Probabilistic Graphical Models
Exercise Sheet No. 3

Due Date: November 30th, 10 am

Hand in: By e-mail to (jlange[at]mpi-inf.mpg.de). Your submission must include your source code files. Begin the subject of your e-mail with [pgm].

1 Binary Image Denoising

Points: 3

In binary image denoising we are given an observed black-and-white noisy image y and want to recover the original uncorrupted image x as good as possible. This can modeled as an undirected graphical model with random variables $x_i, y_i \in \{-1, 1\}$ for each pixel $i = 1, \ldots, N$, where $x_i = 1$ means pixel i is white. The x_i are arranged in a grid graph and each y_i is connected by an edge with x_i. The joint distribution $p(x, y) \propto \exp(-E(x, y))$ is described by the energy function

$$E(x, y) = h \sum_{i=1}^{N} x_i - \beta \sum_{i \sim j} x_i x_j - \eta \sum_{i=1}^{N} x_i y_i. \tag{1}$$

The constants $\beta, \eta \geq 0, h \in \mathbb{R}$ are model parameters and $i \sim j$ denotes that the variables x_i and x_j are horizontally/vertically adjacent.

a) What are the roles of the three terms hx_i, $-\beta x_i x_j$ and $-\eta x_i y_i$? For each of them explain how a high (respectively low) parameter value influences the model.

2 Iterated Conditional Modes

Points: 7

Download the noisy image y and the ground truth image x^{GT} from the course website. In order to recover an original image x given a noisy observation y, we need to compute the MAP estimate

$$x^* = \operatorname*{argmax}_x p(x \mid y). \tag{2}$$

b) Rewrite (2) as an energy minimization problem.

A simple heuristic method to tackle (2) is the *Iterated Conditional Modes* (ICM) algorithm. It works by iteratively optimizing one variable x_i at a time while keeping all other variables fixed.

b) Implement Iterated Conditional Modes (ICM) to compute a heuristic solution to (2).

c) Set the parameters of the model (1) to $h = 0.04$, $\beta = 0.93$ and $\eta = 0.75$. Compute an estimate for x^* by ICM starting from the noisy image y with an update order of your choice. Your program must report the fraction of incorrectly recovered pixels

$$L(x, x^{\mathrm{GT}}) = \frac{1}{N} \sum_{i=1}^{N} [x_i \neq x_i^{\mathrm{GT}}]$$

for your estimated image x.

2.1 Block-ICM (Bonus Task)

Points: 4

d) Implement Block-ICM where each block is a single row (or column) of the image. For this you need to implement an exact solver for the energy minimization problem on chain graphs (dynamic programming).

e) Repeat task c) with your Block-ICM algorithm.

3 Max Product Linear Programming

Points: 6

d) Implement Max Product Linear Programming (MPLP) to solve the dual problem corresponding to (2).

e) Apply your algorithm to the noisy image y with the same parameter settings as in c). Reconstruct images from your dual solution by naïve rounding and by subsequently running ICM (initialized with your rounding solution). Report the primal-dual gap of your solutions. Compare the loss $L(x, x^{\mathrm{GT}})$ to your results from the previous task.