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# Body Models 3

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Max Planck Institute for Informatics

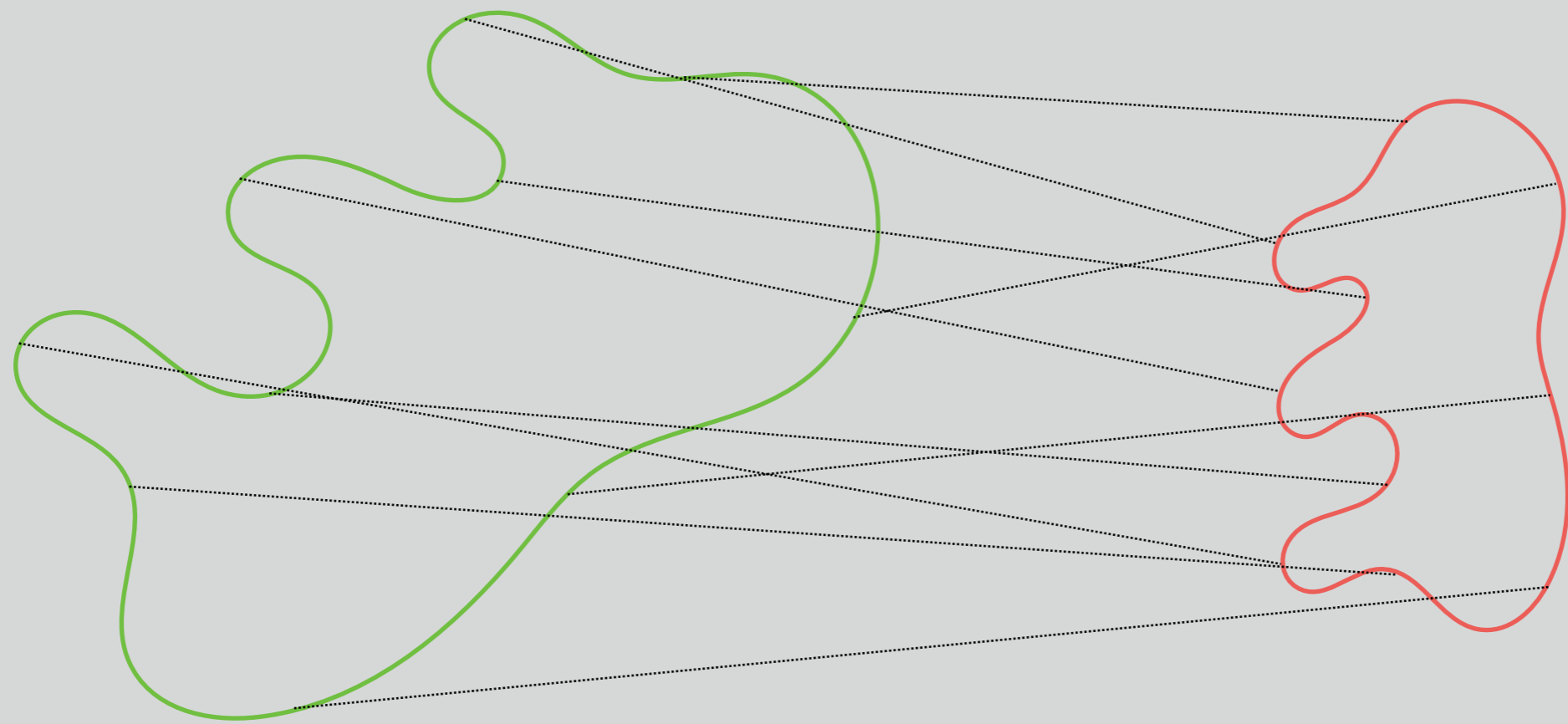
January 9, 2019

# Schedule

17.10.2018	An Optimization Perspective
24.10.2018	Introduction to probabilities and
31.10.2018	An Optimization Perspective
07.11.2018	An Optimization Perspective
14.11.2018	An Optimization Perspective
21.11.2018	An Optimization Perspective
12.12.2018	Body Models 1
19.12.2018	Body Models 2
<b>09.01.2019</b>	<b>Body Models 3</b>
<del>16.01.2019</del> 11.01.2019	Sampling and Tracking
23.01.2019	Graphical Models in Computer Vision
06.02.2019	Wrap-up

# What have we learned so far about bodies?

- BM1: Procrustes for rigid alignment

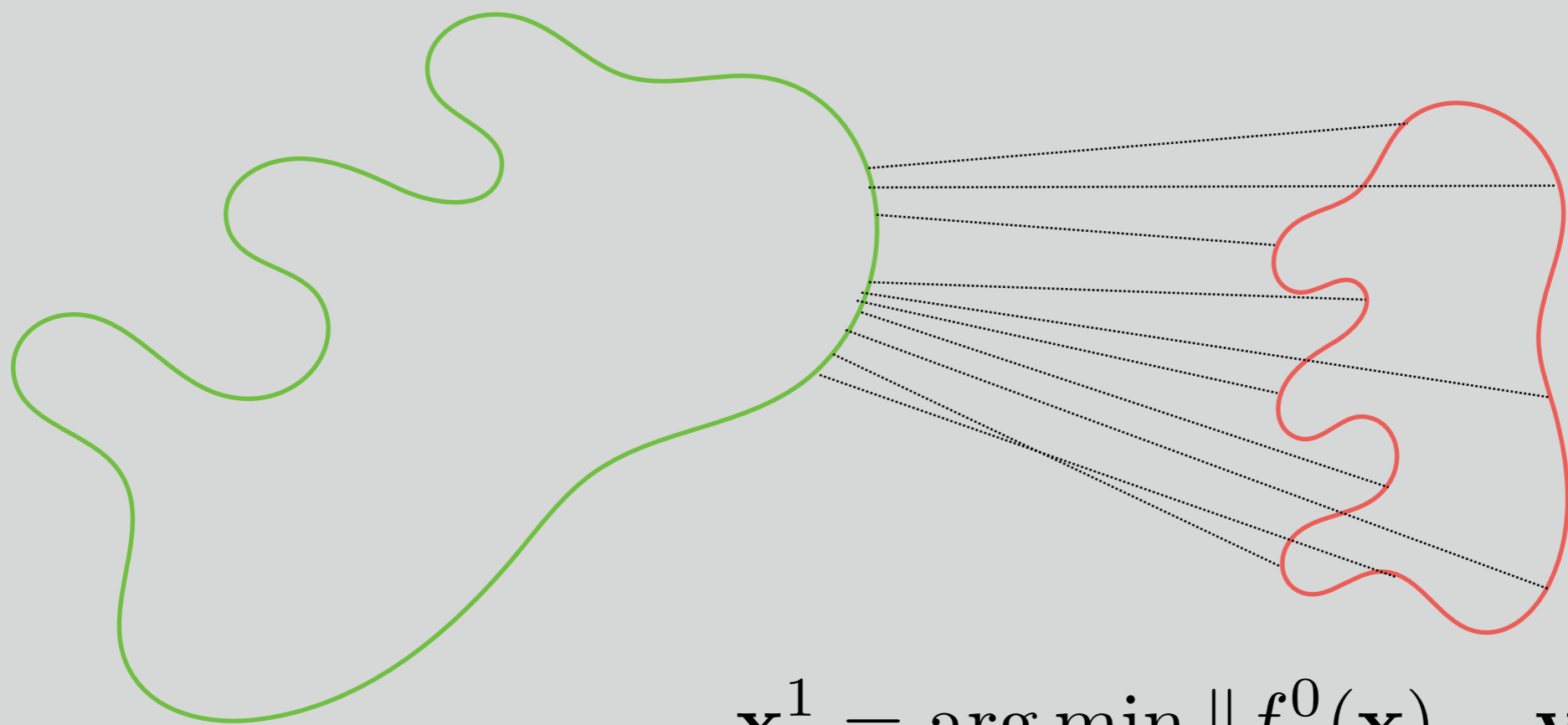


solve with procrustes  
single step

$$\rightarrow f = \arg \min_f \sum_i \|f(\mathbf{x}_i) - \mathbf{y}_i\|^2$$

# What have we learned so far about bodies?

- BM1: Procrustes for rigid alignment
- BM2: ICP, gradient-based ICP

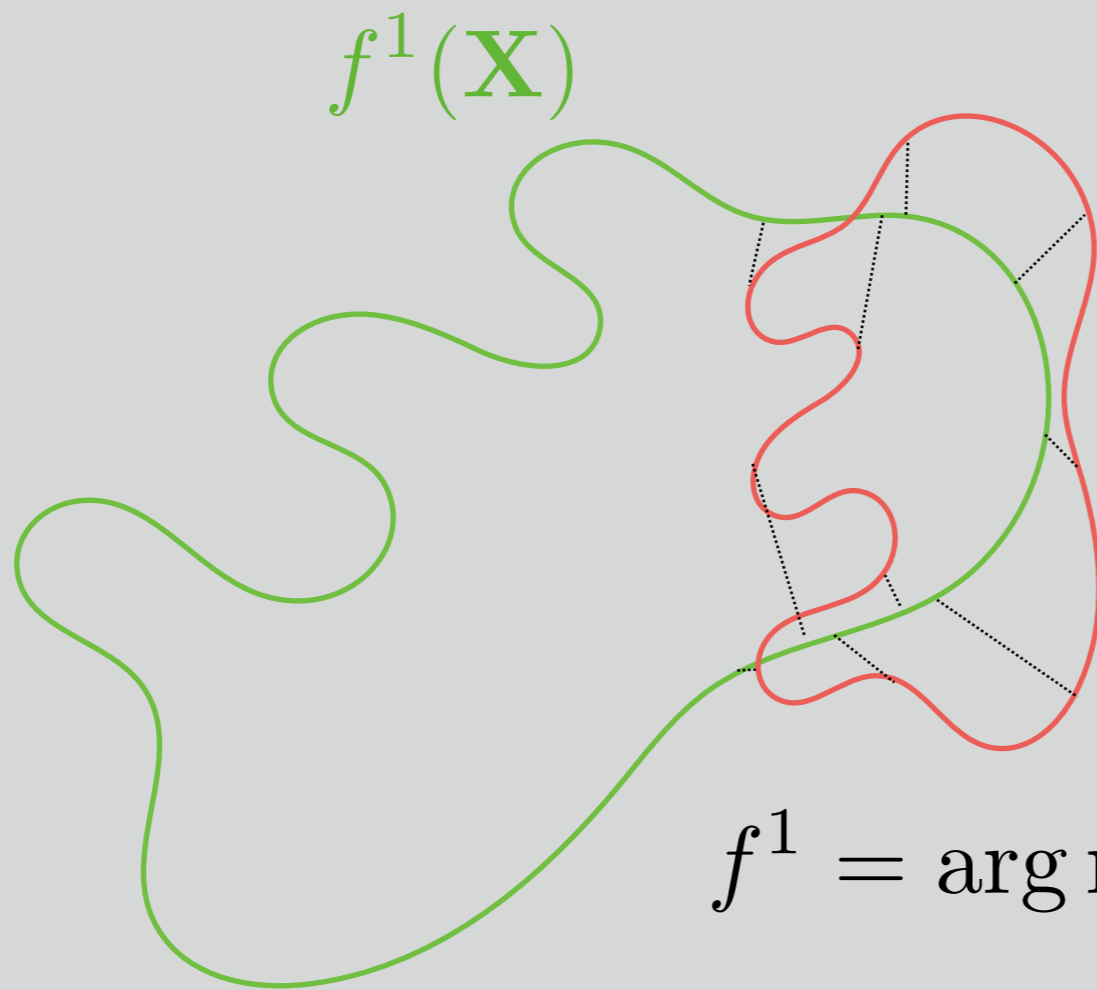


$$\mathbf{x}_i^1 = \arg \min_{\mathbf{x} \in \mathbf{X}} \|f^0(\mathbf{x}) - \mathbf{y}_i\|^2$$

solve with procrustes  
or gradient-based

$$\rightarrow f^1 = \arg \min_f \sum_i \|f(\mathbf{x}_i^1) - \mathbf{y}_i\|^2$$

and iterate!



$$f^1 = \arg \min_f \sum_i \|f(\mathbf{x}_i^1) - \mathbf{y}_i\|^2$$

$$\mathbf{x}_i^2 = \arg \min_{\mathbf{x} \in \mathbf{X}} \|f^1(\mathbf{x}) - \mathbf{y}_i\|^2$$

and iterate!

$f^j(\mathbf{X})$



$$f^j = \arg \min_f \sum_i \|f(\mathbf{x}_i^j) - \mathbf{y}_i\|^2$$

$$\mathbf{x}_i^{j+1} = \arg \min_{\mathbf{x} \in \mathbf{X}} \|f^j(\mathbf{x}) - \mathbf{y}_i\|^2$$



and iterate!

$f^j(\mathbf{X})$



$$f^j = \arg \min_f \sum_i \|f(\mathbf{x}_i^j) - \mathbf{y}_i\|^2$$

$$\mathbf{x}_i^{j+1} = \arg \min_{\mathbf{x} \in \mathbf{X}} \|f^j(\mathbf{x}) - \mathbf{y}_i\|^2$$

and iterate!

$f^j(\mathbf{X})$



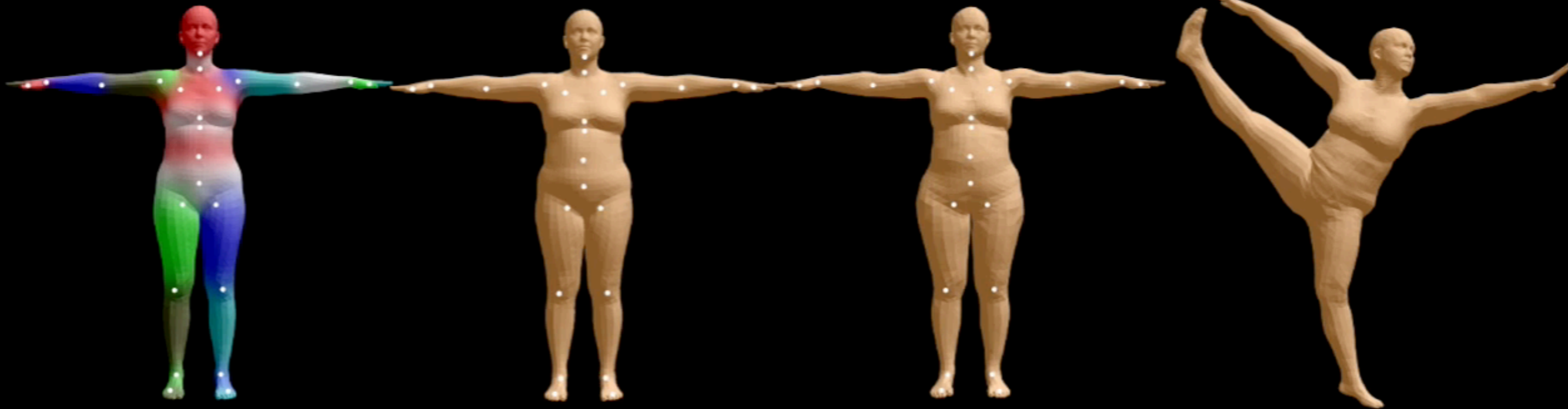
$$f^j = \arg \min_f \sum_i \|f(\mathbf{x}_i^j) - \mathbf{y}_i\|^2$$

$$\mathbf{x}_i^{j+1} = \arg \min_{\mathbf{x} \in \mathbf{X}} \|f^j(\mathbf{x}) - \mathbf{y}_i\|^2$$

# What have we learned so far about bodies?

- BM1: Procrustes for rigid alignment
- BM2: ICP, gradient-based ICP
- BM3: Articulated models, Blendshapes, SMPL

# SMPL Model Pipeline



Template Mesh

Shape  
Blend Shapes

Pose  
Blend Shapes

Final Mesh

# Parameterized Skinning

Standard skinning  $W(\mathbf{T}, \mathbf{J}, \mathcal{W}, \vec{\theta}) \mapsto \text{vertices}$

SMPL model

$M(\vec{\theta}, \vec{\beta}) = W(\mathbf{T}_F(\vec{\beta}, \theta), \mathbf{J}(\vec{\beta}), \mathcal{W}, \vec{\theta}) \mapsto \text{vertices}$

SMPL is skinning parameterized by pose  $\vec{\theta}$   
and shape  $\vec{\beta}$

# What is missing: today

- How do we fit SMPL to meshes without correspondences?
- Where is the color in those meshes?
- Autodiff in images? OpenDR
- Fitting bodies to images

# Fitting SMPL to a scan/mesh

- Problem: Given a registration, find the model pose and shape.

some distance function between the two meshes

$$\vec{\theta}, \vec{\beta} = \arg \min_{\vec{\theta}, \vec{\beta}} d(M(\vec{\theta}, \vec{\beta}) - \mathbf{V})^2$$

Model

Scan

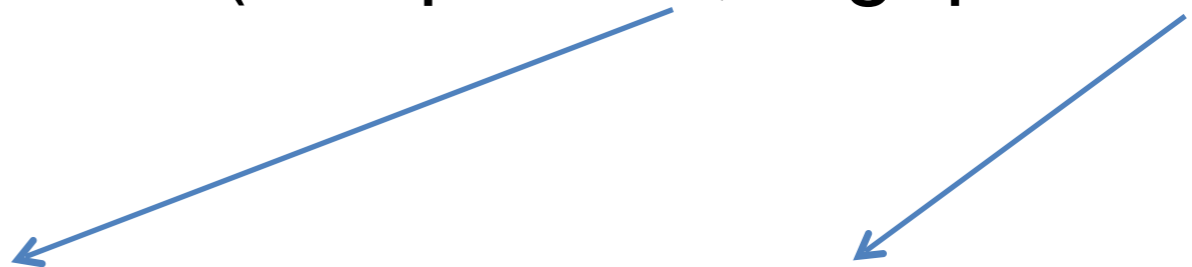
# Fitting SMPL to a scan/mesh

- Problem: Given a registration, find the model pose and shape.

```
from smpl.serialization import load_model
sm = load_model(path_to_downloaded_model)
ch.minimize(point2point_squared(dst_pts=sm, org_pts=Xch),
x0=[sm.betas, sm.pose])
```

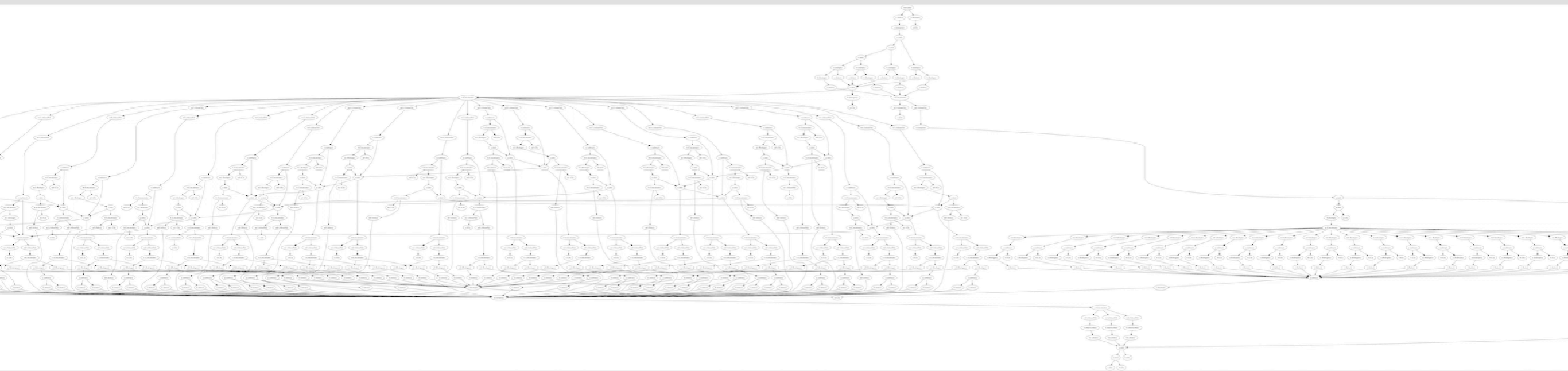
Model

Scan





# SMPL tree: `sm.show_tree()`



- Chumpy minimizes the **sum of squares** of a **vector valued error** function

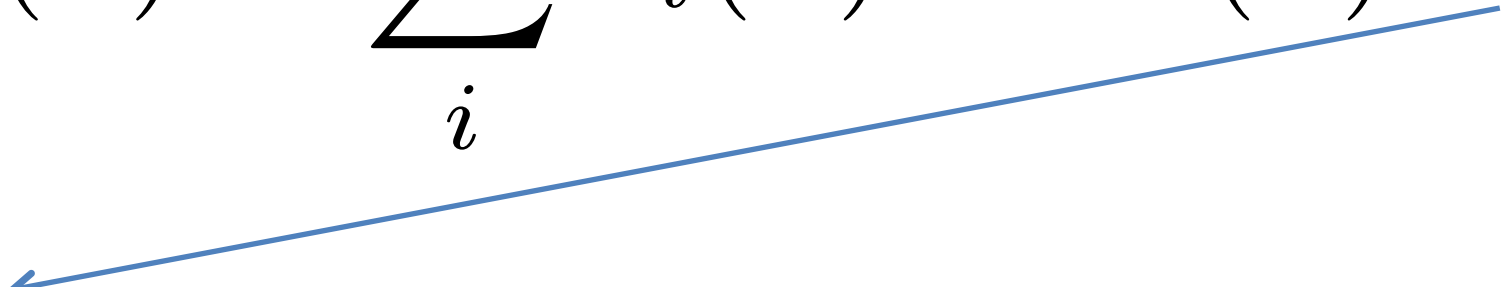
Optimization variables (vector)

$$e(\mathbf{x}) = \sum_i \mathbf{e}_i(\mathbf{x})^2 = \mathbf{e}(\mathbf{x})^T \mathbf{e}(\mathbf{x})$$

