Machine Learning
Boosting

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Lecture 15, 10.12.2018

Roadmap of today and next lecture

Classification methods:

- **Boosting**,
- Decision Trees,
- Neural Networks (aka Deep Learning),
- **Nearest Neighbor Methods, Parzen-Window**,

Roadmap of today

Boosting

- **Given:** a set of (bad) classifiers.
  Question: Is there a way to combine them so that they yield a reasonable classifier ?
- Boosting “boosts” existing bad/weak classifiers.
- Developed by Schapire and Freund (1996) - but concept is older.
- Final classification can often be implemented quite efficiently - interesting for real time application e.g. face detection on a video sequence.

Boosting belongs to the ensemble methods

History of Boosting

- Freund and Schapire propose Adaboost in 1996,
  Folklore result: Adaboost does not overfit !
- Friedman, Hastie and Tibshirani (2000) provide interpretation of Adaboost in terms of mimimization of empirical exponential loss
  $\Longrightarrow$ provides a general scheme for boosting which leads to emergence of new variants.
- Lugosi and Vayatis (2004) prove for a regularized boosting variant that it is Bayes consistent.
- Bartlett and Traskin (2007) prove that Adaboost is Bayes consistent.
- still ongoing discussion about interpretation after more than 10 years of boosting (recent controversial JMLR article).

Boosting II

General scheme for boosting methods

1. For each training point (X_i, Y_i) one has a weight γ_i.
2. A step of boosting method involves the following steps
    1. one trains a classifier f_k using a base method (weak learner) with the weighted training data (X_i, Y_i, γ_i),
    2. one re-computes the weights γ, where usually the weights of wrongly classified training points are increasing and the weights of correctly classified points are decreasing.
3. One aggregates the classifiers f_k to the final classifier $F(x) = \mathrm{sign}(\sum_{k=1} \alpha_k f_k)$, where the coefficients α_k are either one or depend on the error of the classifier f_k.

Variants of Boosting

Caution

- Several boosting methods have been proposed (huge literature),
- Differences are often very subtle,
- Different requirements on the properties of the weak learner.

Note: Boosting heavily depends on the weak learner
$\Longrightarrow$ two boosting methods with different weak learner cannot be compared !

Today: **Adaboost** and **GentleBoost**.

Properties

- Adaboost stands for "Adaptive Boosting"
- Many variants - we discuss Adaboost.M1 proposed by Freund and Schapire in 1996.
- Weak learner has to be **binary-valued** !
- Depending on the weak learner - final classifier allows interpretation (boosted decision stumps).

Adaboost

Input: Training data $(X_i, Y_i)_{i=1}^n$, binary-valued Weak Learner, Number of iterations T.
Initialize weights: $\gamma_i^1 = \frac{1}{n}$, $i = 1, \ldots, n$ For $t = 1, \ldots, T$, 1) Fit the weak learner, $f_t : \mathcal{X} \to \{-1, 1\}$, with weights γ_i. The weak learner uses the weighted zero-one loss $L(f) = \sum_{i=1}^n \gamma_i \mathbb{1}_{f(X_i) \neq Y_i}$ 2) compute the weighted error of f_t, $\mathrm{err}_t = \sum_{i=1}^n \gamma_i \mathbb{1}_{f(X_i) \neq Y_i}$, 3) define $c_t = \log\left(\frac{1-\mathrm{err}_t}{\mathrm{err_t}}\right)$, 4) update the weights γ_i^t, $\gamma_i^{t+1} = \gamma_i^t \exp(c_t \mathbb{1}_{Y_i \neq f_t(X_i)})$, 5) renormalize so that $\sum_{i=1}^n \gamma_i^{t+1} = 1$.
Output: final classifier $f(x) = \mathrm{sign}(\sum_{t=1}^T c_t f_t(x))$.

Adaboost II

Idea of Adaboost:

- in each step: fit weak learner to data,
- data which is misclassified gets higher weight
  $\Longrightarrow$ next weak learner will try harder to fit misclassified points,
- contribution c_t of each weak classifier f_t to the final hypothesis,

$$c_t = \log\left(\frac{1 - \mathrm{err}_t}{\mathrm{err}_t}\right)$$

  $\Longrightarrow$ strictly monotonically decreasing with err_t.
- final classifier: $f(x) = \mathrm{sign}\left(\sum_{t=1}^{T} c_t f_t(x)\right)$,
  $\Longrightarrow$ Adaboost learns a point in the vector space of functions generated by the weak learner.

