Pieter Kleer **Winter 2020-2021**

Topics in Algorithmic Game Theory and Economics, Exercise Sheet 2.5

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter20/game-theory

Total Points: - Due: **Will not be graded (Tutorial on Jan. 4)**

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, using your own words. ***Always explain your answers.*** *Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of the total points on exercise sheets to be admitted to the exam. Send your solutions in* ***PDF*** *format directly to Golnoosh (**gshahkar@mpi-inf.mpg.de**). You get the first 3 points if you hand in typed solutions.*

Exercise 1 — - points

Consider the following two-player game where Alice and Bob both have strategy set $\{C, D, E\}$:

Alice \ Bob	C	D	E
C	$(1, -3)$	$(-1, 3)$	(∞, ∞)
D	$(-1, 2)$	$(1, -2)$	(∞, ∞)
E	(∞, ∞)	(∞, ∞)	$(10, 10)$

Table 1: You may replace ∞ with a larger number (like 100) if you like.

a) Give all pure Nash equilibria.

b) Give a mixed Nash equilibrium that is a not pure Nash equilibrium. (*Hint: You could use support enumeration to find an MNE, but you might be able to guess an appropriate support.*)

c) Show that

$$\sigma = \frac{1}{20}\begin{pmatrix} 2 & 2 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

is a correlated equilibrium. Can you interpret σ in terms of your solutions in a) and b)?

Exercise 2 — - points

Consider the following two-player game where Alice and Bob both have strategy set $\{C, D, E\}$:

Alice \ Bob	C	D	E
C	$(1, -1)$	$(-1, 1)$	(∞, ∞)
D	$(-1, 1)$	$(1, -1)$	(∞, ∞)
E	(∞, ∞)	(∞, ∞)	$(10, 10)$

Table 2: You may replace ∞ with a larger number (like 100) if you like.

a) Show that

$$\sigma = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is a coarse correlated equilibrium. Is it also a correlated equilibrium?

Exercise 3 - points

Consider the game dynamics as given on Slide 19 of Lecture 6, and assume that Alice has $m \in \mathbb{N}$ strategies.

Show that for the regret $\alpha(T)$ (defined by means of the "best choices in hindsight"), no matter how Alice chooses her probability distributions $p^{(t)}$, the adversary can always choose cost functions so that $\alpha(T) \geq 1/m$ in expectation for any T.