Sum of squares  
(scalar)

Residuals  
(vector valued error function)

- Chumpy minimizes the **sum of squares** of a **vector valued error** function

$$e(\mathbf{x}) = \sum_i \mathbf{e}_i(\mathbf{x})^2 = \mathbf{e}(\mathbf{x})^T \mathbf{e}(\mathbf{x})$$


```
ipdb> p2p_yx = point2point_squared(org_pts=Xch, dst_pts=sm)
```

```
ipdb> print(p2p_yx)
```

```
[ 0.001  0.    0.001 ...,  0.012  0.012  0.012]
```

```
ipdb> p2p_yx.shape
```

```
(6890,)
```

as many elements as correspondences between model and scan

Jacobian of the vector valued error function:

$$J_{\mathbf{e}}(\mathbf{x}) = \frac{d\mathbf{e}(\mathbf{x})}{d\mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{e}_1}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \mathbf{e}_1}{\partial \mathbf{x}_P} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{e}_N}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \mathbf{e}_N}{\partial \mathbf{x}_P} \end{bmatrix}$$

P parameters

N residuals

$$J_e(\mathbf{x}) = \frac{d\mathbf{e}(\mathbf{x})}{d\mathbf{x}} = \begin{bmatrix} \frac{\partial e_1}{\partial x_1} & \cdots & \frac{\partial e_1}{\partial x_P} \\ \vdots & \ddots & \vdots \\ \frac{\partial e_N}{\partial x_1} & \cdots & \frac{\partial e_N}{\partial x_P} \end{bmatrix}$$

N residuals

P parameters

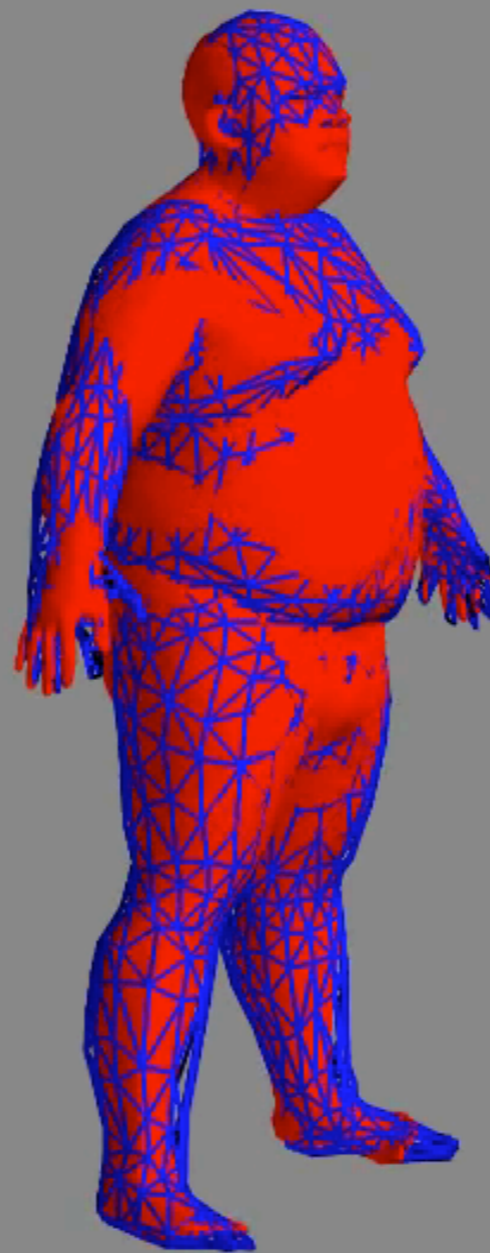
```
ipdb> print(p2p_yx.dr_wrt(sm.betas).shape)
```

```
(6890, 10)
```

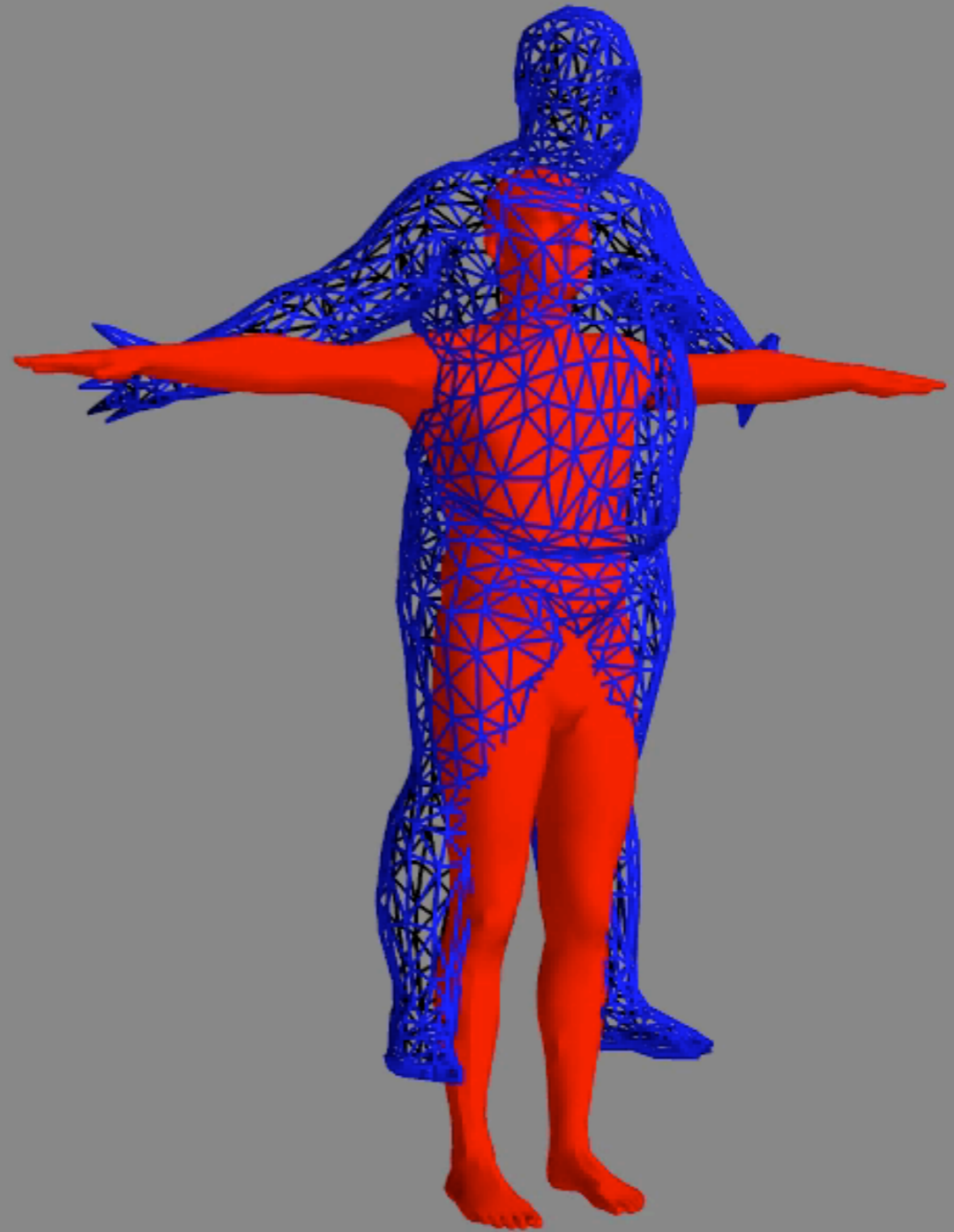
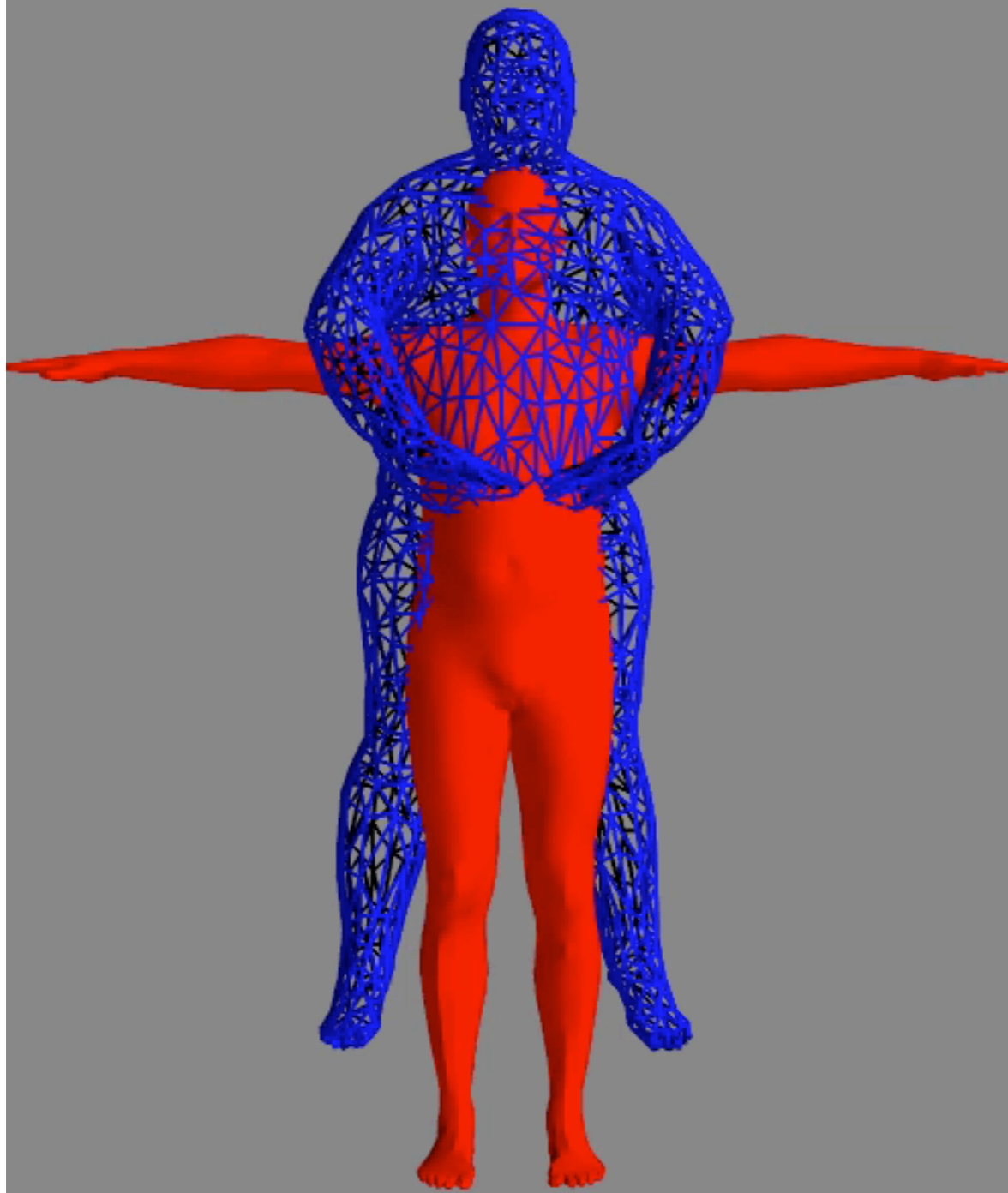
```
ipdb> print(p2p_yx.dr_wrt(sm.betas)[:5, :5].todense())
```

```
[[ -1.144e-04  -1.148e-04   3.350e-05  -2.048e-05   8.550e-06]
 [  3.490e-04  -4.617e-05  -1.243e-04  -7.371e-05   3.262e-05]
 [  5.642e-04  -1.518e-04  -2.017e-04  -1.487e-04   9.339e-05]
 [  2.437e-04  -2.448e-04  -9.368e-05  -1.272e-04   9.360e-05]
 [  8.284e-04  -1.090e-04  -2.925e-04  -1.700e-04   9.579e-05]]
```

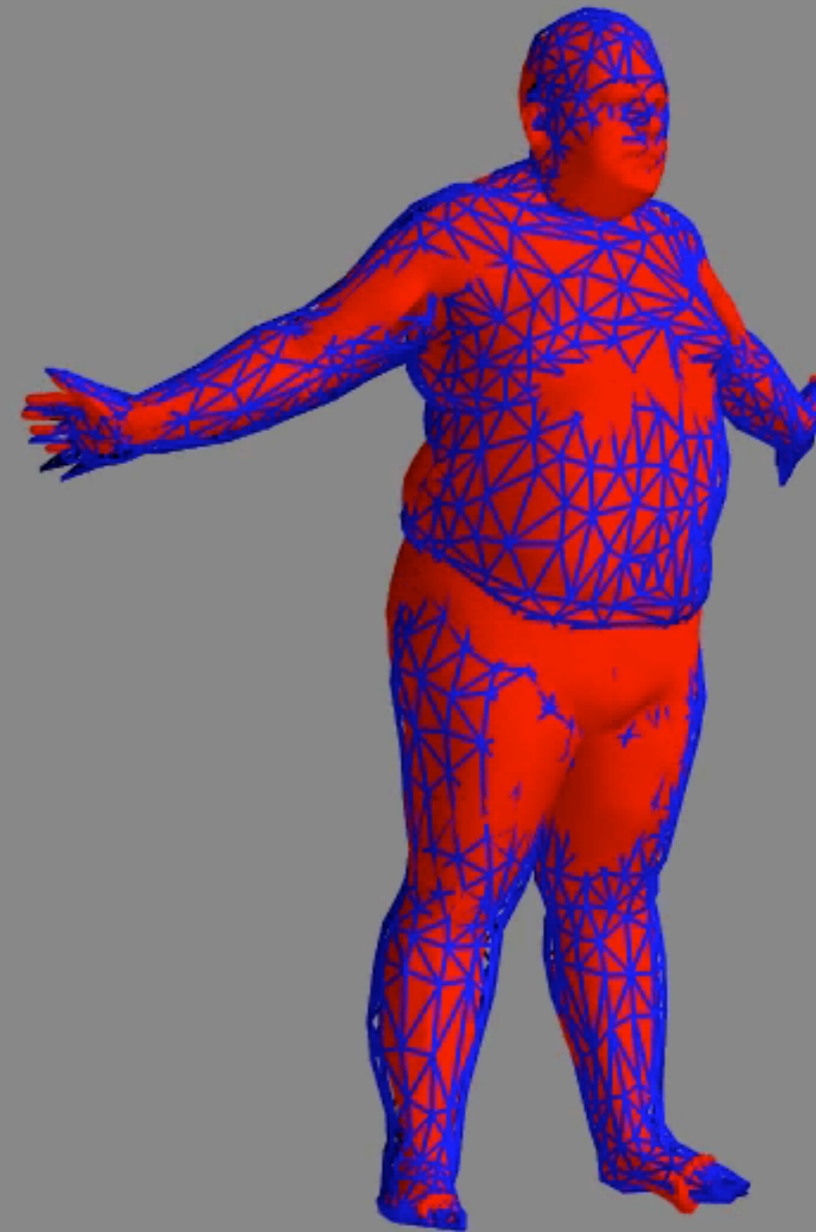
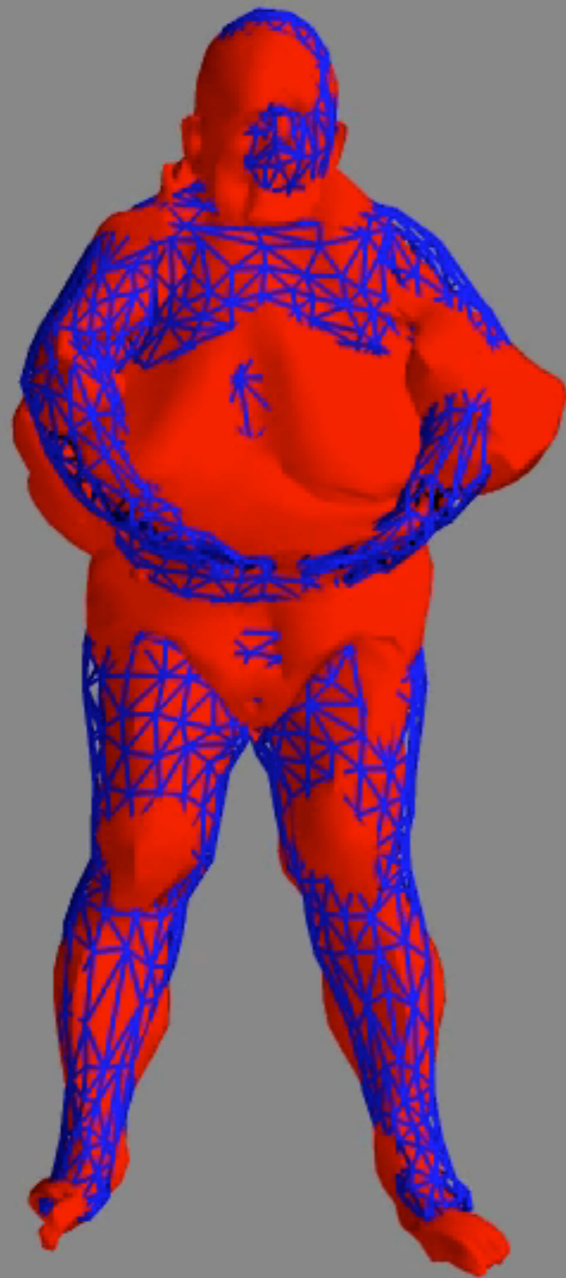
# Try it!



# Which one will fail?



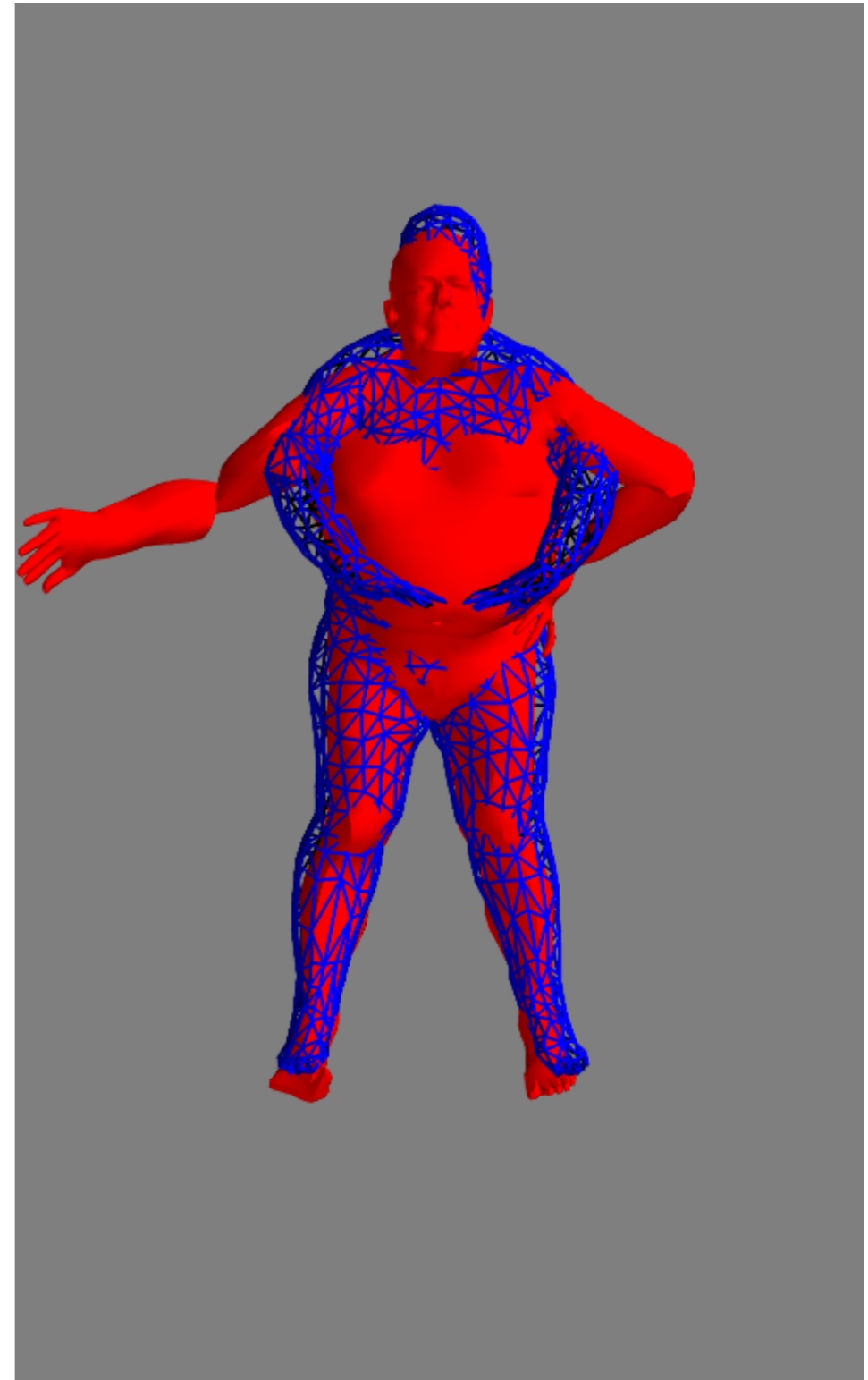
# Which one will fail?





