Exercise 1

Consider the graph depicted below. How many vertices have to be removed from this graph to decrease its treewidth by one? Justify your answer.

![Graph Image]

Exercise 2

A tree decomposition of some graph $G$ is given below. What is the width of this decomposition? Give a nice tree decomposition of the same width for $G$.

![Tree Decomposition Image]

Exercise 3

Given a graph $G$, the Independent Dominating Set problem asks for a set $S$ of vertices (of any size) that is both an independent set and a dominating set. That is, vertices in $S$ are not adjacent to each other and every vertex not in $S$ has a neighbor in $S$. Use Courcelle’s Theorem to show that Independent Dominating Set is FPT parameterized by the treewidth of $G$.

Exercise 4

Show that if a graph $G$ has a vertex cover of size $k$, then the treewidth of $G$ is at most $k$.

Exercise 5

Given a graph $G$ and an integer $k$, the Odd Cycle Transversal problem asks for a set $S$ of at most $k$ vertices such that $G - S$ is bipartite. It can be shown using a dynamic programming approach that the problem is FPT parameterized by the treewidth of a given tree decomposition.

(a) What would be the subproblems in the dynamic programming?

(b) How many subproblems do you need to solve in total?

(Note: You do not have to give a complete algorithm, just answer these questions.)