SIC Saarland Informatics Campus

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Parameterized Algorithms, Exercise Sheet 3

cms.cispa.saarland/paramalg

Total Points: 50 Due: Tuesday, **December 07**, 2020

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Philipp (*philipp.schepper@cispa.de*).*

Exercise 1 — 10 points

Let Π be a parameterized problem that admits an FPT algorithm with running time $8^k \cdot n^{O(1)}$, where k is the parameter and n is the size of the input instance. Show that Π admits a kernel of size 2^k.

Exercise 2 — 10 points

In the Point Line Cover problem, we are given n points on the plane, and the goal is to cover these points with at most k lines. A line covers a point if the point lies on the line. Design a polynomial kernel for the Point Line Cover problem parameterized by k.

Make sure to show that all the reduction rules in your proposed kernelization algorithm are safe and can be applied in polynomial time.

Exercise 3 — 5+5 points

In the k-Path problem *parameterized by vertex cover*, we are given as input an undirected graph G, a vertex cover C of G and a positive integer k. The goal is to determine if G has a path of length k. The parameter is $|C|$.

a) Design a kernel for the above problem with $|C|^{O(1)}$ vertices without using the Expansion Lemma.

b) Improve the size of the above kernel to $O(|C|^2)$ vertices using the Expansion Lemma.

Exercise 4 — 10 points

In the Max Leaf Subtree problem, given a graph G and integer k the goal is to find a sub-tree with at least k leaves. Show that this problem does not admit a polynomial kernel.

Exercise 5 — 10 points

Show that the Connected Vertex Cover problem parameterized by the solution size k, does not admit a polynomial kernel. You may assume that Set Cover parameterized by the size of the universe doesn't admit a polynomial kernel.

Recall that the Set Cover problem is defined as follows: given a universe U of elements, a family $\mathcal{F}$ containing subsets of U and a positive integer k, the goal is to decide if there exists $\mathcal{F}' \subseteq \mathcal{F}$ such that $|\mathcal{F}'| \leq k$ and $\cup_{F \in \mathcal{F}'} F = U$.