Enumerating important cuts)**.** Complete missing proofs from the lecture.

1. We explained half of the proof of theorem 8.11. Provide a full proof of it. Of course you can read it first, however you have to write your own understanding.
2. The proof of 8.11 was a constructive proof. Turn it to an algorithm with running time $O(4^k P(n))$ for some polynomial function P.