Dániel Marx and Roohani Sharma Winter 2021/22

Parameterized Algorithms, Exercise Sheet 2

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Total Points: 50 Due: Tuesday, **November 16**, 2021

*You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, **using your own words**. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Philipp (*philipp.schepper@cispa.de*).*

Exercise 1 — **10** points

Recall the Triangle Free Deletion problem from Lecture 1. Here, given an n-vertex undirected graph G and a positive integer k, the goal is to determine if there exists at most k vertices whose deletion results in a graph with no triangles (cycles of length three). We saw that this problem can be solved using a simple branching algorithm in time $3^k \cdot n^{\mathcal{O}(1)}$. Design an algorithm for Triangle Free Deletion that runs in time $(3-\epsilon)^k \cdot n^{\mathcal{O}(1)}$, for some $\epsilon > 0$, using Iterative Compression. *Hint:* Recall from Lecture 1, that Vertex Cover can be solved in $1.4656^k \cdot n^{\mathcal{O}(1)}$ time.

Exercise 2 — **10** points

The Hamiltonian Path problem is defined as follows: given an n-vertex undirected graph G, does there exist a path on n vertices in G? A simple brute force algorithm that enumerates all possible n-length paths in G, gives a $n^{\mathcal{O}(n)}$ time algorithm for Hamiltonian Path. Design a $2^n \cdot n^{\mathcal{O}(1)}$ time algorithm for Hamiltonian Path using dynamic programming over subsets.

Exercise 3 — **5 + 5** points

Suppose that we know that parameterized problem B can be solved in time $3^k \cdot n^{O(1)}$. Furthermore, there is a parameterized reduction from problem A to problem B that runs in polynomial time. What running time bound we can give for problem A if we know that the reduction creates new instances with parameter

(a) at most $4k$,
(b) at most $2k^2$?

Exercise 4 — **10** points

Either show that the following problem is FPT or show that it is W[1]-hard.

Distance-4 Independent Set	
Input:	A graph G, an integer k
Parameter:	k
Question:	Is there a set S of k vertices in G such such that the distance of any two vertices in S is at least 4?

Exercise 5 — **10** points

In the Set Packing problem the input consists of a family $\mathcal{F}$ of subsets of a finite universe U and an integer k, and the question is whether one can find k pairwise disjoint sets in F. Prove that Set Packing is W[1]-hard when parameterized by k.