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Randomized Algorithms and Probabilistic Analysis of Algorithms, Exercise Sheet 1

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/winter22/random

Total Points: 100 Due: Wednesday, **November 9**, 2022

You are allowed to collaborate on the exercise sheets. Justify your answers. Cite all external sources that you use (books, websites, research papers, etc.). You need to collect at least 50% of all points on exercise sheets to be admitted to the exam.
Please hand in your solutions before the lecture on the day of the deadline.

Exercise 1 — **10 + 10** points

Suppose that a randomized algorithm $\mathcal{A}$ has a one-sided error and a success probability of $p \in (0, 1)$.

a) How often do you need to run $\mathcal{A}$ to achieve a success probability of at least $1/2$?

b) How often do you need to run $\mathcal{A}$ to achieve a success probability of at least $b \in (0, 1)$?

Exercise 2 — **10 + 20** points

Consider the following problem.

PIT: Given two univariate polynomials $P(x) = \prod_{i=1}^{d}(x - a_i)$ and $Q(x) = \sum_{i=0}^{d} c_i x^i$, each of degree at most d, check if P and Q are the same polynomial. (For example, the polynomials $(x - 1)(x - 5)$ and $x^2 - 6x + 5$ are equal, but the polynomials $x(x - 9)$ and $x^2 - 9x + 9$ are not.)

a) Give a deterministic algorithm for PIT. What is the running time of your algorithm?

b) Give a randomized algorithm that solves PIT in time $O(d)$ with a success probability of at least $1/2$.

Exercise 3 — **25** points

Recall the simple contraction algorithm from the lecture for computing a min-cut in a graph $G = (V, E)$: we pick an edge of G uniformly at random and contract is (deleting self-loops but keeping parallel edges) until G contains exactly 2 vertices.

Show how to implement this algorithm so that it runs in time $O((|V| + |E|) \log |E|)$.

Exercise 4 — **25** points

Consider the following game. Shuffle a standard deck of 52 cards and distribute the card onto 13 piles of 4 cards each. Label the piles with A, 2, 3, …, 10, J, Q, K. Now take the top card from the pile “K”. Next, continue to take the top card from pile “X”, where X is the value of the previous card you took. You win if you manage to collect all cards, you loose otherwise.

What is the probability that you win this game?