# Problems?

- Unlikely pose



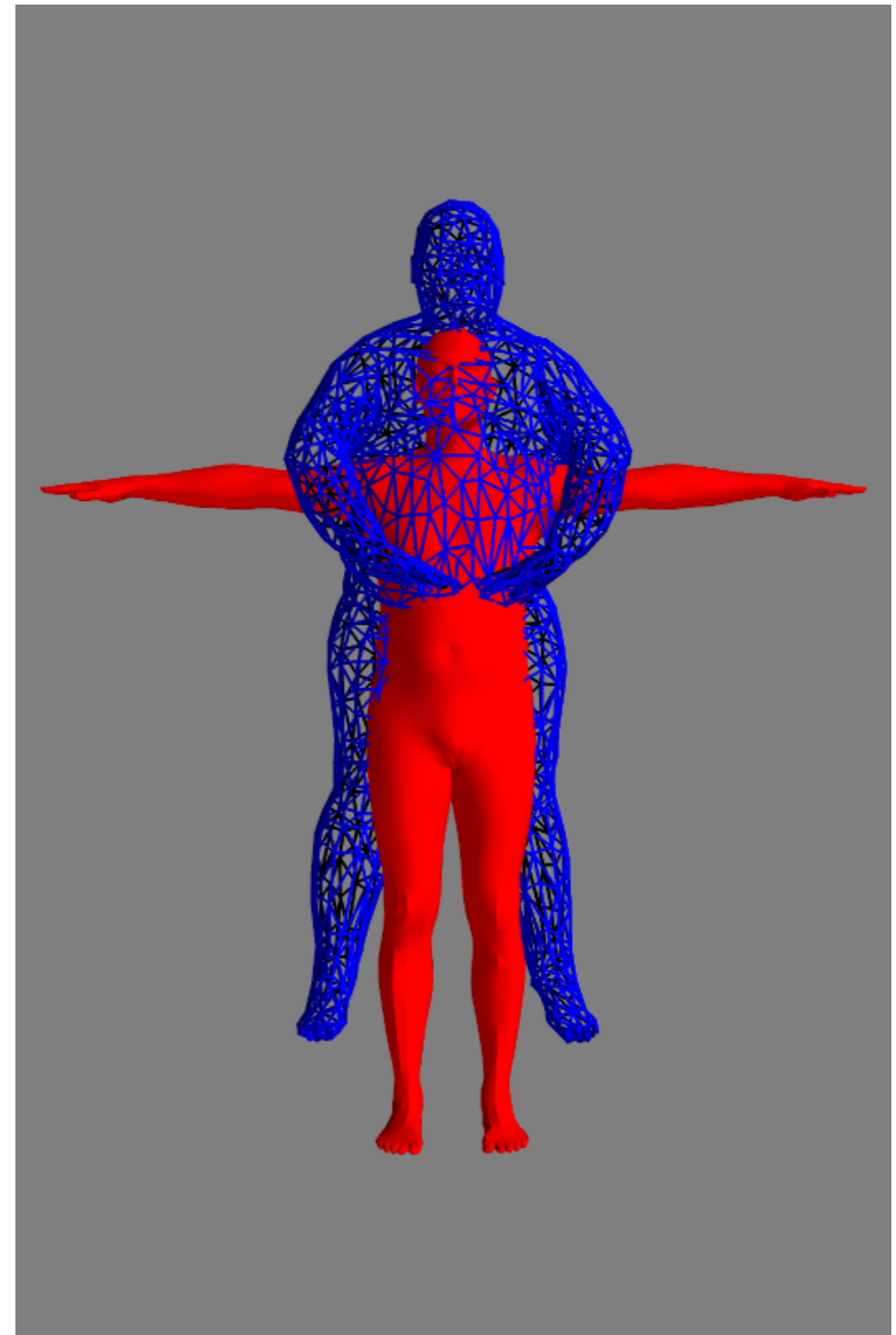
# Problems?

- Unlikely pose
- Unlikely shape



# Problems?

- Difficult pose
- Difficult shape
- Bad initialization

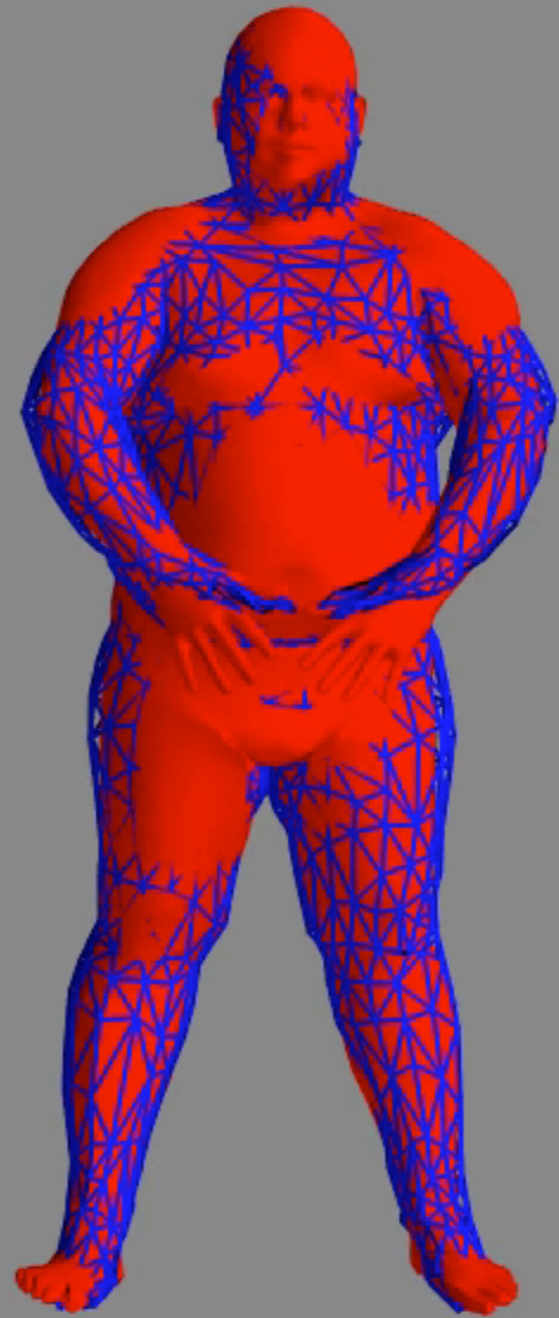


# Fitting SMPL to a scan/mesh

$$\vec{\theta}, \vec{\beta} = \arg \min_{\vec{\theta}, \vec{\beta}} \|M(\vec{\theta}, \vec{\beta}) - \mathbf{V}\|^2$$

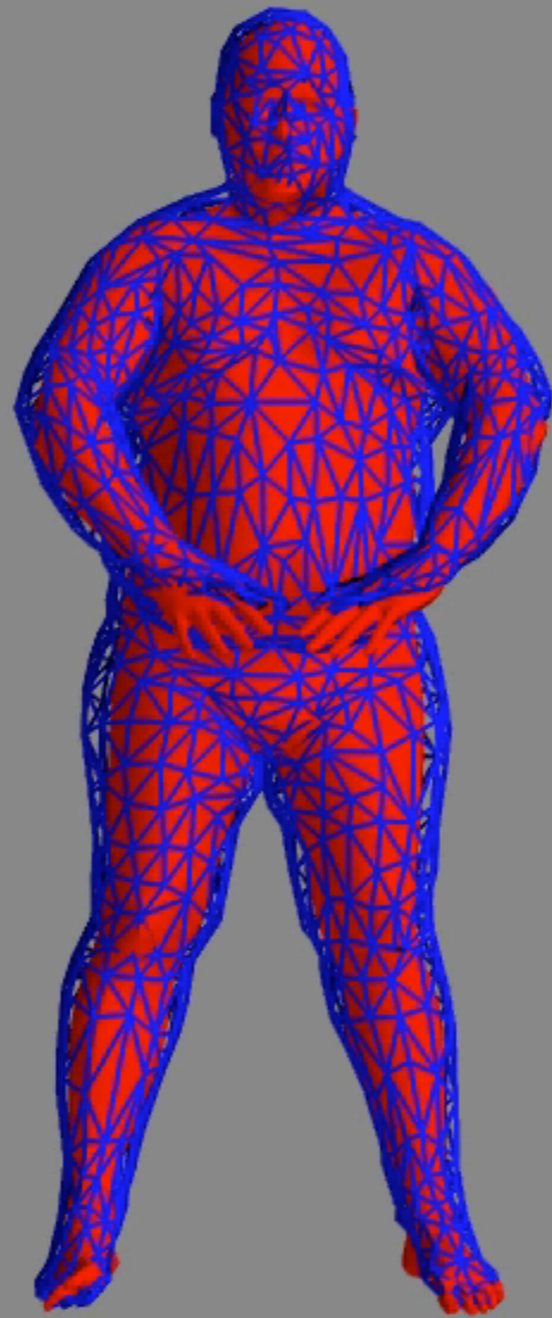
# Fitting SMPL to a scan/mesh

$$\begin{aligned} \vec{\theta}, \vec{\beta} = \arg \min_{\vec{\theta}, \vec{\beta}} & \|M(\vec{\theta}, \vec{\beta}) - \mathbf{V}\|^2 \\ & + E_{\theta}(\vec{\theta}) \end{aligned}$$



# Fitting SMPL to a scan/mesh

$$\begin{aligned} \vec{\theta}, \vec{\beta} = \arg \min_{\vec{\theta}, \vec{\beta}} & \|M(\vec{\theta}, \vec{\beta}) - \mathbf{V}\|^2 \\ & + E_{\theta}(\vec{\theta}) \\ & + E_{\beta}(\vec{\beta}) \end{aligned}$$





# Fitting SMPL to a scan/mesh

$$\vec{\theta}, \vec{\beta} = \arg \min_{\vec{\theta}, \vec{\beta}} \|M(\vec{\theta}, \vec{\beta}) - \mathbf{V}\|^2$$

$$+ E_{\theta}(\vec{\theta})$$

$$+ E_{\beta}(\vec{\beta})$$

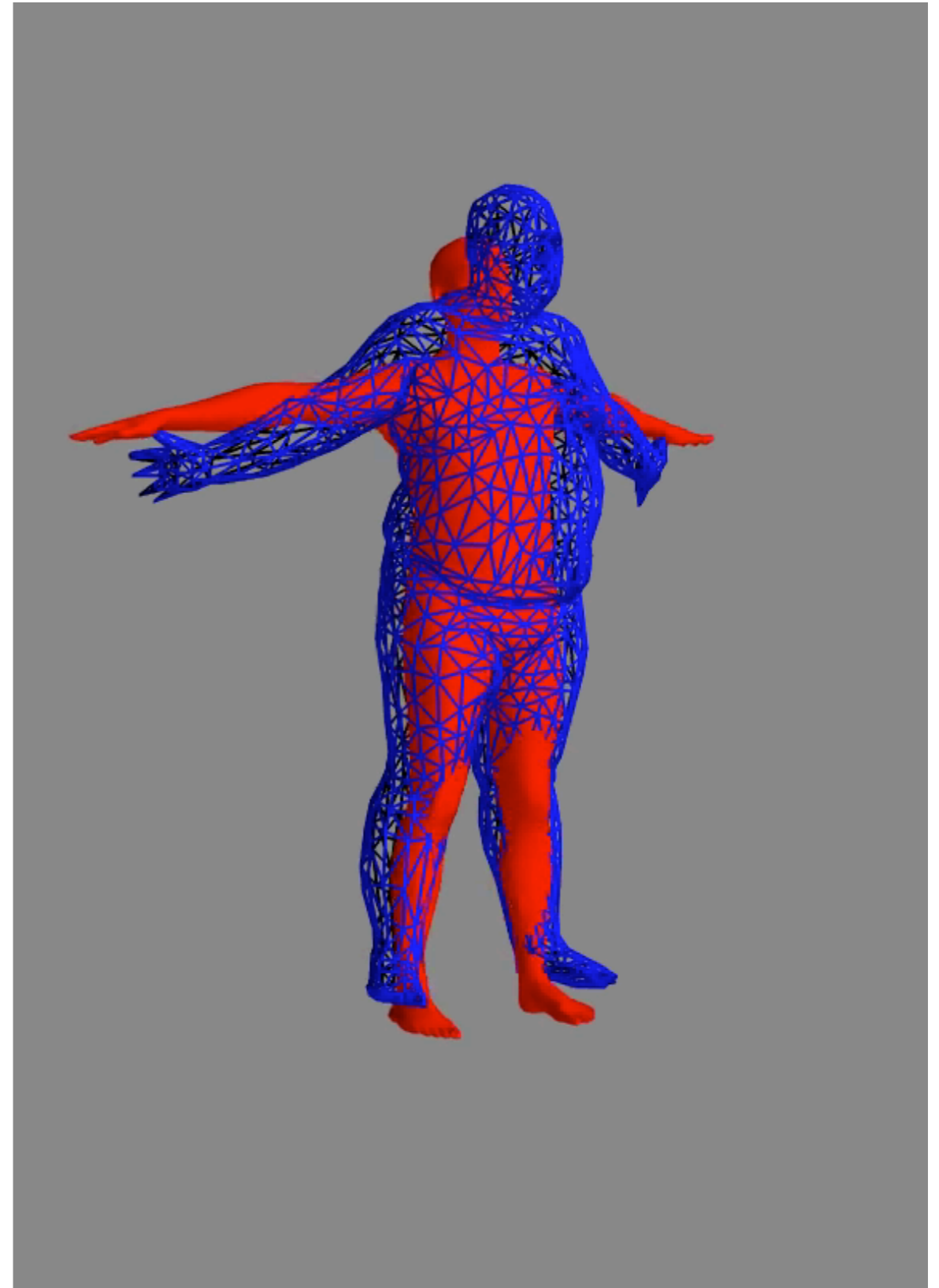
Mahalanobis distance  
induced by distribution  $\mathcal{N}(\vec{\mu}_{\theta}, \Sigma_{\theta})$

$$E_{\theta}(\vec{\theta}) \equiv (\vec{\theta} - \vec{\mu}_{\theta})^T \Sigma_{\theta}^{-1} (\vec{\theta} - \vec{\mu}_{\theta})$$

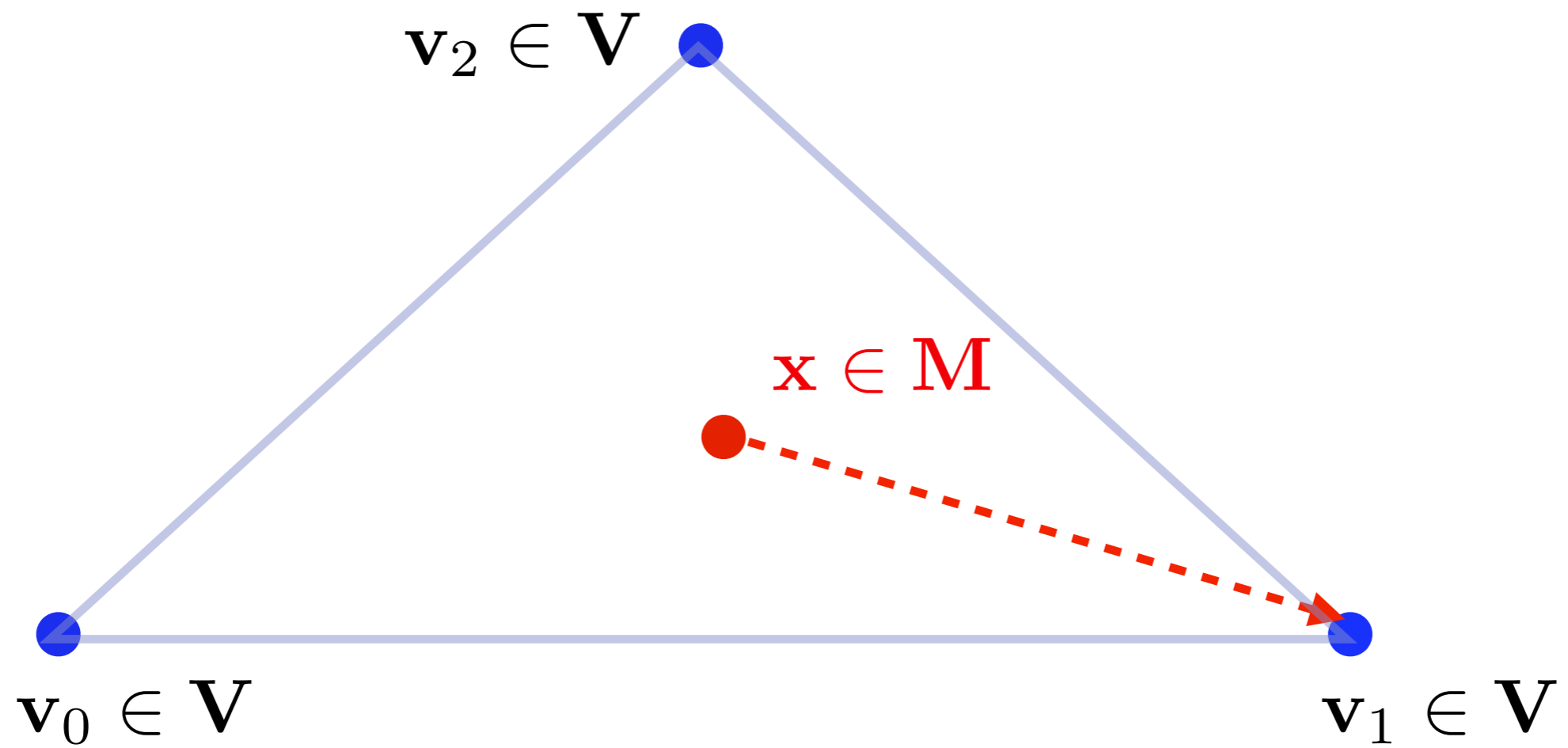
$$E_{\beta}(\vec{\beta}) \equiv (\vec{\beta} - \vec{\mu}_{\beta})^T \Sigma_{\beta}^{-1} (\vec{\beta} - \vec{\mu}_{\beta})$$

# Fitting SMPL to a scan/mesh

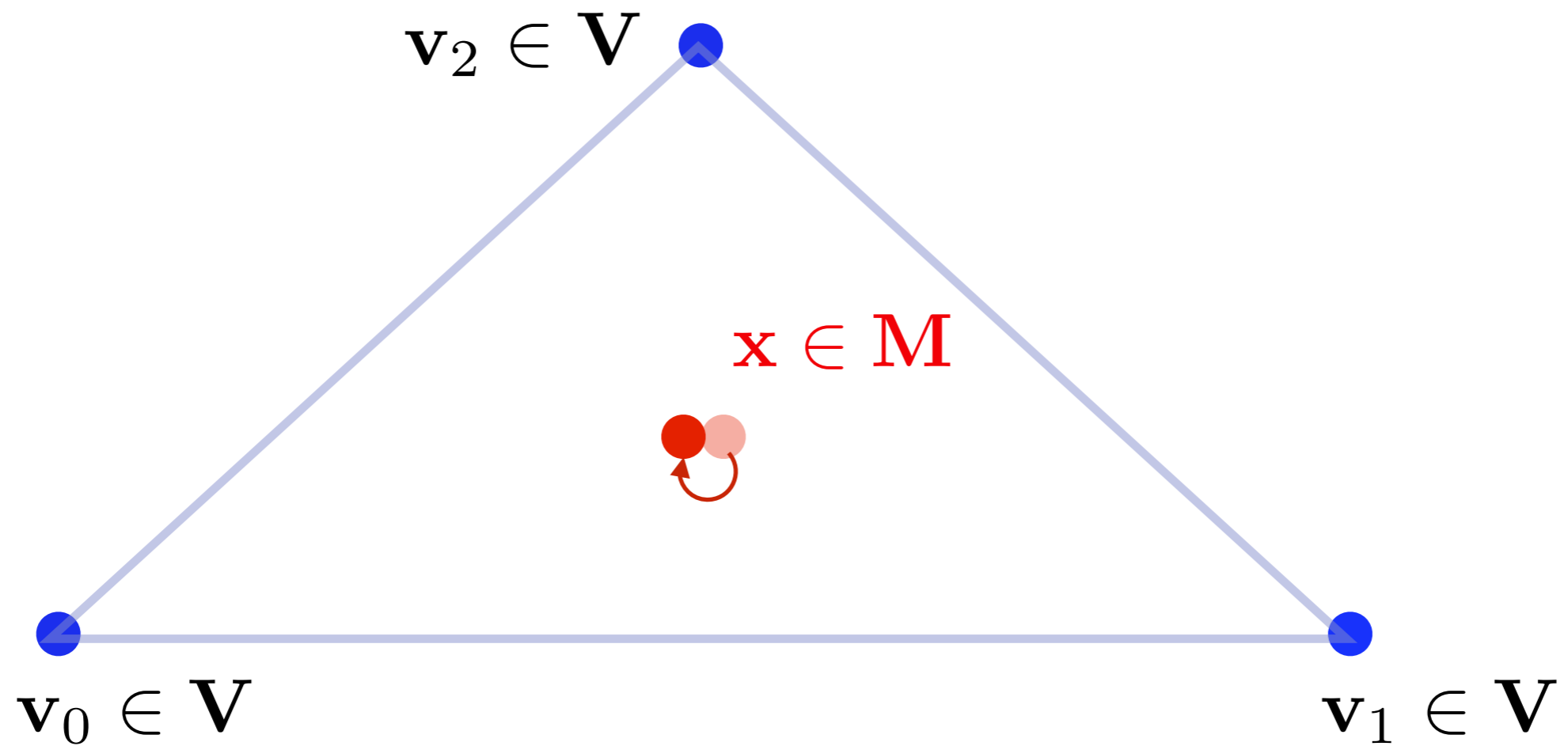
- What makes it so jumpy?
  - Correspondences change abruptly!