Adaboost in action

Decision stump as weak learner:

$$f_t(x) = 2\mathbb{1}_{x_i+b>0} - 1 \quad \text{and} \quad f_t(x) = 2\mathbb{1}_{-x_i+b>0} - 1.$$

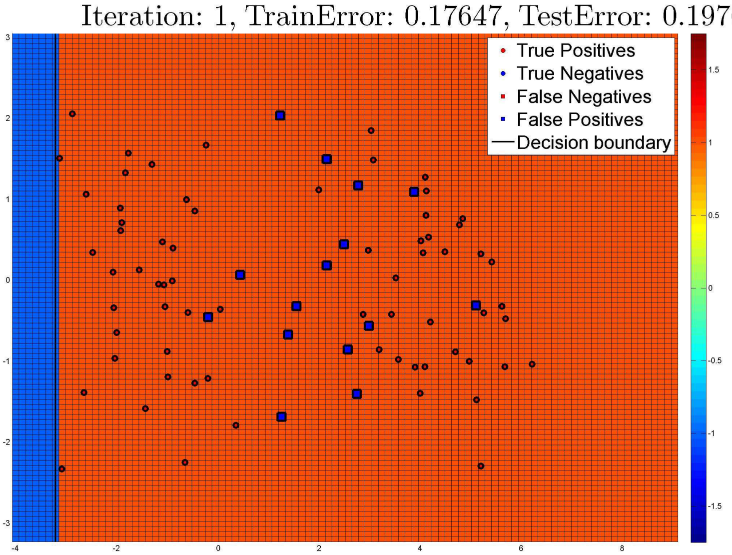
or more generally,

$$f_t(x) = 2\mathbb{1}_{\langle w,x\rangle+b>0} - 1.$$

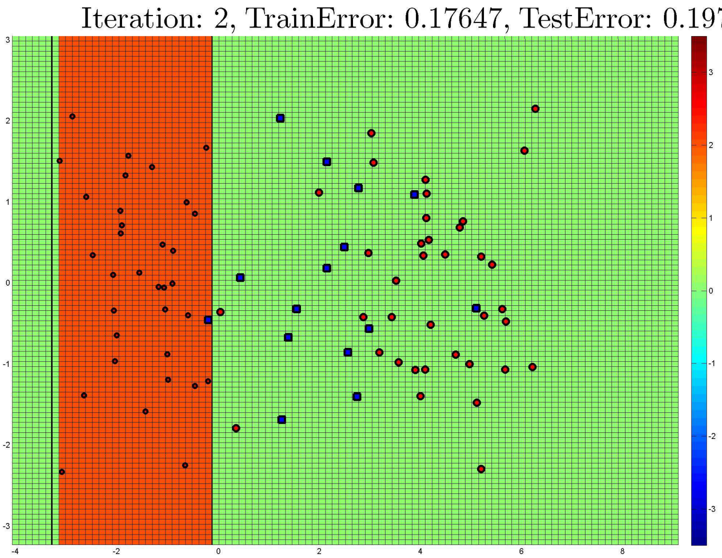
Selection of the weak learner:

- random (e.g. randomly select coordinate or direction w)
- compute for all choices weak learner (if possible) and take the one with the smallest weighted error.

Adaboost in action



Adaboost in action



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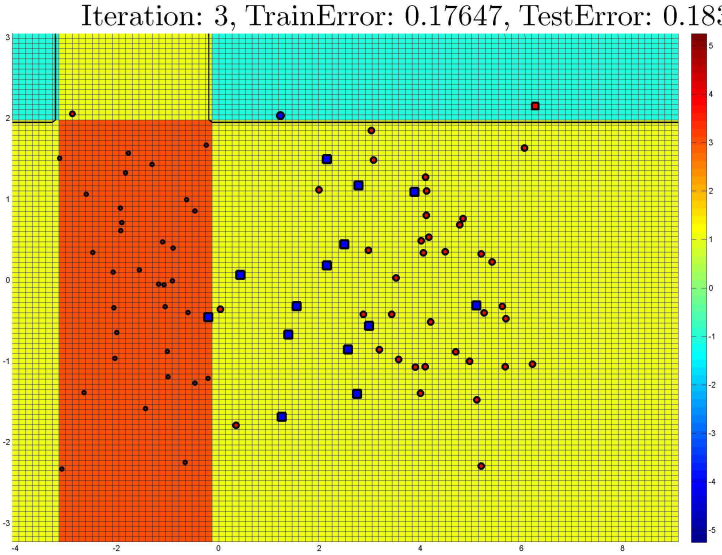

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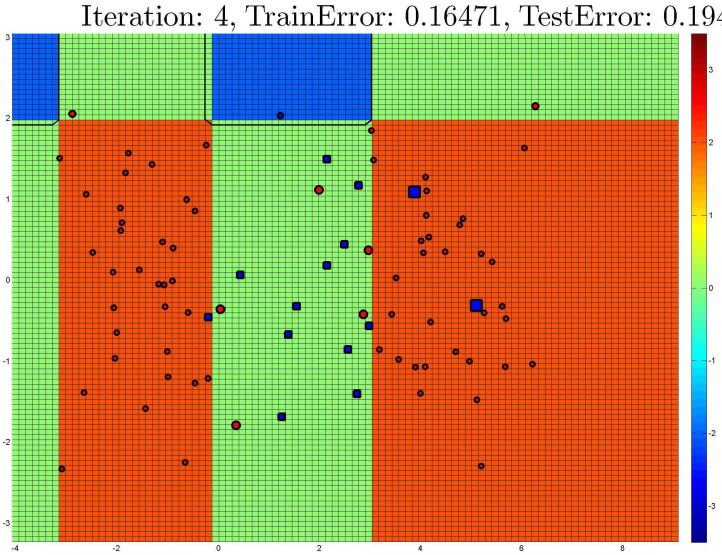

