SIC Saarland Informatics Campus

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Parameterized Algorithms, Exercise Sheet 5

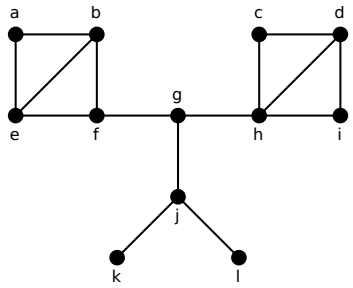
cms.cispa.saarland/paramalg

Total Points: 50 Due: Tuesday, **February 1**, 2022

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, ***using your own words****. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam. Please send your solutions directly to Philipp (*philipp.schepper@cispa.de*).*

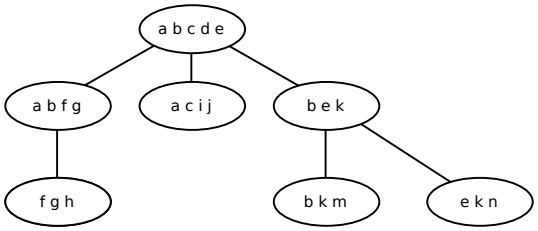
Exercise 1 — 10 points

Consider the graph depicted below. How many vertices have to be removed from this graph to decrease its treewidth by one? Justify your answer.



Exercise 2 — 10 points

A tree decomposition of some graph G is given below. What is the width of this decomposition? Give a *nice* tree decomposition of the same width for G.



Exercise 3 — 10 points

Given a graph G, the Independent Dominating Set problem asks for a set S of vertices (of any size) that is both an independent set and a dominating set. That is, vertices in S are not adjacent to each other and every vertex not in S has a neighbor in S. Use Courcelle's Theorem to show that Independent Dominating Set is FPT parameterized by the treewidth of G.

Exercise 4 — 10 points

Show that if a graph G has a vertex cover of size k, then the treewith of G is at most k.

Exercise 5 — 5 + 5 points

Given a graph G and an integer k, the Odd Cycle Transversal problem asks for a set S of at most k vertices such that $G - S$ is bipartite. It can be shown using a dynamic programming approach that the problem is FPT parameterized by the treewidth of a given tree decomposition.

(a) What would be the subproblems in the dynamic programming?
(b) How many subproblems do you need to solve in total?

(Note: You do not have to give a complete algorithm, just answer these questions.)