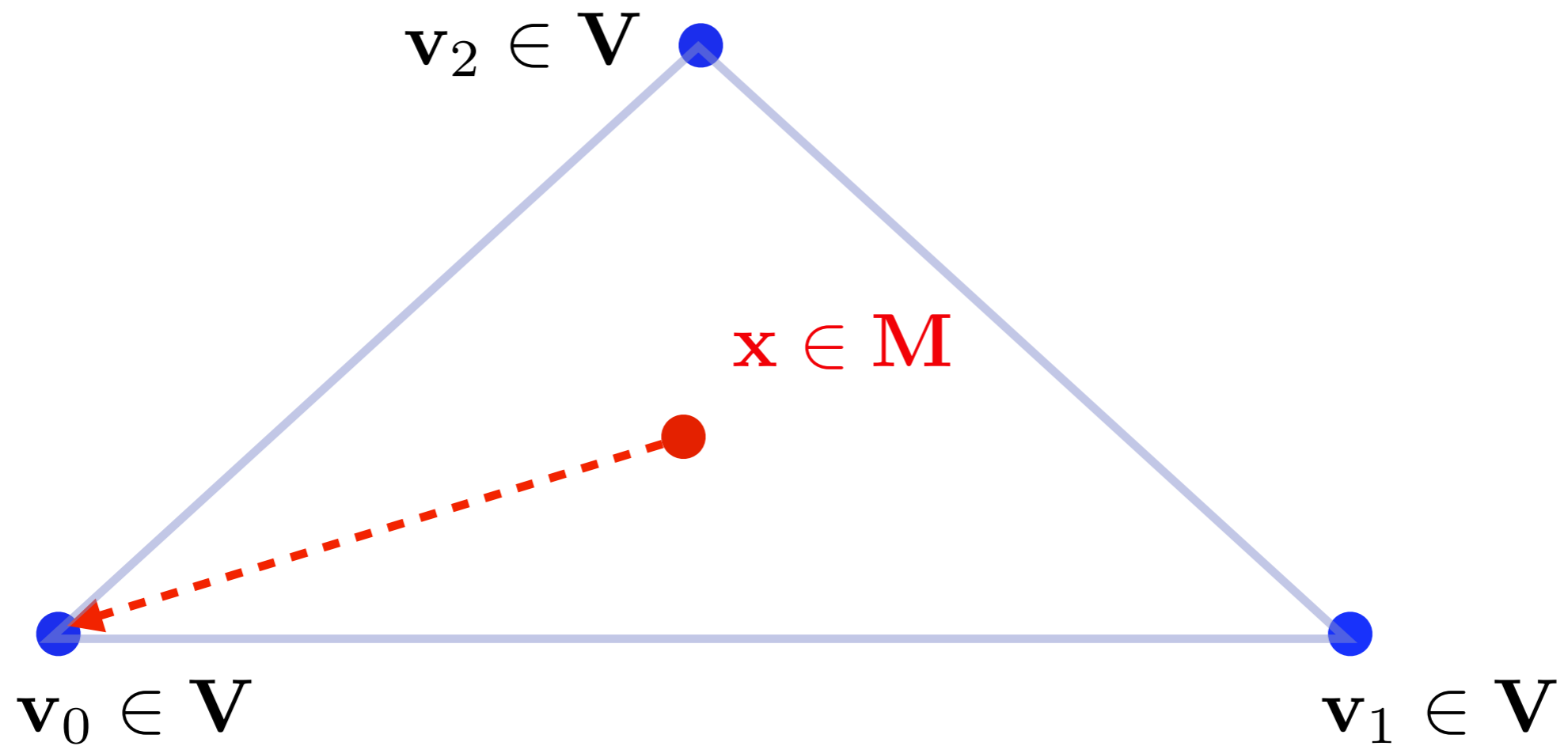
# Point-to-point distance



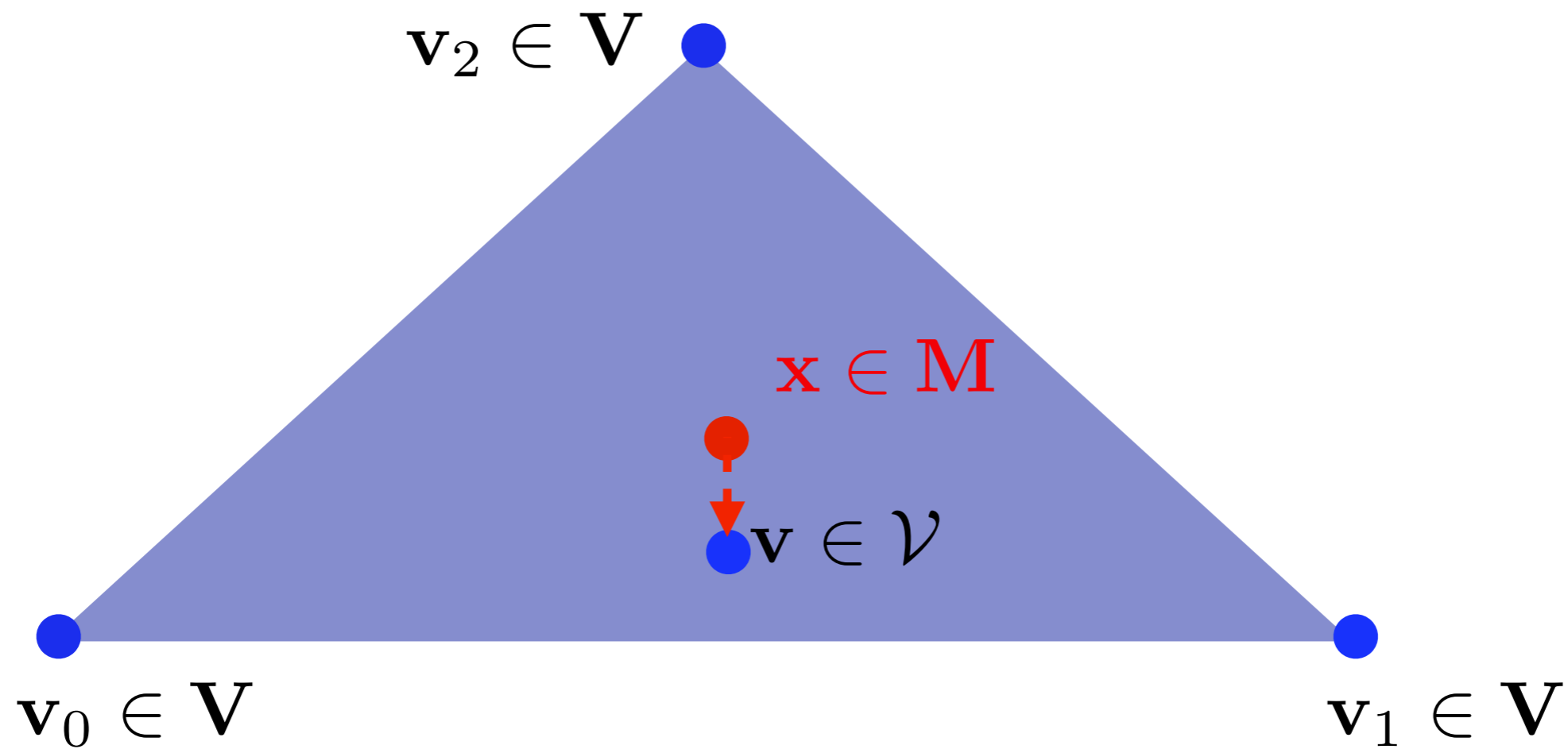
# Point-to-point distance



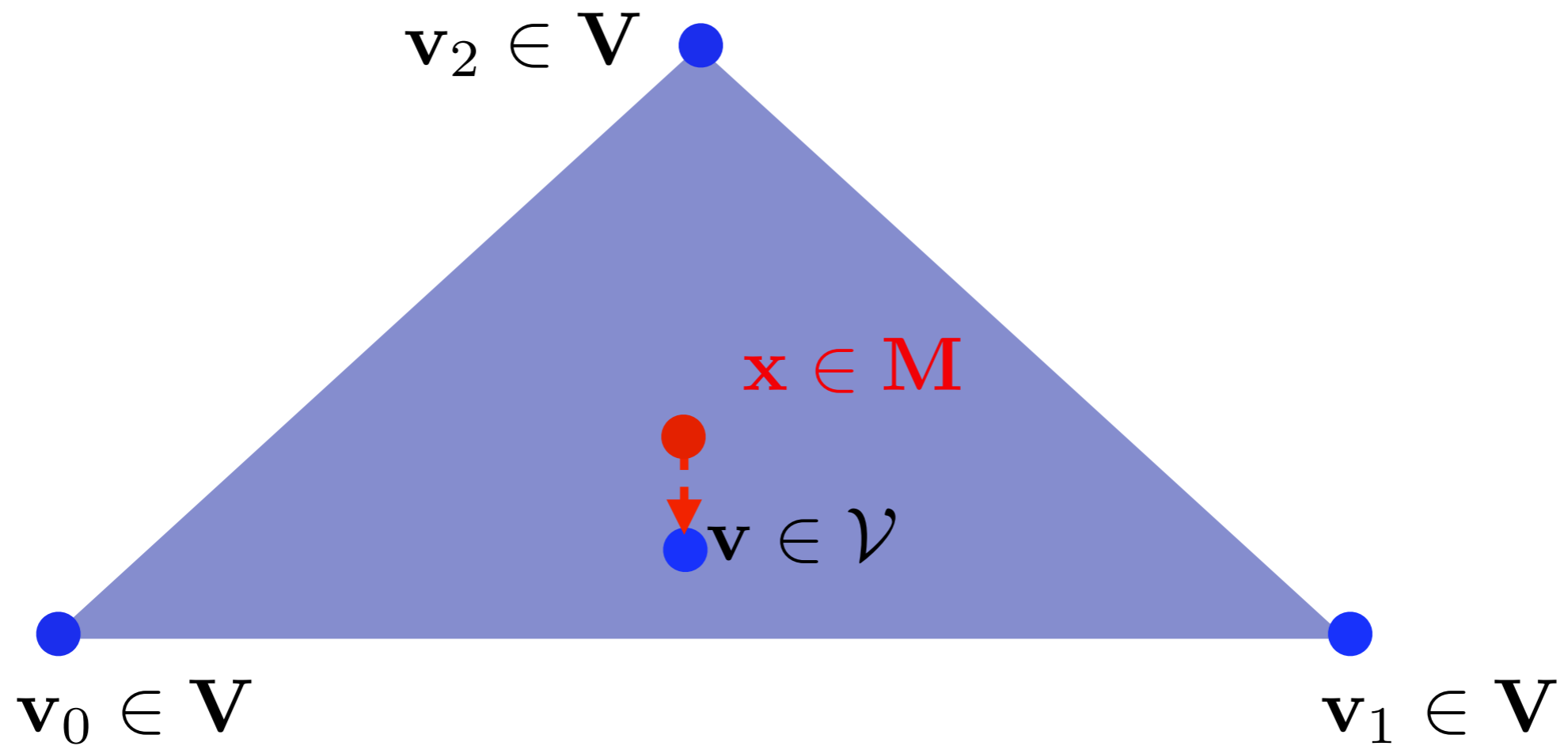
# Point-to-point distance



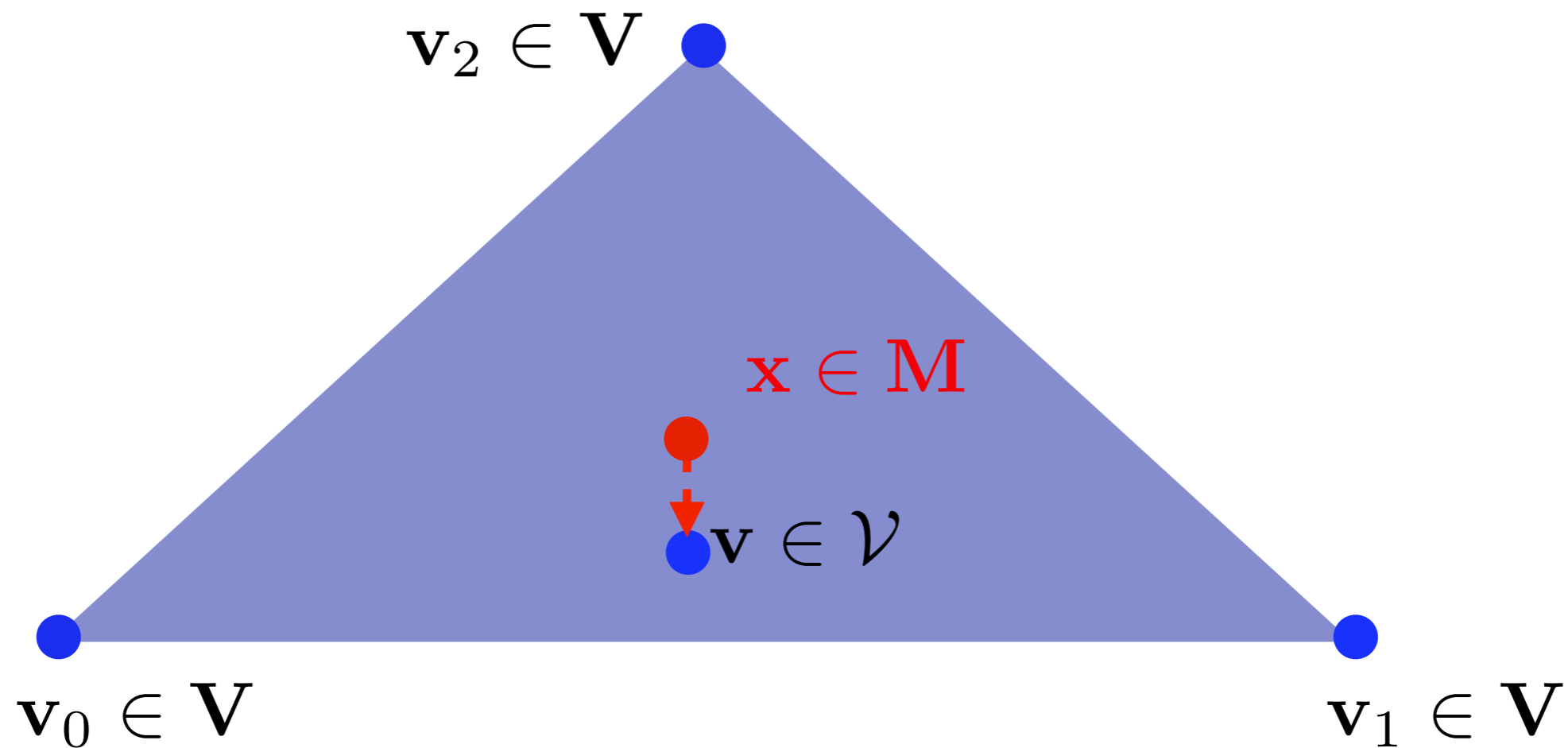
# Point-to-surface distance



# Point-to-surface distance

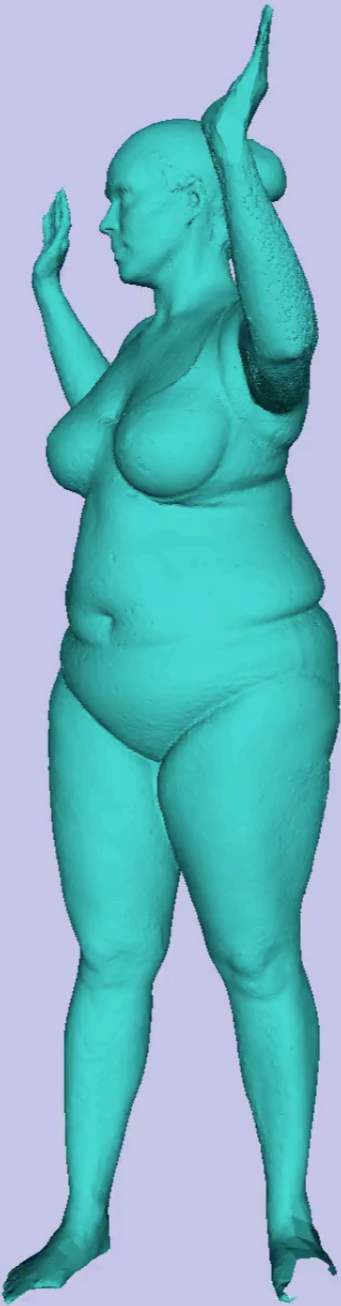


# Point-to-surface distance



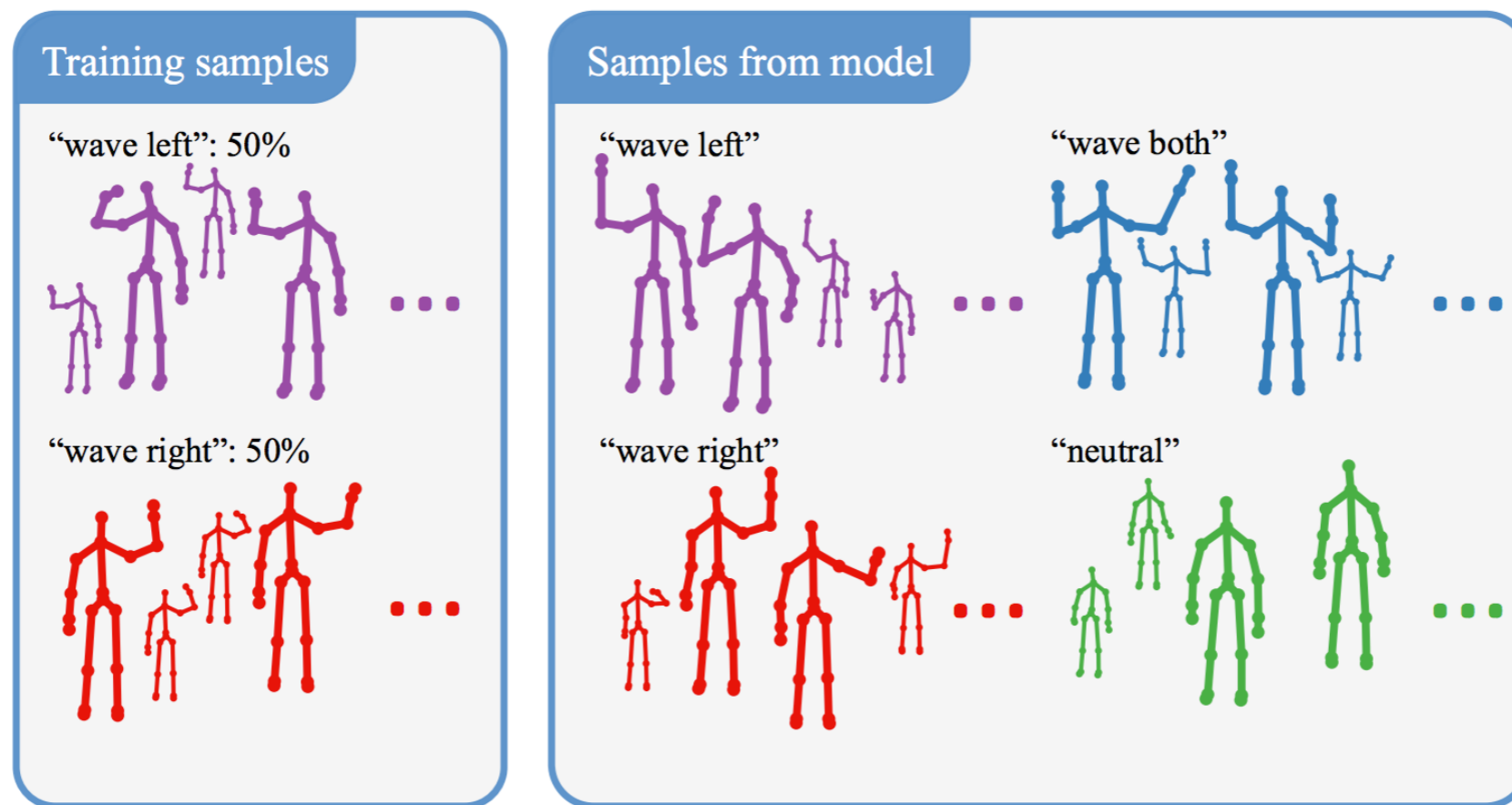
Implementation requires taking care of special cases  
when  $v$  falls in edges or points