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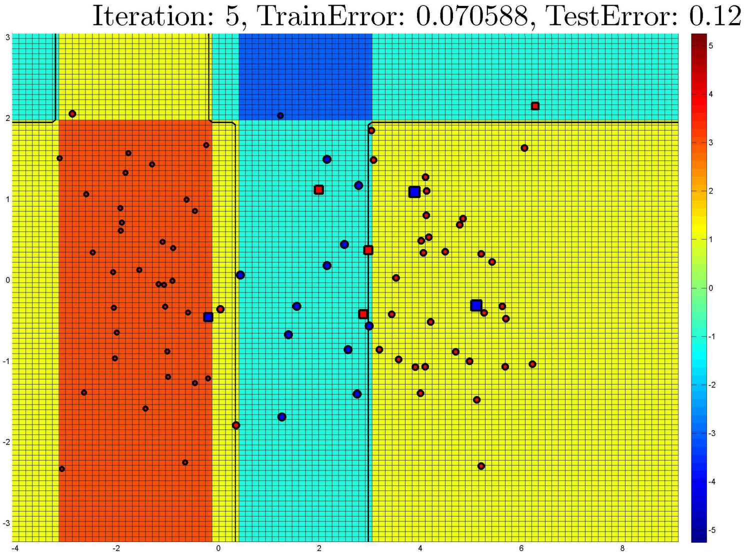

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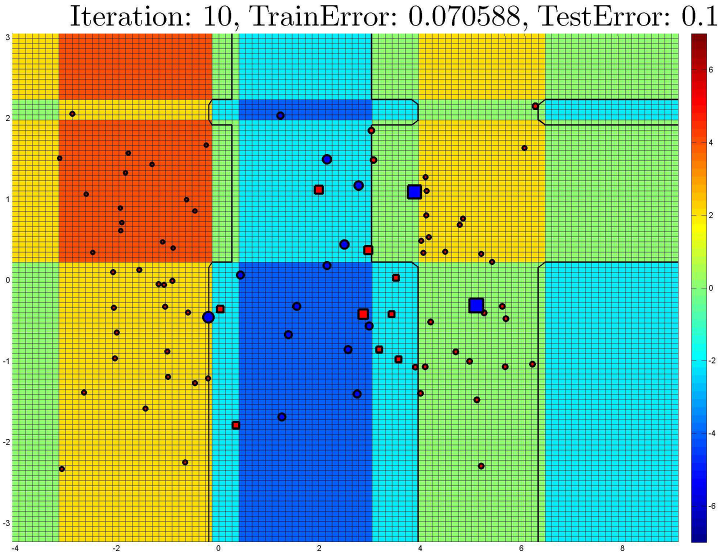

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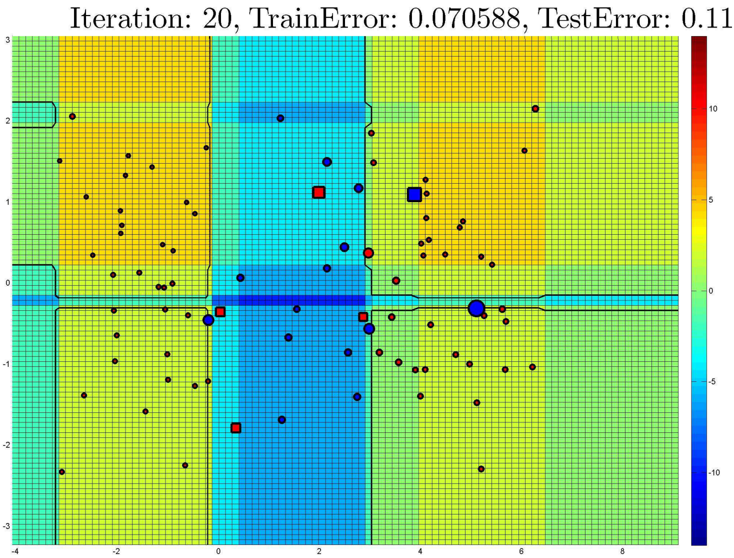

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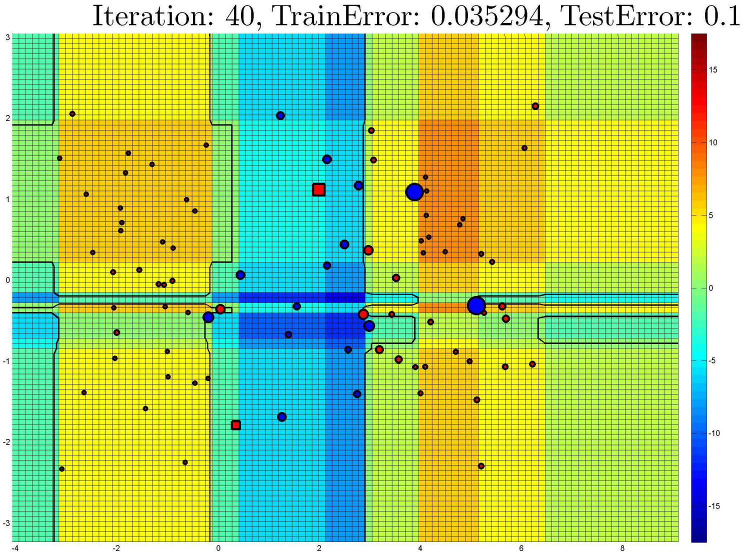

