Probabilistic Graphical Models
Exercise Sheet No. 2

Due Date: November 16th, 10 am

Hand in: At beginning of tutorial OR at office 629 (Jan-Hendrik) OR by e-mail to (jlange[at]mpi-inf.mpg.de). Begin the subject of your e-mail with [pgm].

Please do not forget to put your name on the submission.

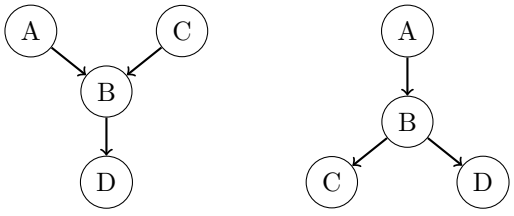
1 Belief Networks & Markov Networks

Points: 10

Assume that the day of the week that cats are born on, x, is independent of the day of the week, y that dogs are born on. However, assume that a pet's character (either a or b) is dependent on the day of the week that the pet is born on. Further, suppose whether or not a cat and a dog can live together in the same household, indicated by h, depends strongly on the pets' characters, but is independent of x and y if their characters are known.

1. Draw a belief network that can represent the described setting.
2. What can be said about the (graphical) dependency between the days of the week that a cat and a dog are born on, given that the pets cannot live in the same household?

Consider the following two belief networks:



3. For each network determine whether there is another Markov equivalent belief network. If yes, draw one. If no, give reason why there cannot be such a network.

Consider the pairwise Markov network given by

$$p(x) = \phi(x_1, x_2)\phi(x_2, x_3)\phi(x_3, x_4)\phi(x_4, x_1).$$

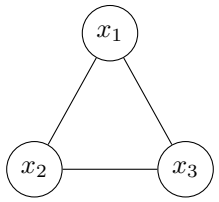
4. Draw a graph that represents p.

5. Express the probability $p(x_4 \mid x_1, x_3)$ purely in terms of the function ϕ.

2 Local Polytope Relaxation

Points: 4

Let $x_1, x_2, x_3 \in \{0, 1\}$ be binary variables and consider the energy function $E_\theta(x) = \sum_{i \in V} \theta_i(x_i) + \sum_{\{i,j\} \in E} \theta_{i,j}(x_i, x_j)$ that is defined on the following graph $G = (V, E)$



with potentials θ given by

x_i	$\theta_1(x_1)$	$\theta_2(x_2)$	$\theta_3(x_3)$
0	0	0	0
1	−1	−1	−1

$$\theta_{i,j}(x_i, x_j) = \begin{cases} 2 & \text{if } x_i = x_j = 1 \\ 0 & \text{else.} \end{cases}$$

1. Determine some labeling $x^* \in \{0, 1\}^3$ that minimizes E_θ.

2. Determine some $\mu^* \in L$, where L denotes the local polytope, such that $\langle \theta, \mu^* \rangle < E_\theta(x^*)$.

3 Semiglobal Matching

Points: 2

Consider again the network from the previous task, but replace the unary potentials with

x_i	$\theta_1(x_1)$	$\theta_2(x_2)$	$\theta_3(x_3)$
0	0	1	2
1	−1	0	1

Calculate x_3^* by semiglobal matching, taking into account the lines $x_1 \to x_3$ and $x_2 \to x_3$.