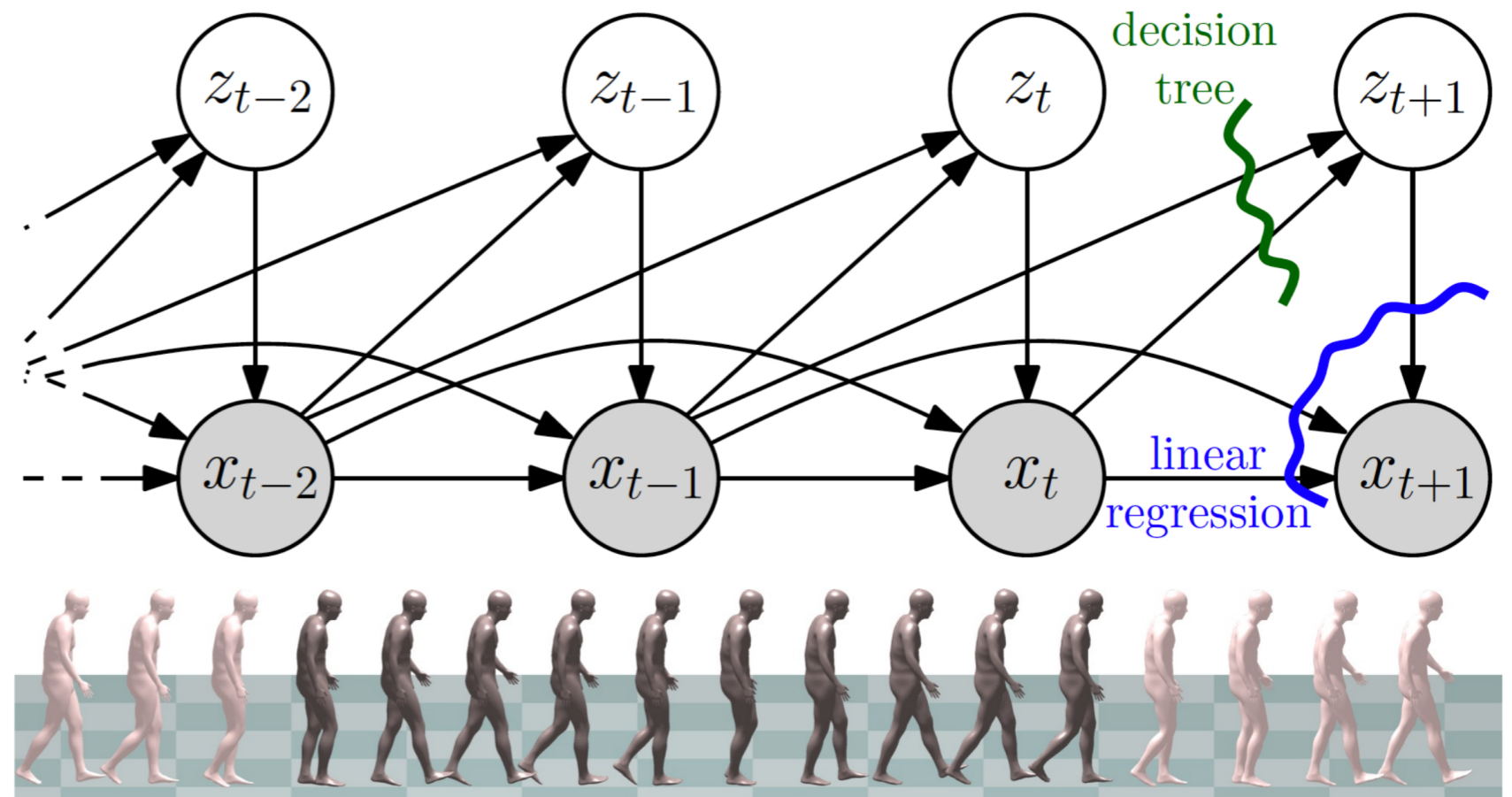
# Advanced registration

- Better pose priors
- Non-parametric



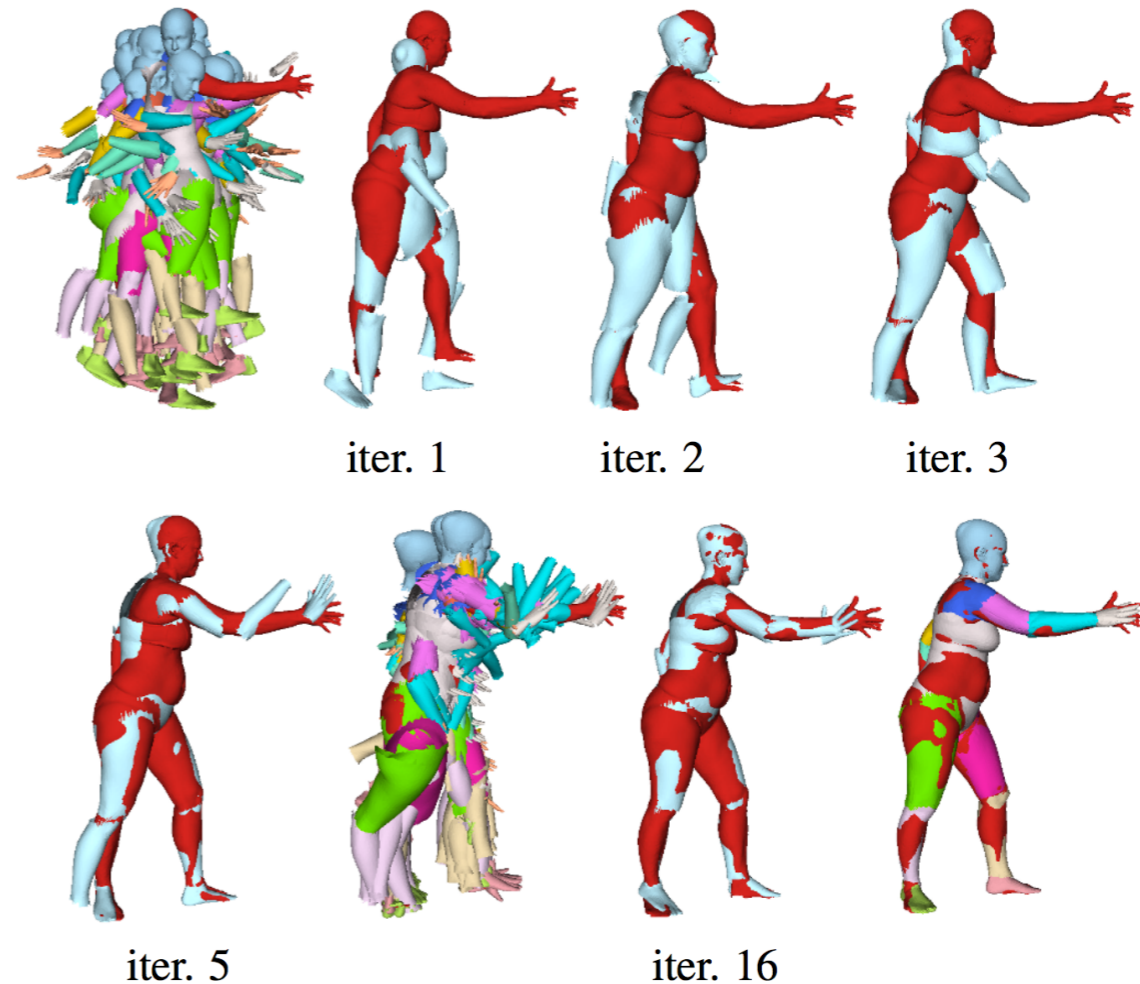
# Fitting SMPL to a scan/mesh

- Better pose priors
- Non-parametric
- Dynamic



# Fitting SMPL to a scan/mesh

- Better pose priors
- Non-parametric
- Dynamic
- Better initialisation



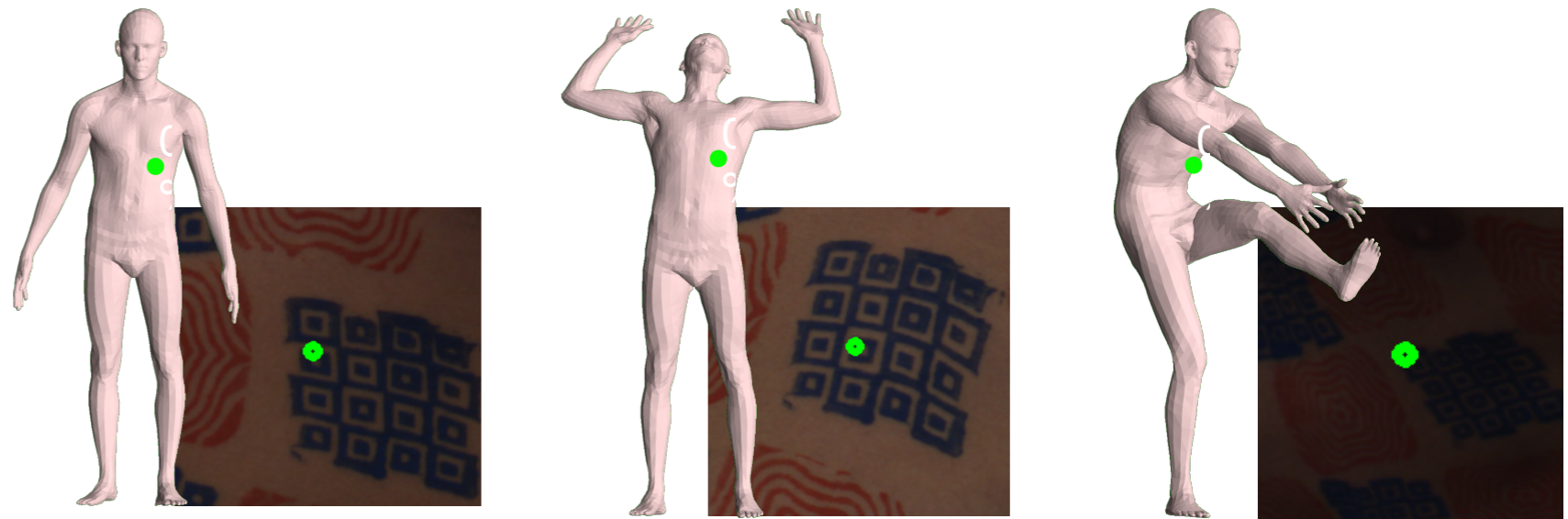
- From previous frame, from discriminative approaches, from graphical models

# Fitting SMPL to a scan/mesh

- Better pose priors

- Non-parametric

- Dynamic



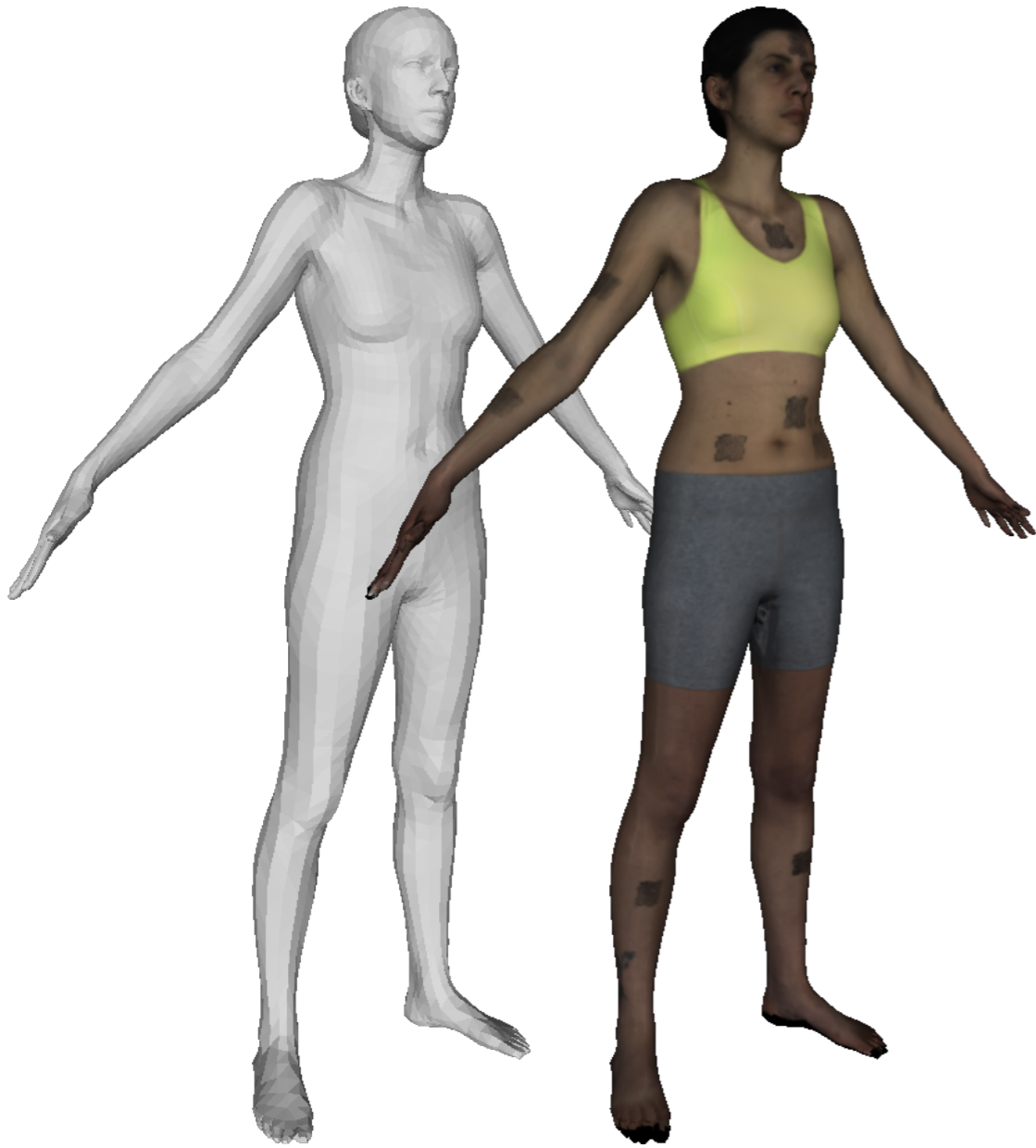
- Better initialisation

- From previous frame, from discriminative approaches, from graphical models

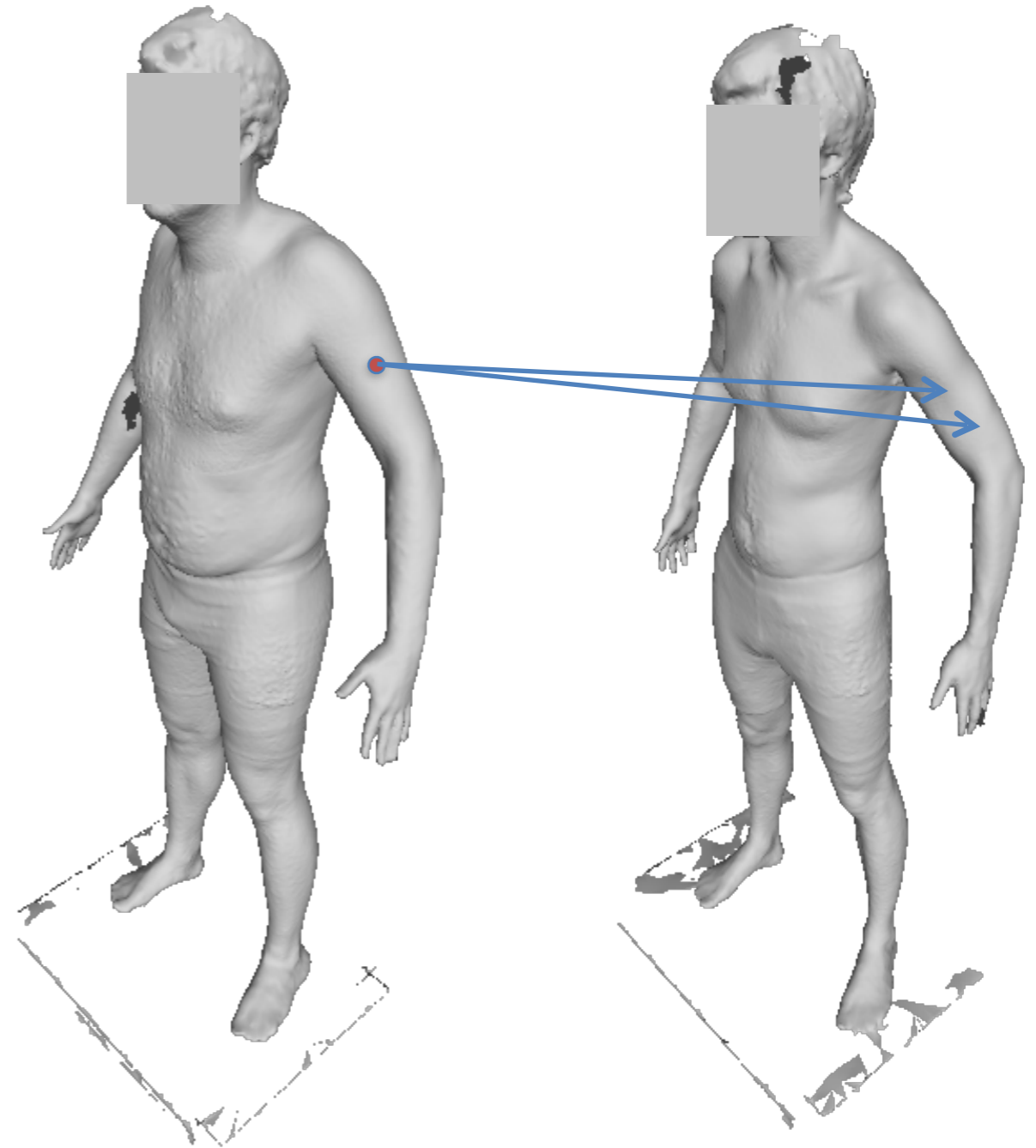
- Other information: appearance (color)!