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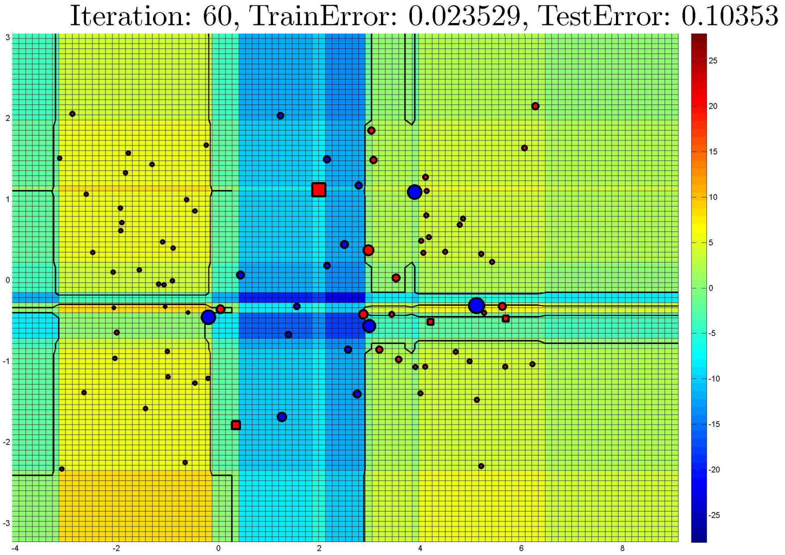

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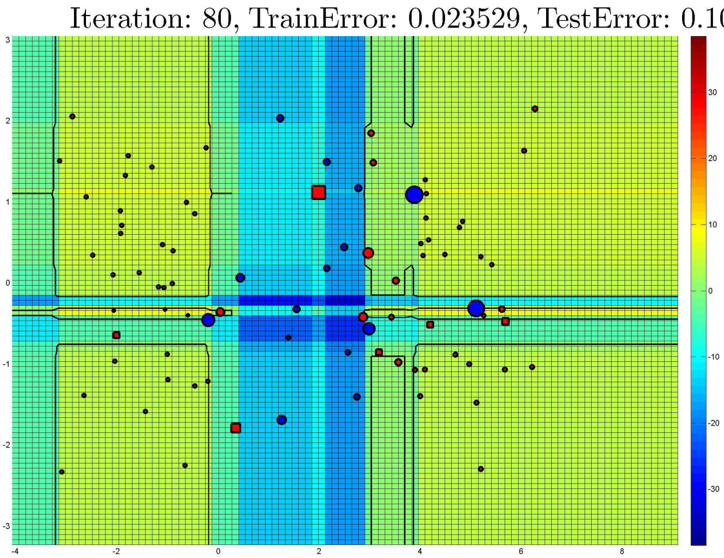

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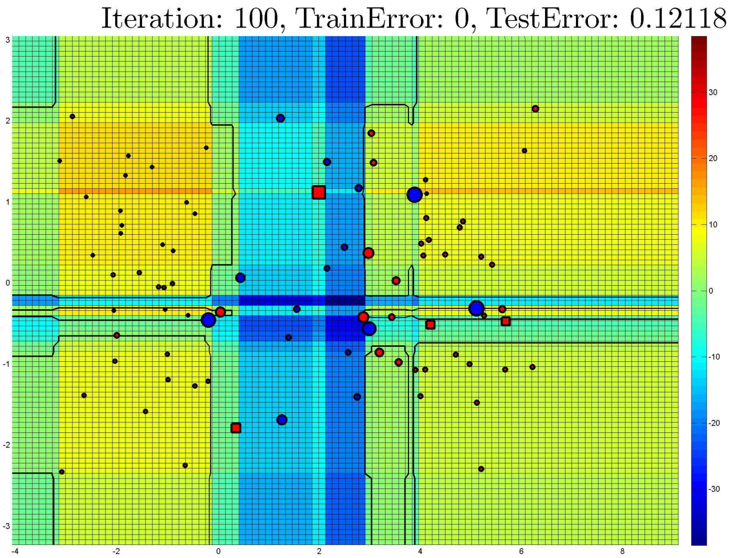


