Exercise 12: ~~Weed~~ Weak Models

Task 1: Hyper, Hyper!

Recall that a hypergraph is a graph where edges may comprise more than 2 nodes. The degree of a hyperedge is the number of nodes in it. Consider a hypergraph of maximum edge-degree d in the port-numbering model (i.e., nodes send messages to ports and receive messages from ports, one per hyperedge).

a) Generalize maximal matchings and vertex covers to hypergraphs.

b) Suppose the hypergraph is d-node-colored such that for each hyperedge, all contained nodes have different colors. Provide an algorithm that computes a maximal matching of the hypergraph. It suffices to show that it terminates eventually and outputs a matching, concrete running time bounds are not required.

   Hint: Use the same strategy as used in the lecture for $d = 2$: have each node of the first color propose one of its hyperedges, then have the nodes of the second color select one out of the proposals they got, then nodes of the third color select, and so on for all colors. Note that only the final color can permanently reject a proposal; other colors only "temporarily" reject an edge.
   Then argue that *some* progress is made.

c) Show how to compute a d^2-approximate vertex cover.

   Hint: Again, use the same strategy as in the lecture: Replace each node by d copies of different color and replace each hyperedge by $d!$ hyperedges – one for each possible coloring of its constituent nodes with different colors. Compute a matching of this new hypergraph and map it to a vertex cover of the original graph. Observe that the matching induces a sub(hyper)graph of maximum node-degree d of the original graph and use this information to derive the approximation guarantee.

Task 2: Colors, all these Beautiful Colors!

a) Show that an edge coloring in the port numbering model is insufficient to obtain a node coloring!

b) Suppose we have an anonymous graph without port numbers, but with a node coloring. Communication is synchronous, by sending a message to each color and receiving a multiset of messages (i.e., the messages by the neighbors of this color, counting multiplicity) from each color. Show that this is insufficient to obtain an edge coloring!

c) Show how to construct an edge coloring from a node coloring in the port numbering model!

Task 3*: …Huh?

a) Meditate on the nature of the course and the insights you gained.

b) Extract questions that will guide the true seeker of knowledge to these insights.

c) Pose your questions in the exercise group for the others to contemplate.[1]

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[1] More profanely said, figure out what was important in the course, come up with viable exam questions, and let us know. If they are good, we may ask them, making everyone's life better and happier.