# Why appearance

More realism

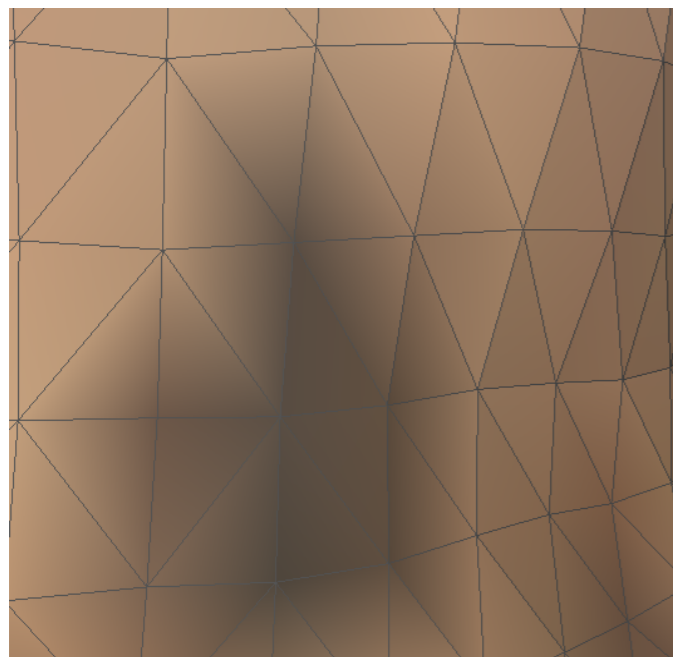
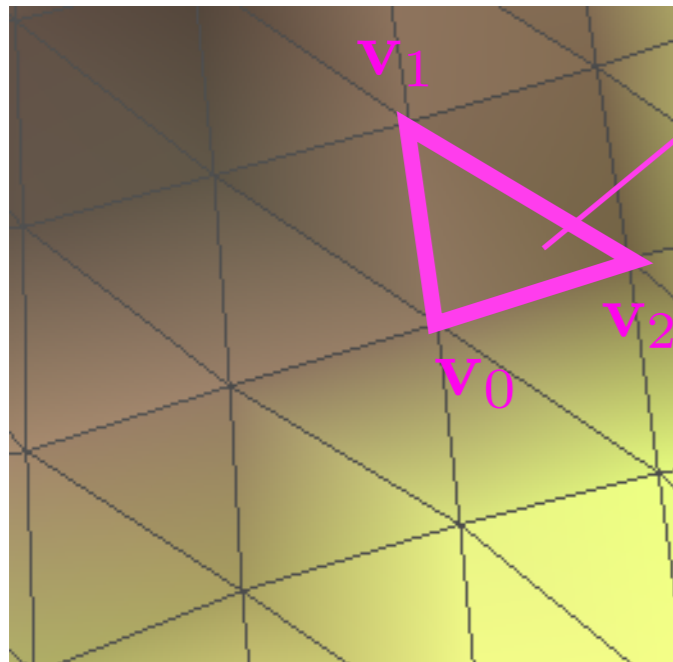


More accurate correspondences



# Representing appearance

## Vertex coloring



$$\mathbf{V} \in \mathbb{R}^{N \times 3}$$

$$\mathbf{F} \in \mathbb{N}^{M \times 3}, \mathbf{F}_{ij} \in [0, N)$$

$$\mathbf{W} \in \mathbb{R}^{N \times 3}, \mathbf{W}_{ij} \in [0, 256)$$

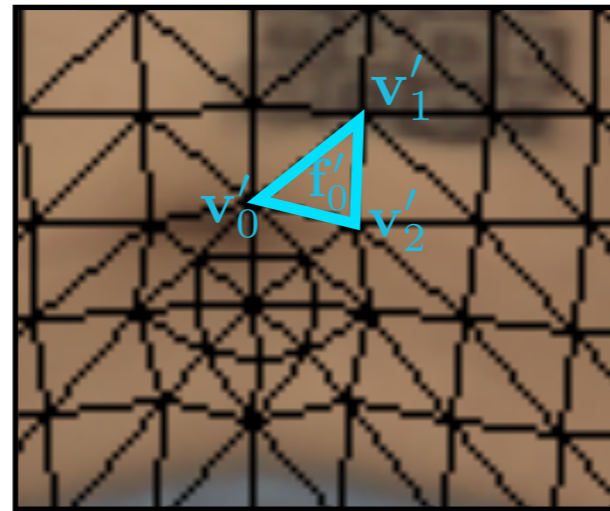
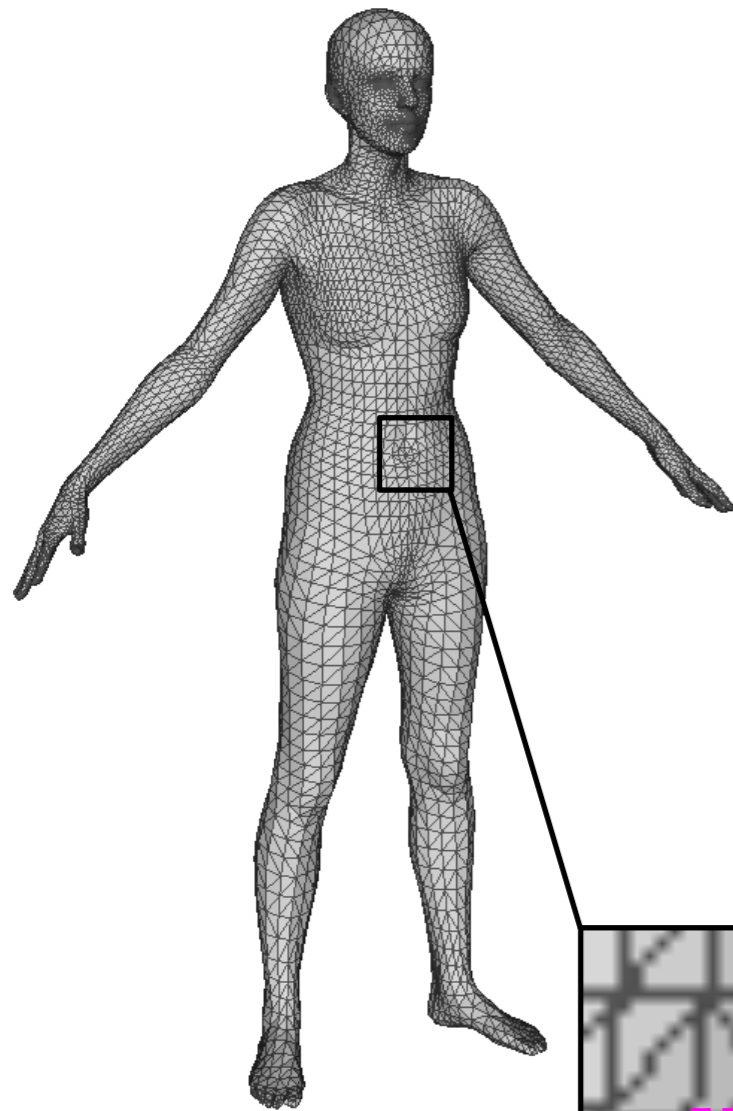
$$\mathbf{f}_0 = [\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2]$$

$$\mathbf{x} \equiv \alpha_0 \mathbf{v}_0 + \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2$$

$$c(\mathbf{x}) = \alpha_0 \mathbf{w}_0 + \alpha_1 \mathbf{w}_1 + \alpha_2 \mathbf{w}_2$$

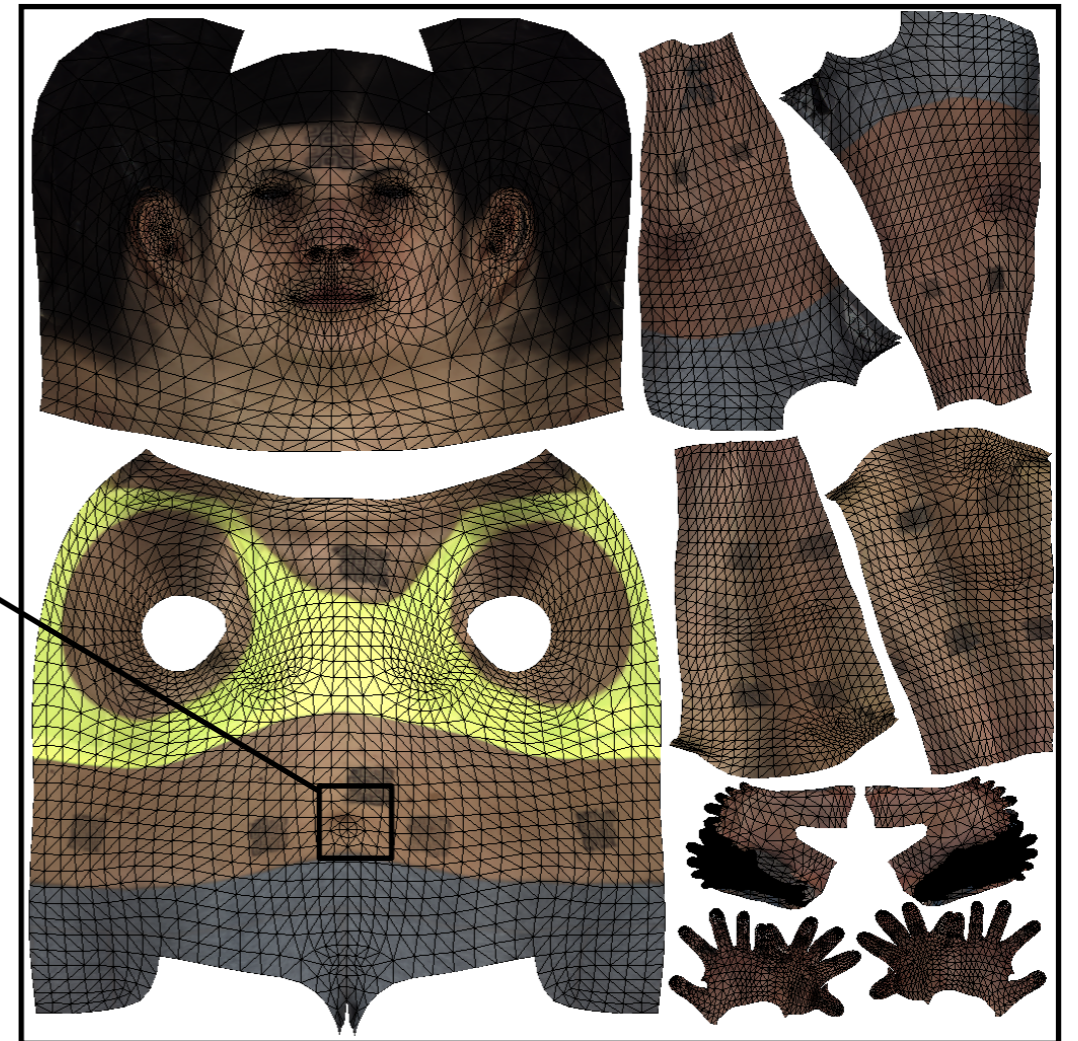
# Decouple geometry and appearance resolution

(1,1)

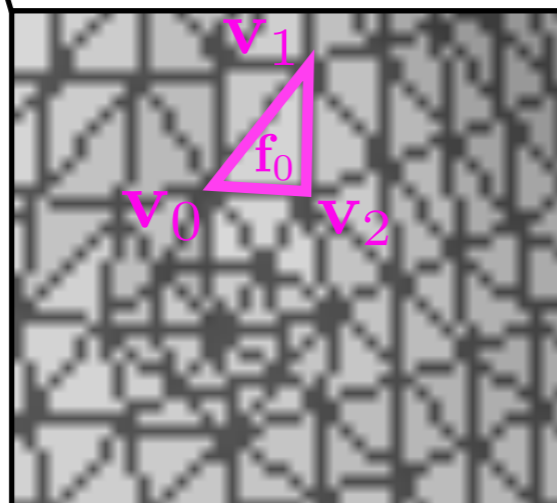


$$\mathbf{v}' \in \mathbb{R}^2, \mathbf{v}'_i \in [0, 1]$$

$$\mathbf{f}' \in \mathbb{N}^3, \mathbf{f}'_i \in [0, N')$$



(0,0)



$$\mathbf{v} \in \mathbb{R}^3$$

$$\mathbf{f} \in \mathbb{N}^3, \mathbf{f}_i \in [0, N)$$

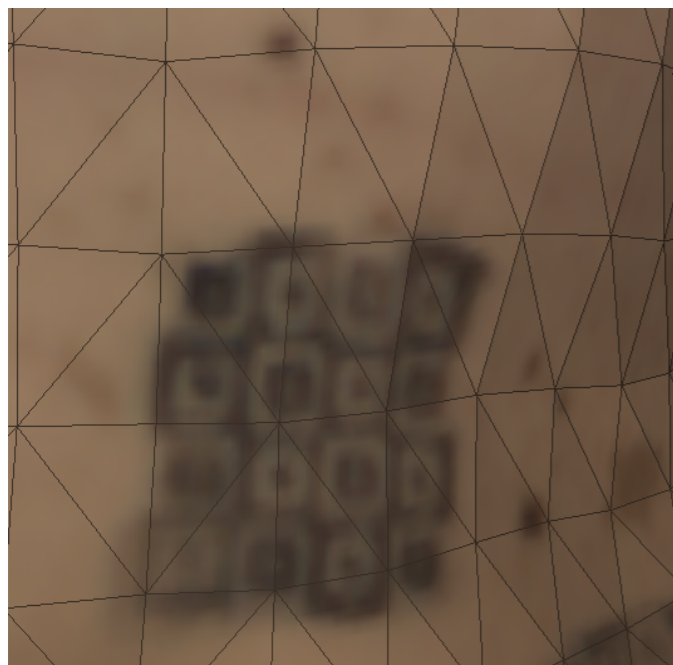
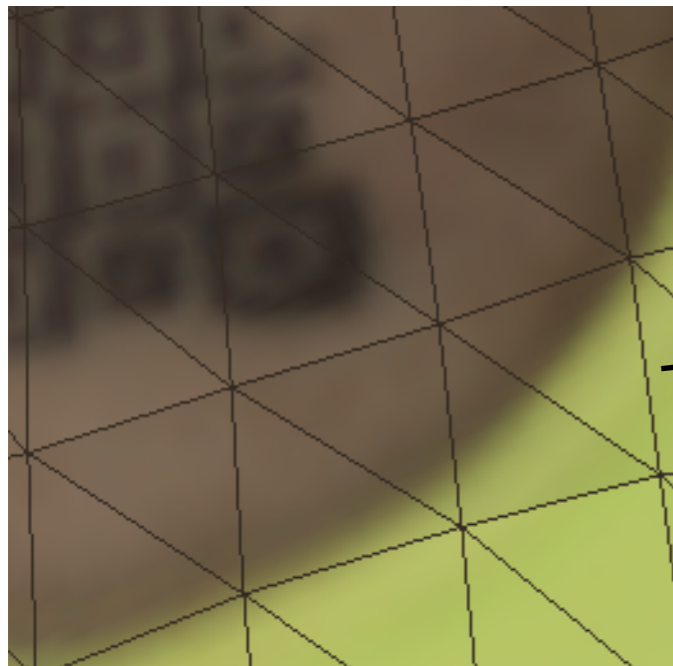
$$\mathbf{f}'_i \equiv \mathbf{f}_i$$

Catmull, PhD Thesis, 1974.



# Representing appearance

## Texture mapping



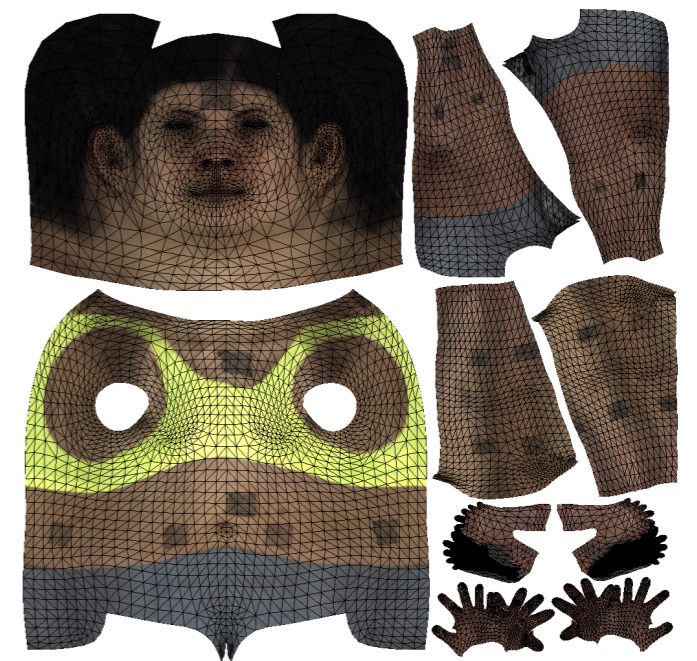
$$\mathbf{V} \in \mathbb{R}^{N \times 3}$$

$$\mathbf{F} \in \mathbb{N}^{M \times 3}, \mathbf{F}_{ij} \in [0, N)$$

$$\mathbf{V}' \in \mathbb{R}^{N' \times 2}, \mathbf{V}'_{ij} \in [0, 1]$$

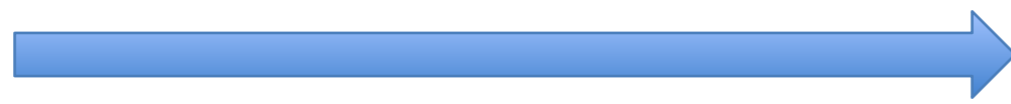
$$\mathbf{F}' \in \mathbb{N}^{M \times 2}, \mathbf{F}'_{ij} \in [0, N')$$

$$\mathbf{U} \in \mathbb{N}^{K \times K \times 3}, \mathbf{U}_{ijk} \in [0, 256)$$

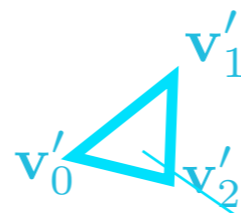
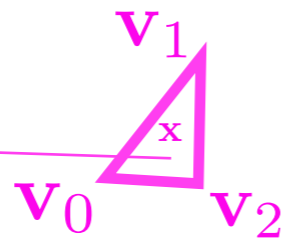
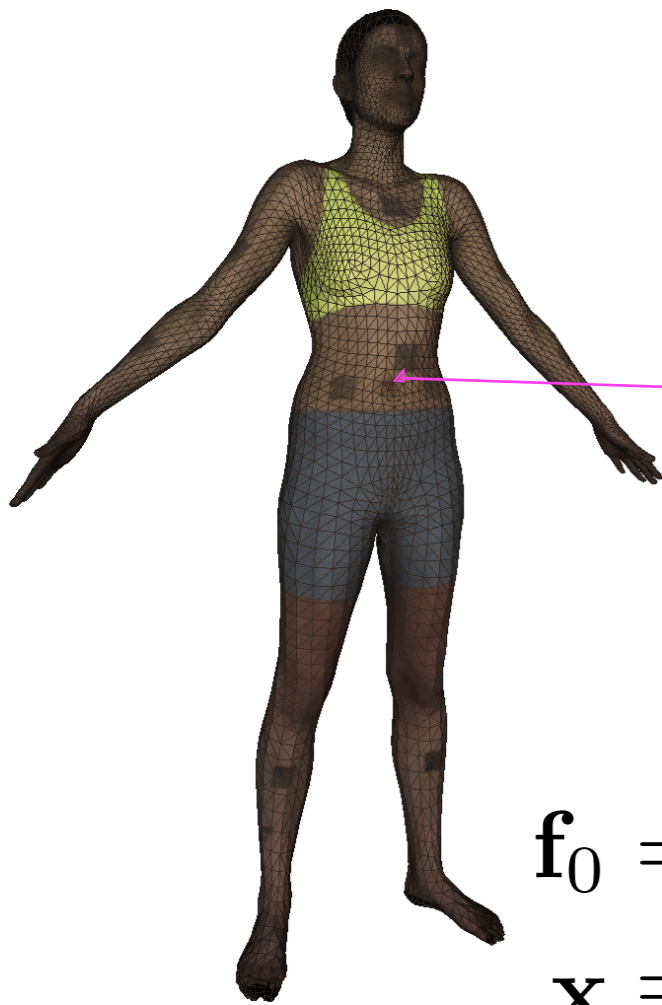


# Texture mapping

3D



UV



$$\mathbf{f}_0 = [\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2]$$

$$\mathbf{x} \equiv \alpha_0 \mathbf{v}_0 + \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2$$

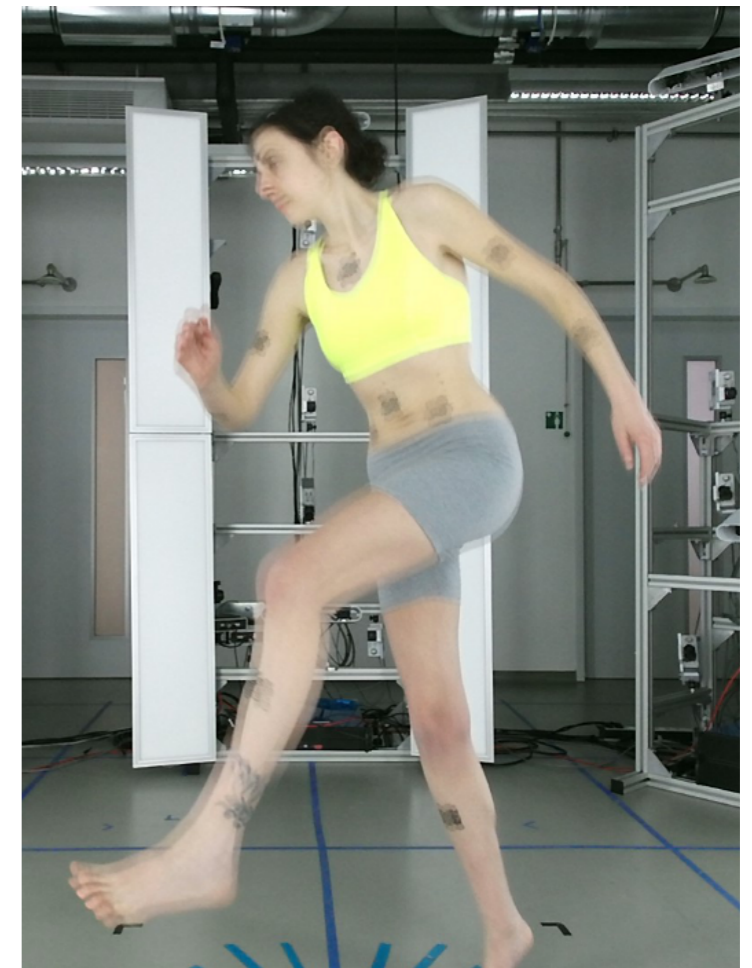
$$\mathbf{f}'_0 = [\mathbf{v}'_0, \mathbf{v}'_1, \mathbf{v}'_2]$$

$$c(\mathbf{x}) = \mathbf{U}[\alpha_0 \mathbf{v}'_0 + \alpha_1 \mathbf{v}'_1 + \alpha_2 \mathbf{v}'_2]$$

How do we create texture maps?