Adaboost in action

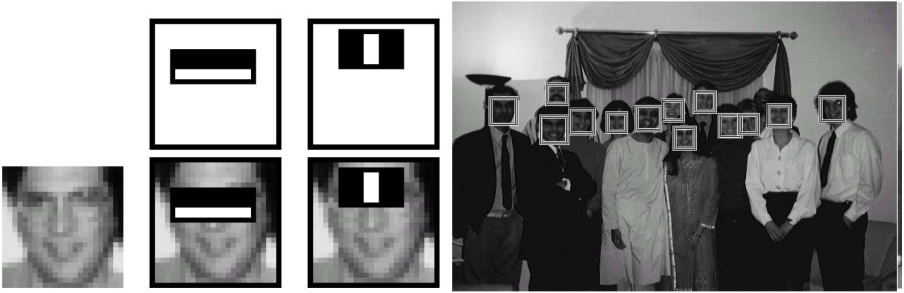
Adaboost in Computer Vision

- final classification using decision stump can be done extremely fast $\Longrightarrow$ very interesting for applications where real-time performance is required.
- Most successful application of Adaboost in face recognition by Viola and Jones (2001).
  They use more generally weak learner of the form,

  $$f_t(x) = 2\mathbb{1}_{\langle w,x\rangle + b > 0} - 1$$

  but coefficients w are restricted to be binary valued or zero (most coefficients of the weight vector w are zero).

Face Recognition



Interpretation of boosting

- Iterative updates,
$$F(x) \;\mapsto\; F(x) + c_t f_t(x),$$
can be understood as a descent in function space, where
  1. f_t is a descent direction based on the current F,
  2. c_t is the stepsize of the descent step.
- The objective which is minimized is the empirical exponential loss of the final classifier F,
$$L(F) = \frac{1}{n}\sum_{i=1}^{n} e^{-Y_i F(X_i)}.$$

Interpretation of boosting

- initiated simultaneously in the statistics and machine learning community,
- following Proposition is a variation of the result of Friedman, Hastie and Tibshirani (2000).

Proposition

Suppose the weak learner f_t is binary-valued, $f_t : \mathcal{X} \to \{-1, 1\}$. The update step $F_{t+1} = F_t + c_t f_t$ of the Adaboost algorithm , where $c_t = \log\left(\frac{1-\mathrm{err}_t}{\mathrm{err}_t}\right)$ and err_t is the weighted zero-one loss, is a descent step in order to minimize the empirical exponential loss.

$\Rightarrow$ proof on blackboard !

Result for Adaboost

Corollary

After each update of the weights, the weighted misclassification error of the most recent weak learner is 50%.

Proof: This follows directly from the optimality condition for the parameter c derived in the last proof,

$$\mathbb{E}[Y\, f(X)\, e^{-Y\, F(X)}\, e^{-Y\, f(X)\alpha}] = 0.$$

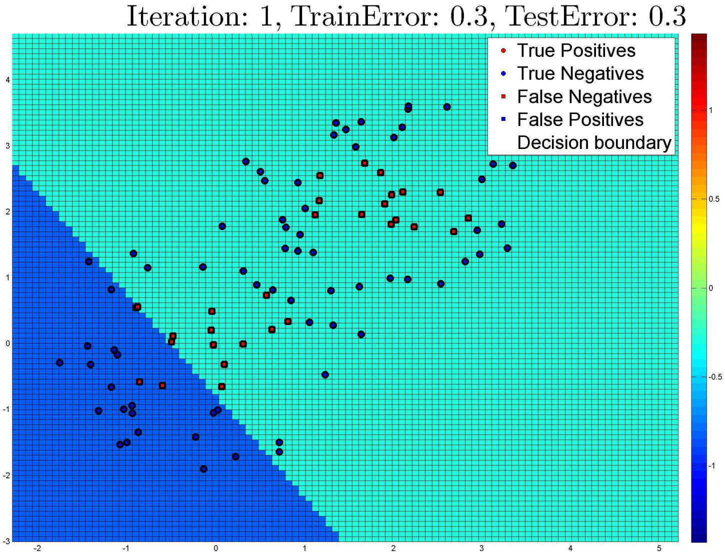
Note, that the second factor corresponds to the new weights (up to normalization) and the weighted zero one loss of f is given as

$$\frac{\frac{1}{2}\mathbb{E}[(1 - Y\, f(X))e^{-Y\, F(X)}\, e^{-Y\, f(X)\alpha}]}{\mathbb{E}[e^{-Y\, F(X)}\, e^{-Y\, f(X)\alpha}]} = \frac{1}{2}.$$

Theory has motivated many variants of boosting $\Longrightarrow$ **GentleBoost**

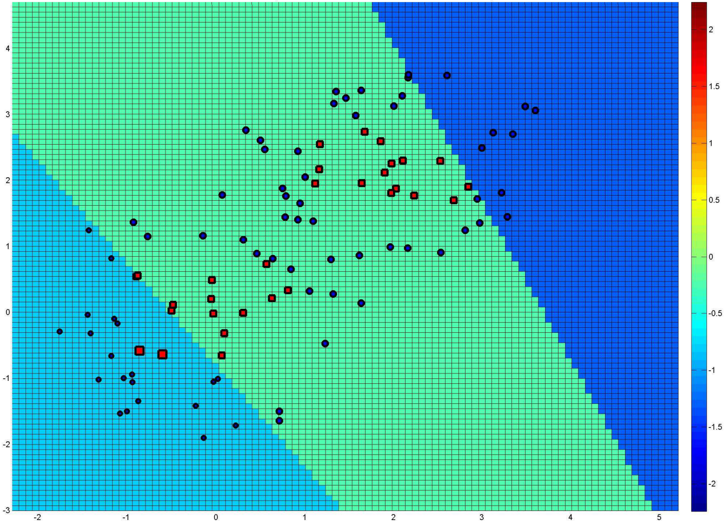
Input: Training data $(X_i, Y_i)_{i=1}^n$, real-valued Weak Learner, Number of iterations T.
Initialize weights: $\gamma_i^1 = \frac{1}{n}$, $i = 1, \dots, n$. For $t = 1, \dots, T$, 1) Fit the weak learner, $f_t : \mathcal{X} \to \mathbb{R}$, with weights γ_i. The weak learner uses weighted squared loss $L(f) = \sum_{i=1}^n \gamma_i (Y_i - f_t(X_i))^2$, 2) update the weights γ_i^t, $\gamma_i^{t+1} = \gamma_i^t \exp(-Y_i f_t(X_i))$, 5) renormalize so that $\sum_{i=1}^n \gamma_i^{t+1} = 1$.
Output: final classifier $f(x) = \mathrm{sign}\left(\sum_{t=1}^T f_t(x)\right)$.

