Exercise 2: Flirting with Synchrony and Asynchrony

Task 1: Growing Balls

In this exercise, we will see how a crucial step of the \( \gamma \)-synchronizer works; specifically, that a desirable partition of the nodes exists.

Denote by \( B(v, r) \) the ball of radius \( r \) around \( v \), i.e., \( B(v, r) = \{ u \in V : \text{dist}(u, v) \leq r \} \). Consider the following partitioning algorithm. Note the difference between intercluster edges and intrachuster edges.

Algorithm 1 Cluster construction. \( \rho \geq 2 \) is a given parameter.

1: \textbf{while} there are unprocessed nodes \textbf{do}
2: \hspace{1em} select an arbitrary unprocessed node \( v \);
3: \hspace{1em} \( r := 0 \);
4: \hspace{1em} \textbf{while} \( |B(v, r + 1)| > \rho |B(v, r)| \) \textbf{do}
5: \hspace{2em} \( r := r + 1 \)
6: \hspace{1em} \textbf{end while}
7: \hspace{1em} \text{makeCluster}(B(v, r)) \quad /\!/ \text{all nodes in } B(v, r) \text{ are now processed}
8: \hspace{1em} \text{remove all cluster nodes from the current graph}
9: \hspace{1em} \textbf{end while}
10: \select intercluster edges

a) Show that Algorithm ?? constructs clusters of radius at most \( \log_\rho n \).

b) Show that Algorithm ?? produces at most \( \rho m \) intercluster edges.

c) For given cluster radius \( k \in \{1, \ldots, \lfloor \log n \rfloor \} \), determine an appropriate choice \( \rho(k) \geq 2 \), proving the precondition of Corollary 2.14!

\textbf{Hint:} As a short-hand, we often don’t write out common terms like \( n \) that are assumed to be globally known. Specifically, \( \rho(k) \) may also depend on \( n \), as if we had written \( \rho(k, n) \). If in doubt, then we weren’t clear enough, so tell us!

Task 2: Showing Dijkstra, and Bellman & Ford the Ropes

a) Show that if the asynchronous Bellman-Ford algorithm from the lecture is executed synchronously, it sends only \( O(|E|) \) messages.

b) Use this to construct an asynchronous BFS tree construction algorithm of time complexity \( O(D) \) that uses \( O(|E|D) \) messages and terminates. You may assume that \( D \) is known here.

c) Can you give an asynchronous Bellman-Ford-based algorithm that sends \( O(|E| + nD) \) messages and runs for \( O(D^2) \) rounds?

\textbf{Hint}: Either answer is feasible, provided it is backed up by appropriate reasoning!
Task 3*: Liaison with Leslie Lamport

a) Look up what the happened-before relation, Lamport clocks, and vector clocks are.

b) Contemplate their relation to synchronizers and what you’ve learned in the lecture.

c) Discuss your findings in the exercise session!