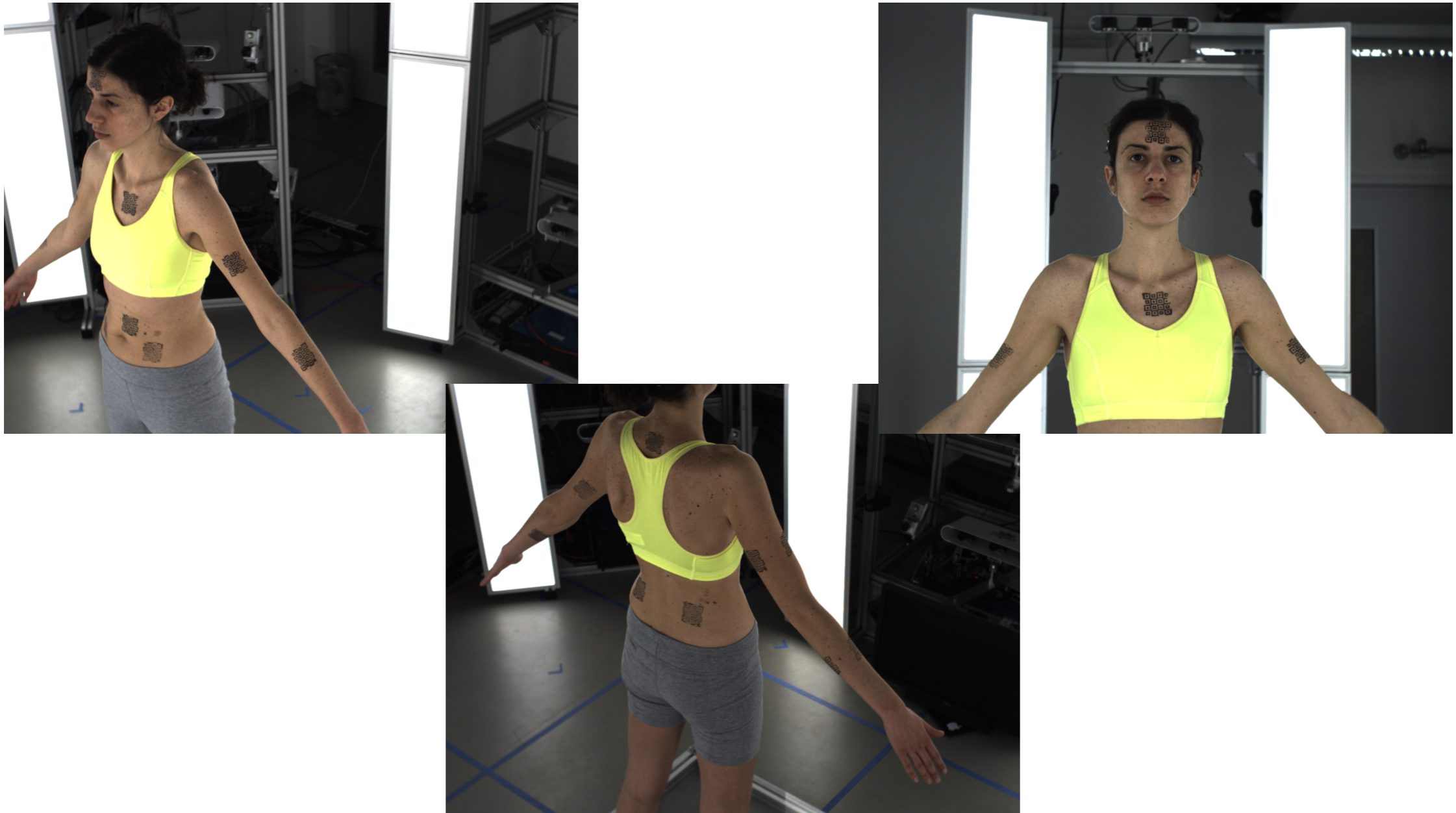
# From 2D images to textures

Problem: combining multiple views of a 3D surface



# From 2D images to textures

Problem: combining multiple views of a 3D surface



# From 2D images to textures

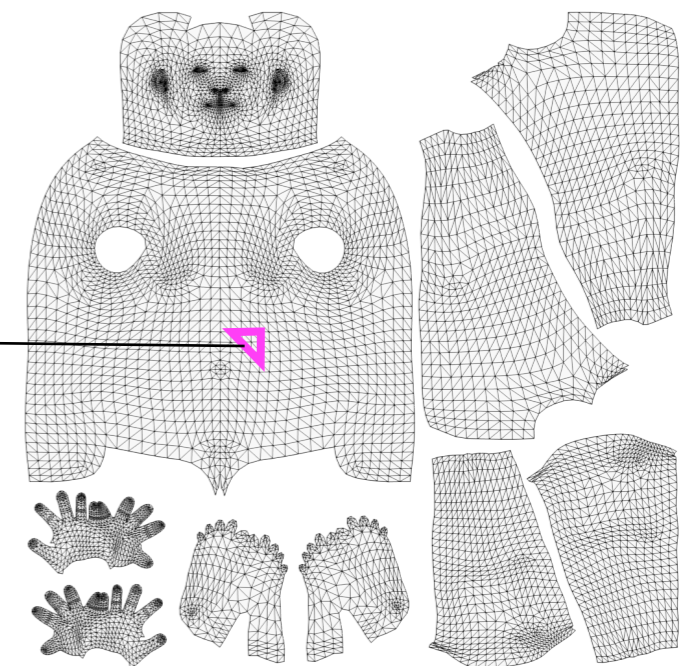
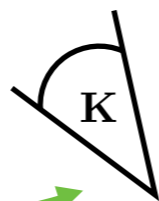
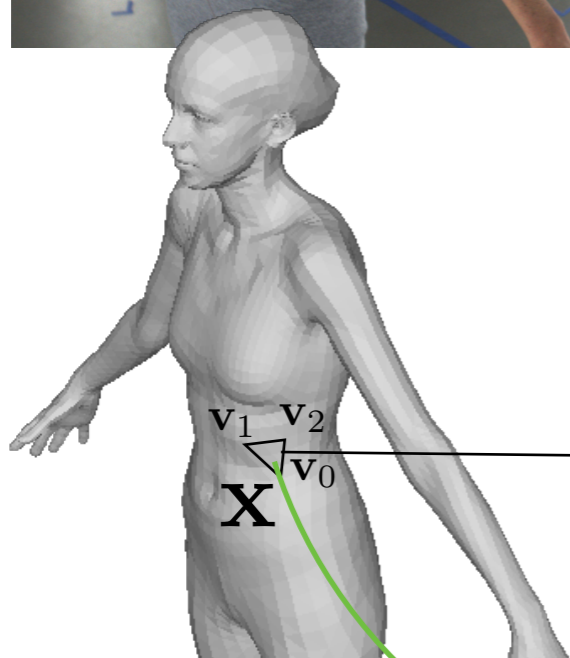


# From 2D images to textures

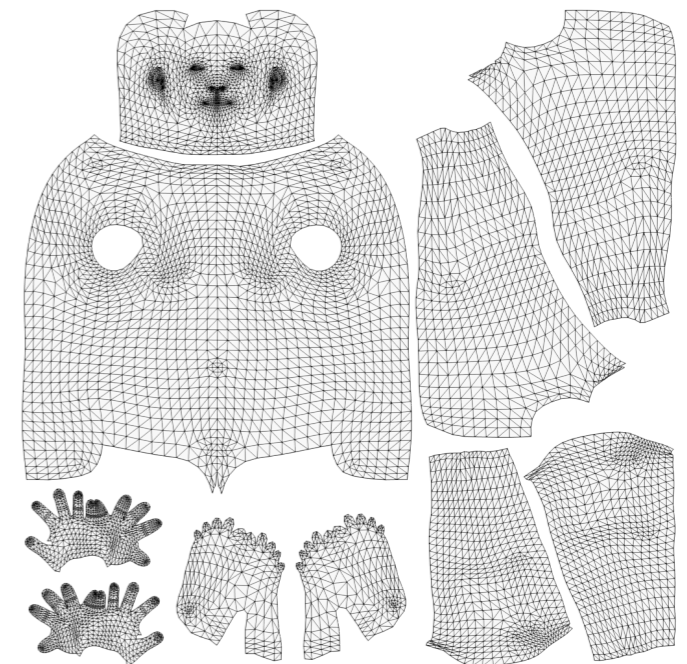
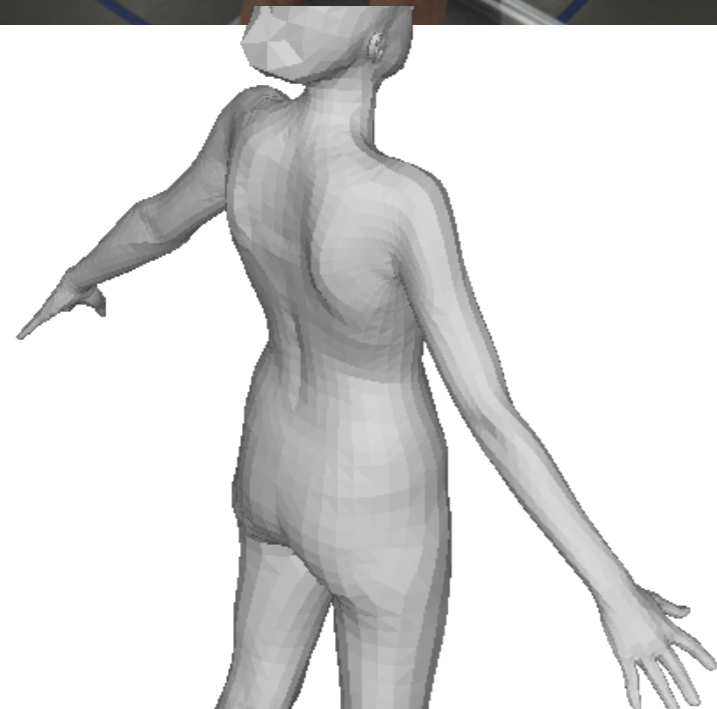
original image

visibility of original pixels in U

original pixels mapped to U

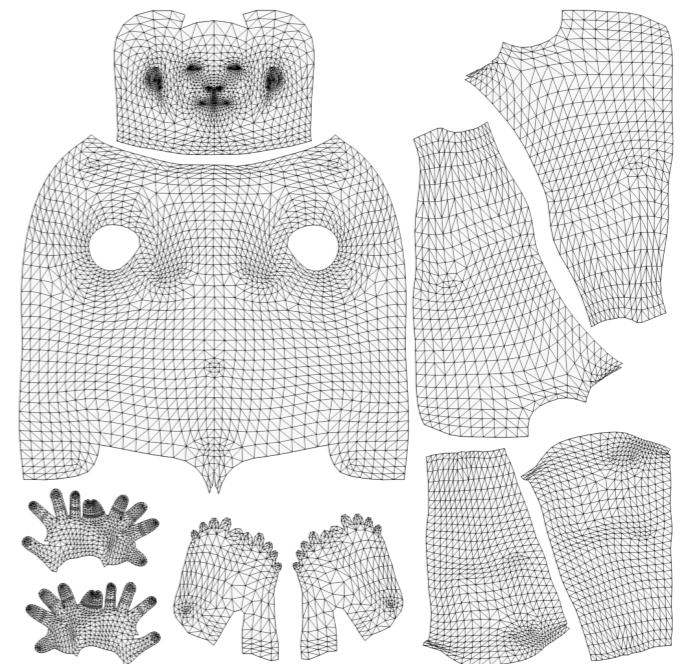
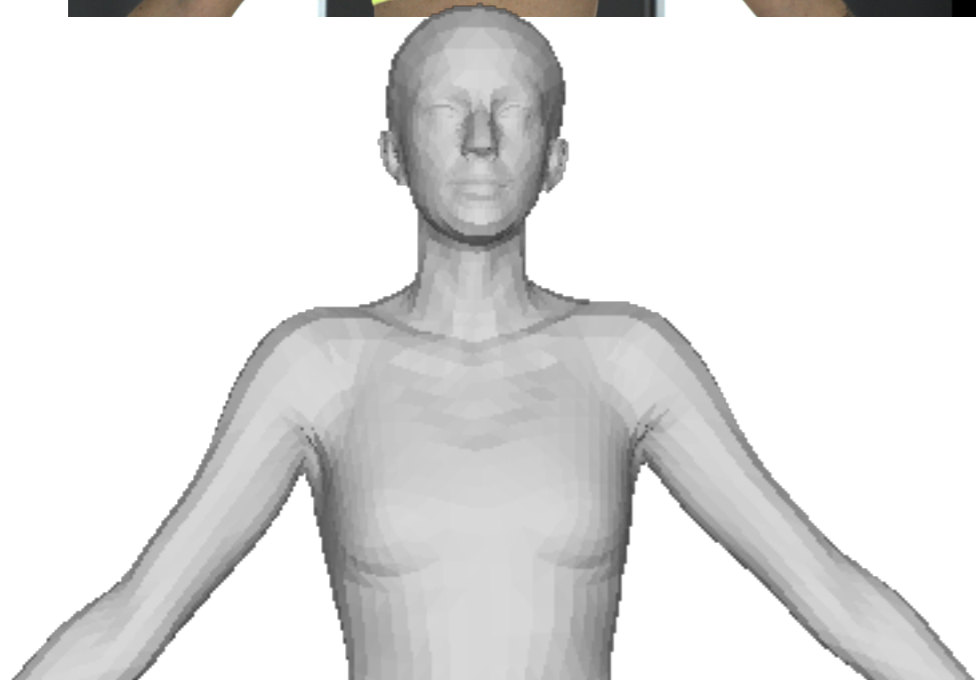
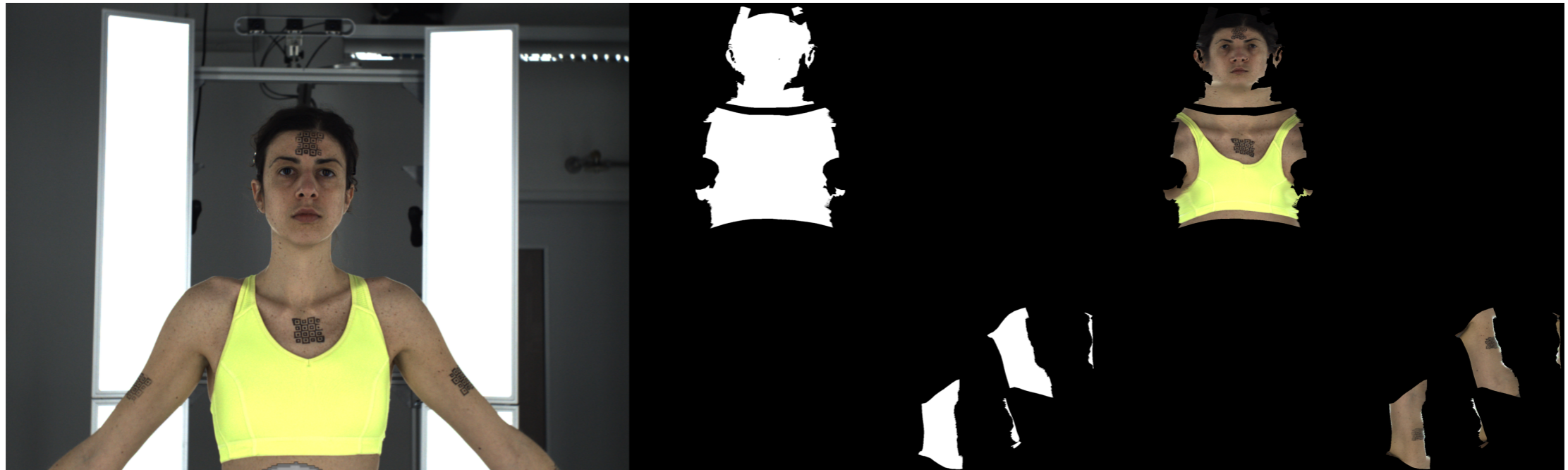