GentleBoost in action

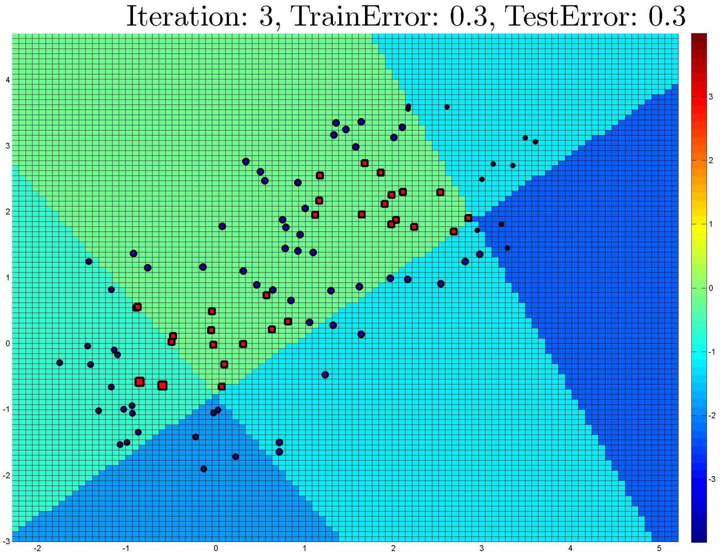


GentleBoost in action

Iteration: 2, TrainError: 0.3, TestError: 0.3



GentleBoost in action



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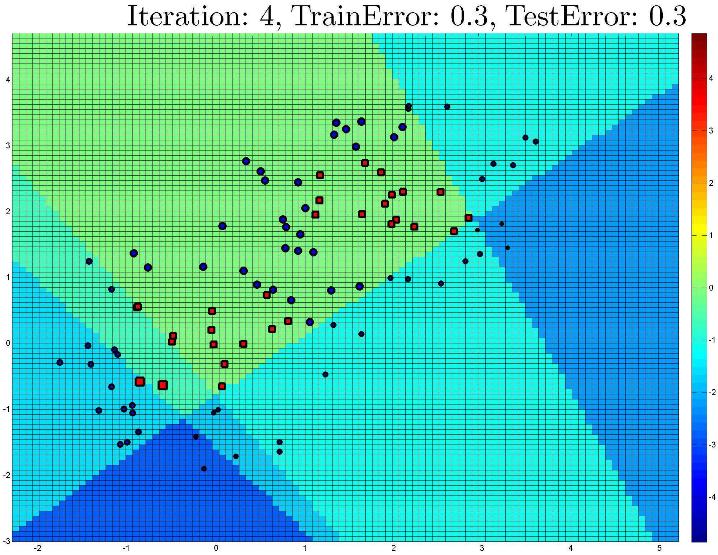

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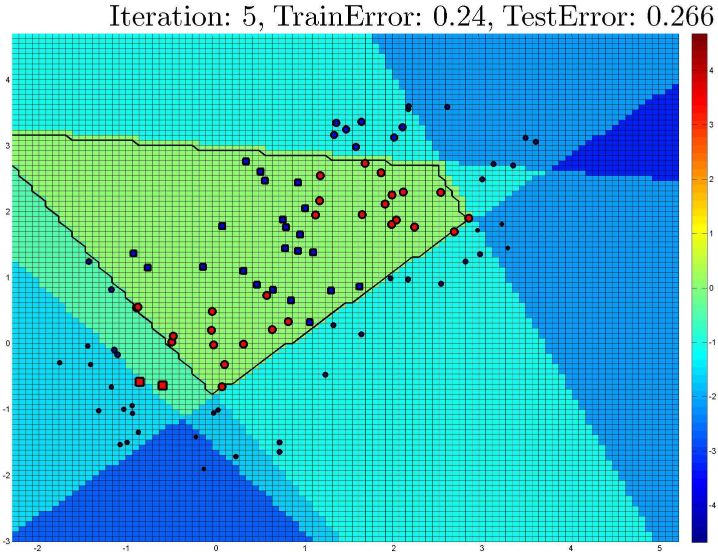

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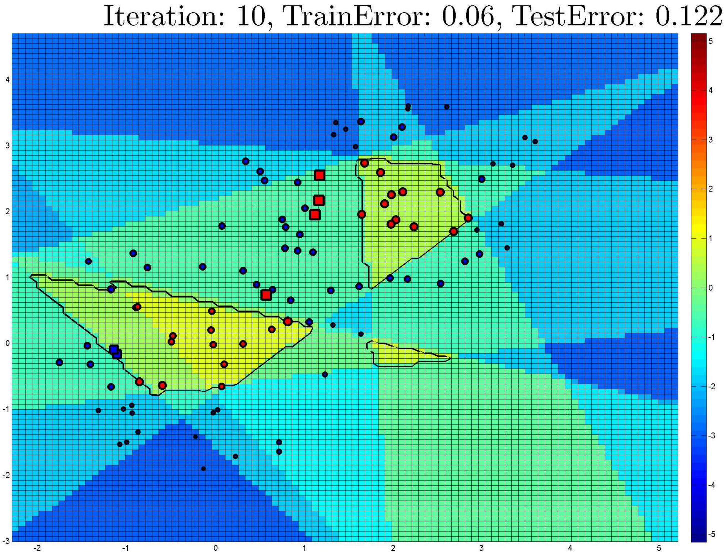

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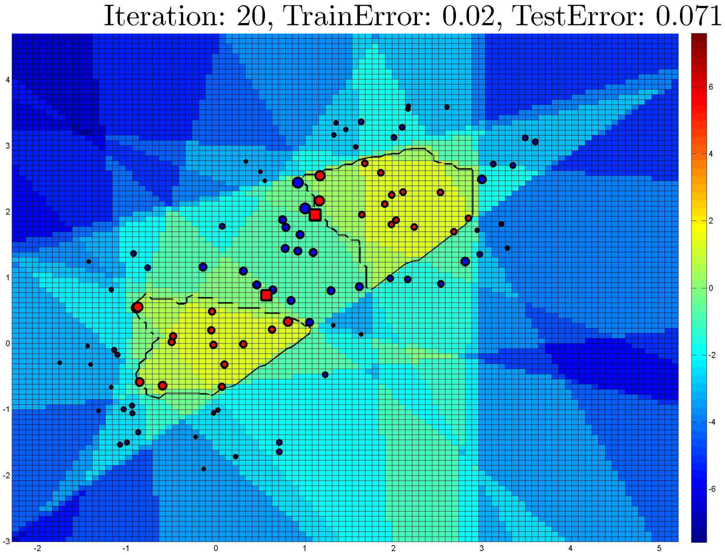

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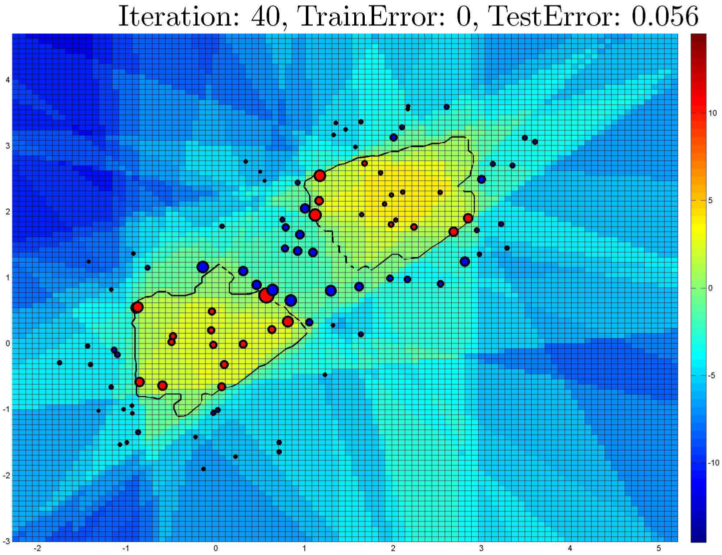

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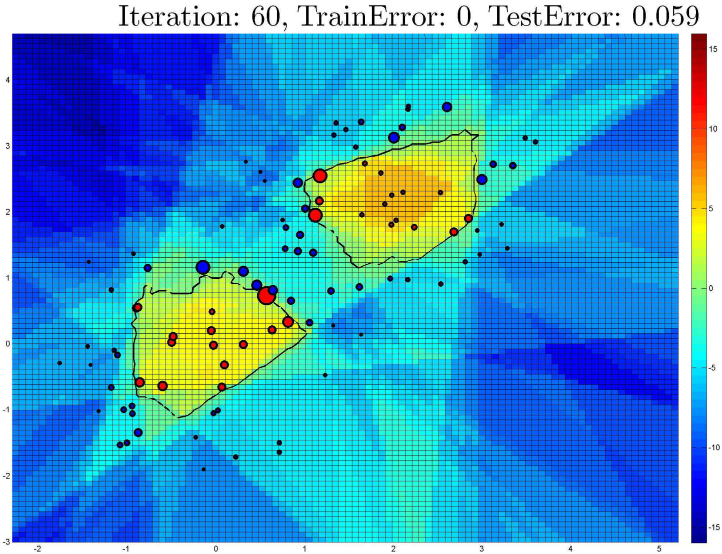

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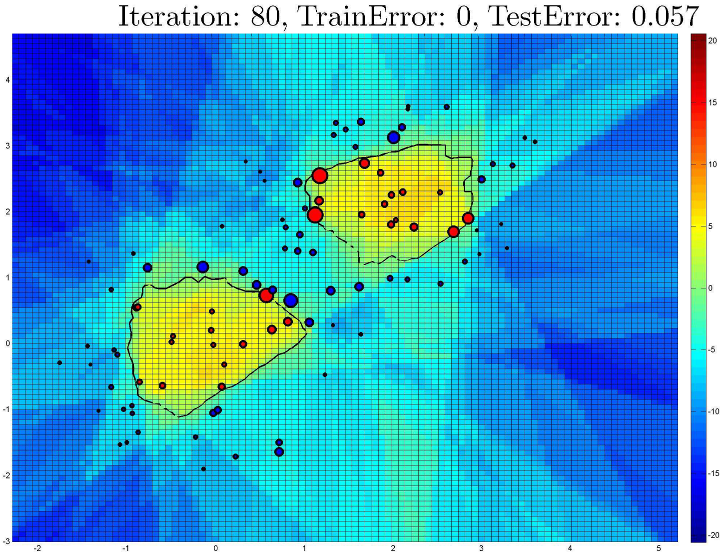

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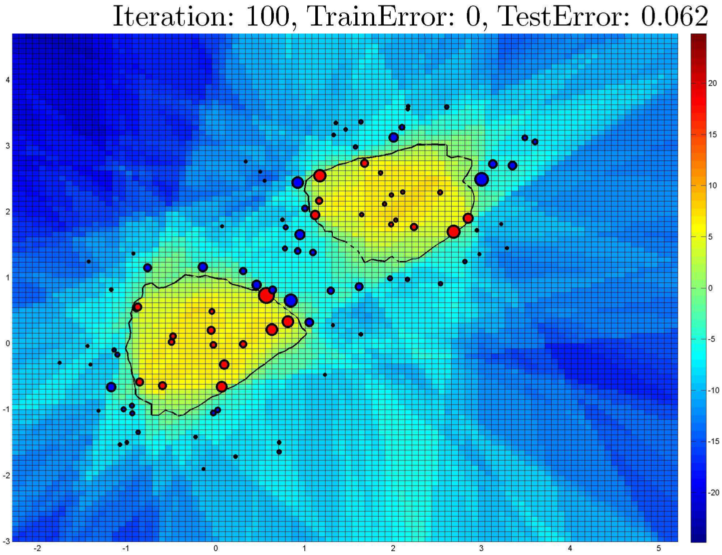

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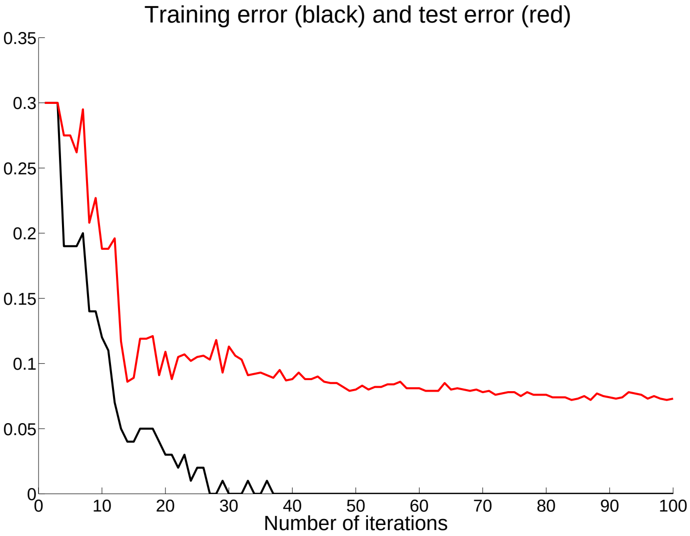

Proposition

Suppose the weak learner f_t is real-valued, $f_t : \mathcal{X} \to \mathbb{R}$. The update step $F_{t+1} = F_t + f_t$ of the GentleBoost algorithm is an approximate Newton step in order to minimize the empirical exponential loss.

Proof: We can expand the risk of $F + f$ up to second order,

$$R(F+f) = \mathbb{E}[e^{-Y(F(X)+f(X))}] \approx \mathbb{E}[e^{-Y F(X)}(1 - Y f(X) + \frac{1}{2} f(X)^2)].$$

The first term does not depend on f so we can modify it without changing the minimizer,

$$\mathbb{E}[e^{-Y F(X)}(\frac{1}{2} - Y f(X) + \frac{1}{2} f(X)^2)] = \frac{1}{2}\mathbb{E}[e^{-Y F(X)}(Y - f(X))^2],$$

which is up to the normalization of the weights $\gamma_i = e^{-Y_i F(X_i)}$ equal to the weighted squared loss minimized by the weak learner $\Rightarrow$ weak learner minimizes in each step a second order approximation of the exponential loss.