# From 2D images to textures

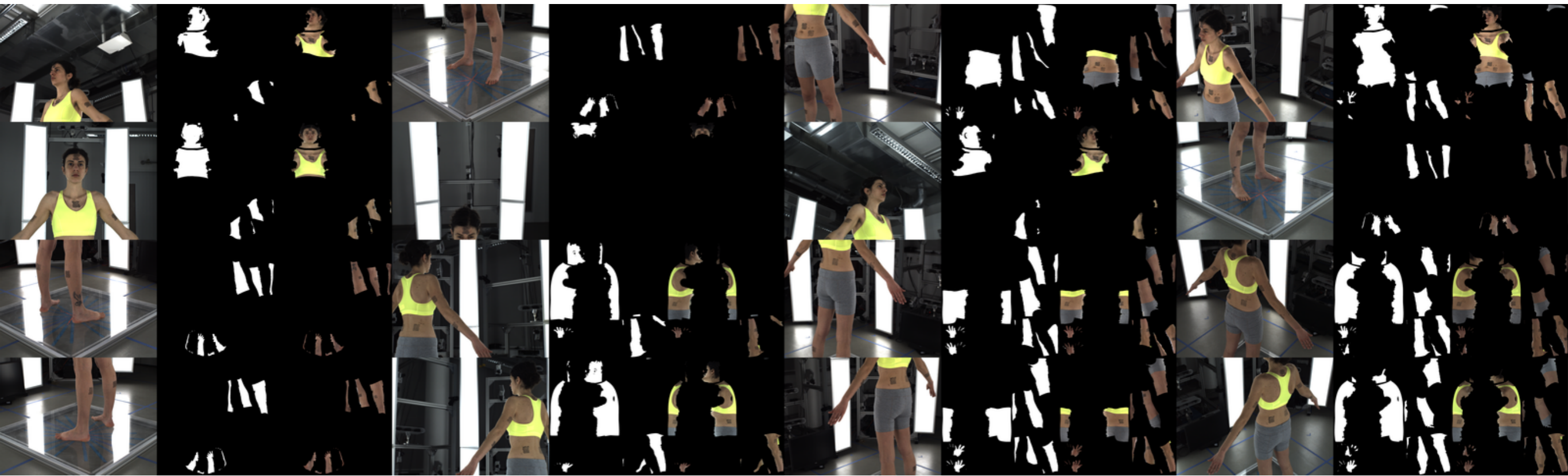


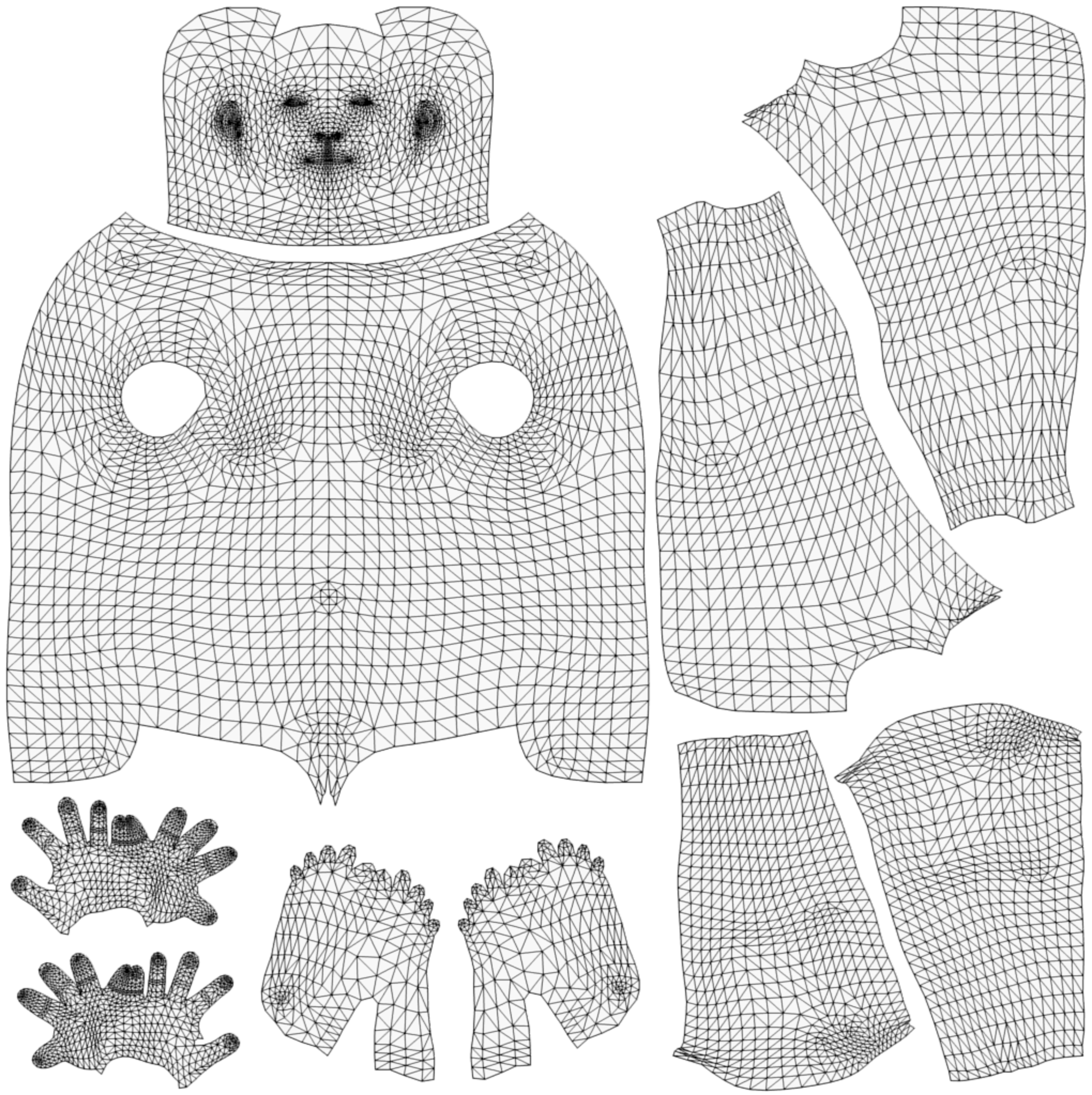


# From 2D images to textures



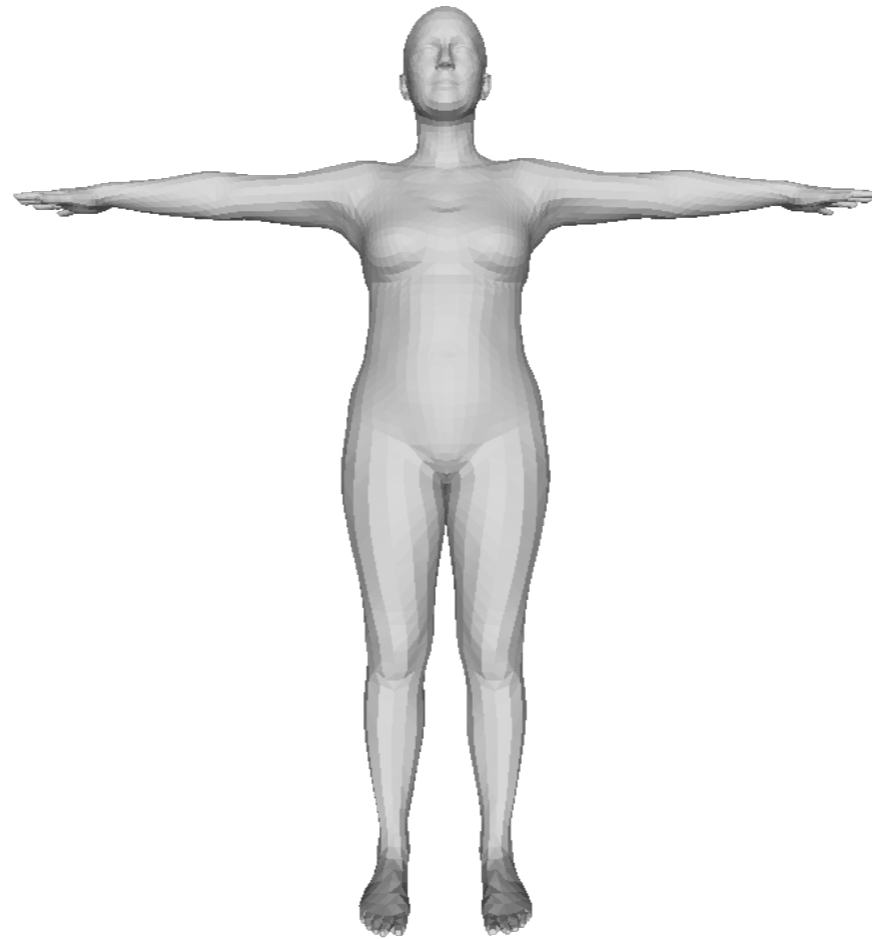
# From 2D images to textures



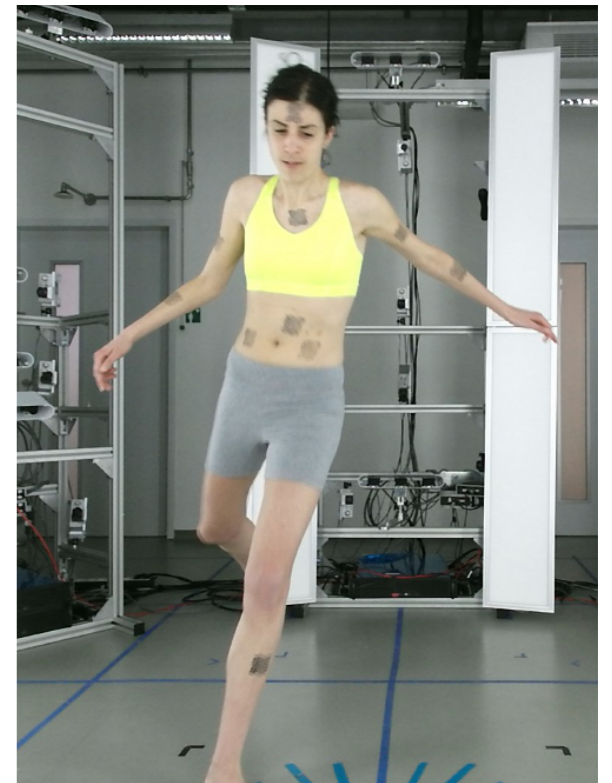




# Generating an image

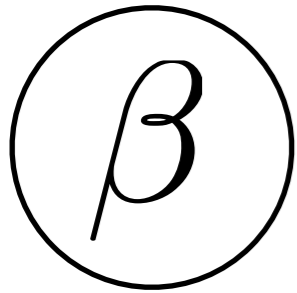


image

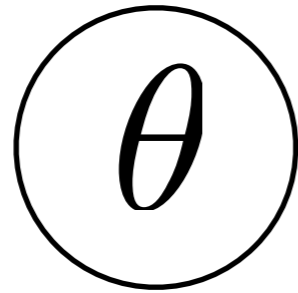


# Generating an image

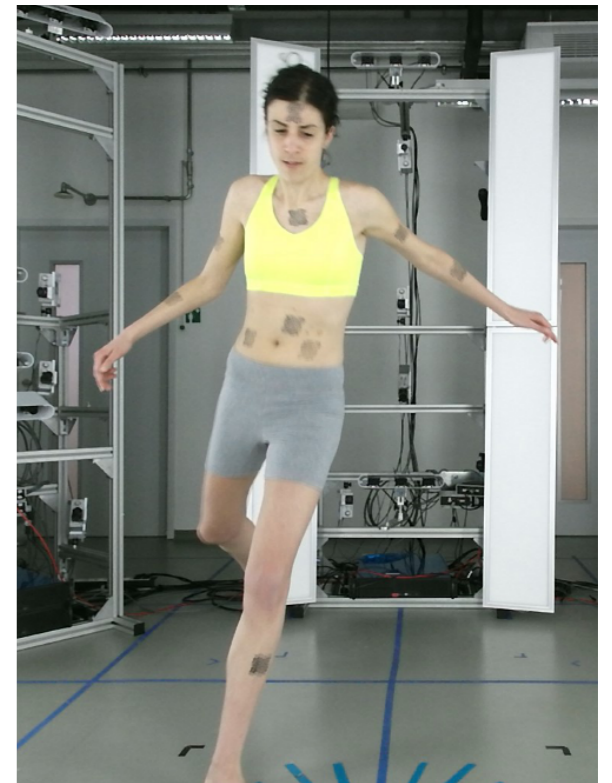
shape



pose

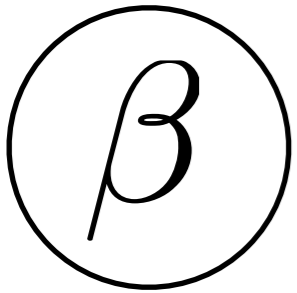


image

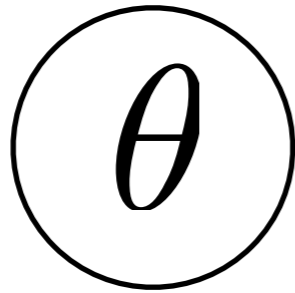


# Generating an image

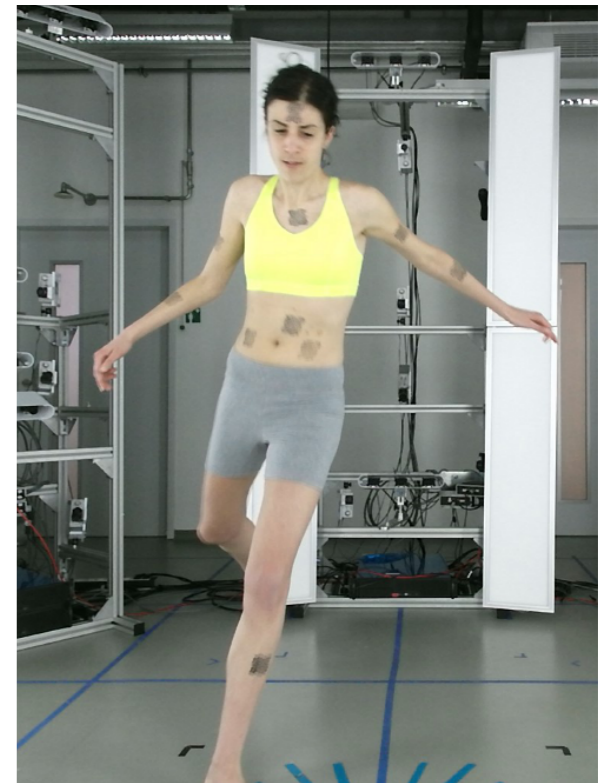
shape



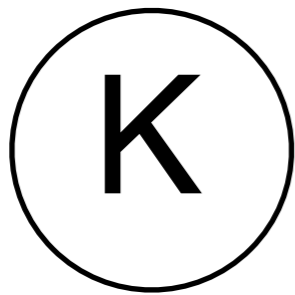
pose



image



camera

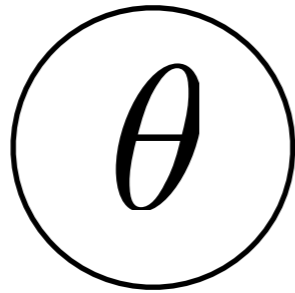


# Generating an image

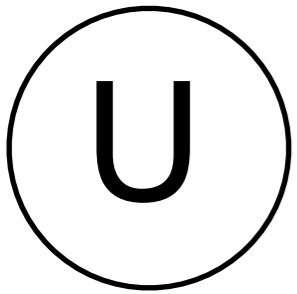
shape



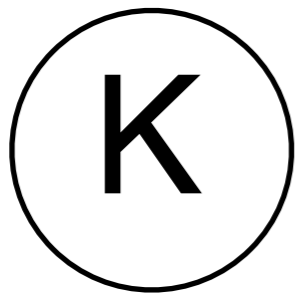
pose



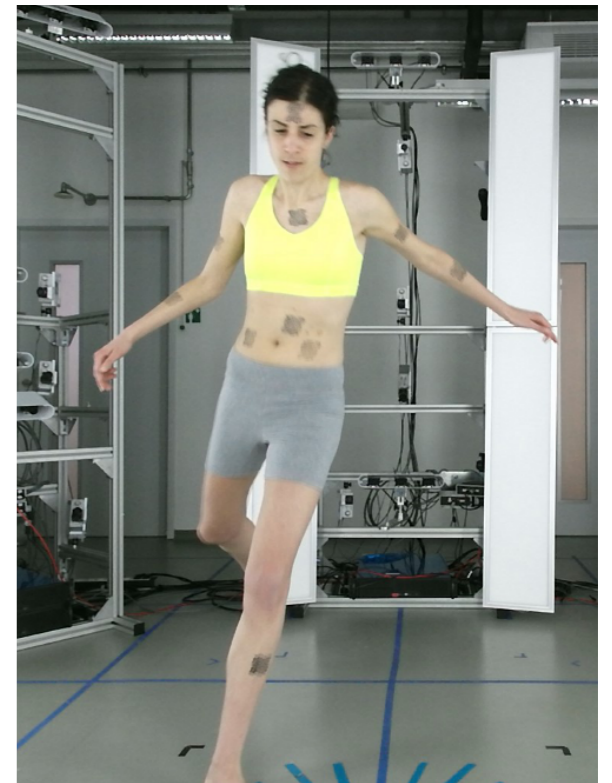
UV map



camera



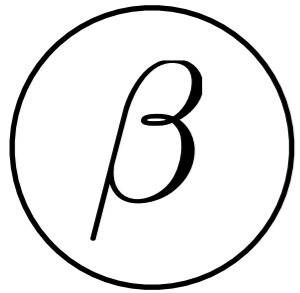
image



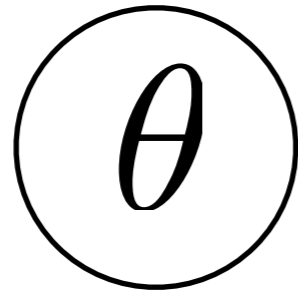


# That's all, no?

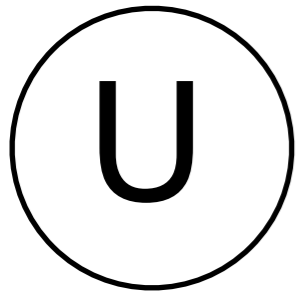
shape



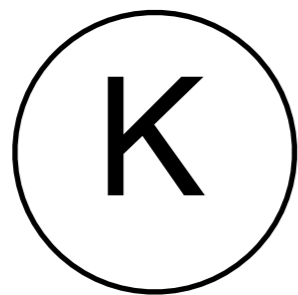
pose



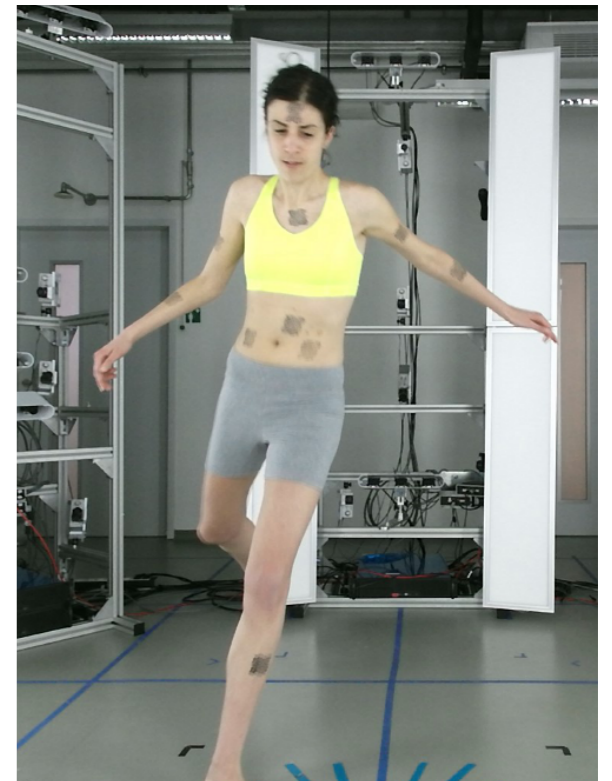
UV map



camera

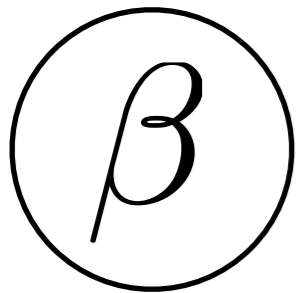


image

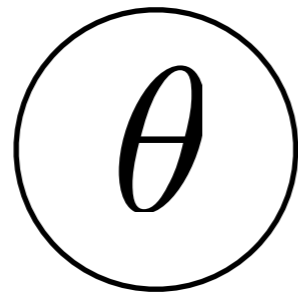


# This slide is wrong: have all the vertices the same shading ?

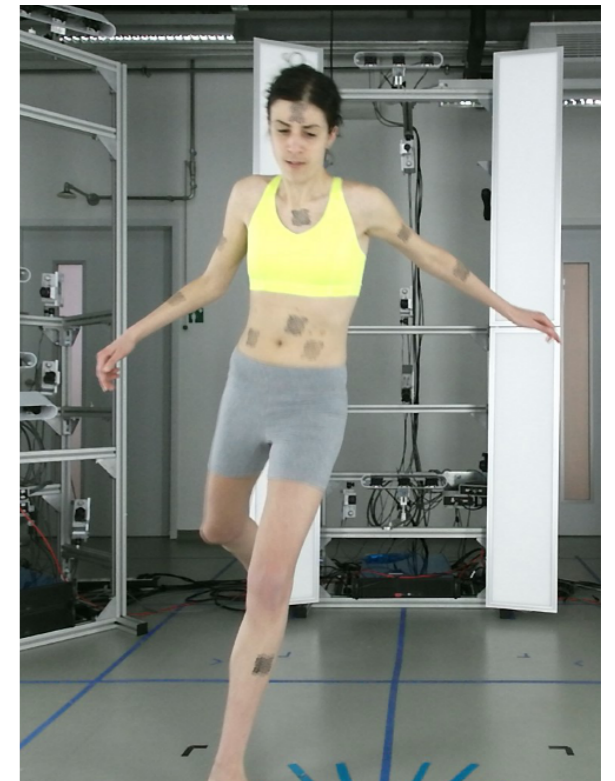
shape



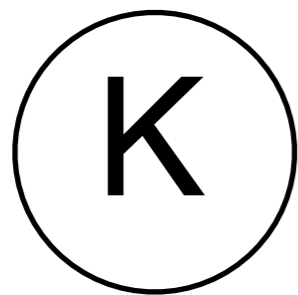
pose



image

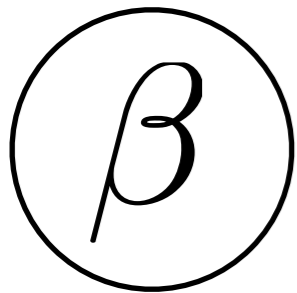


camera

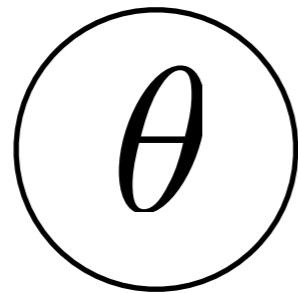


# This one has a single shading

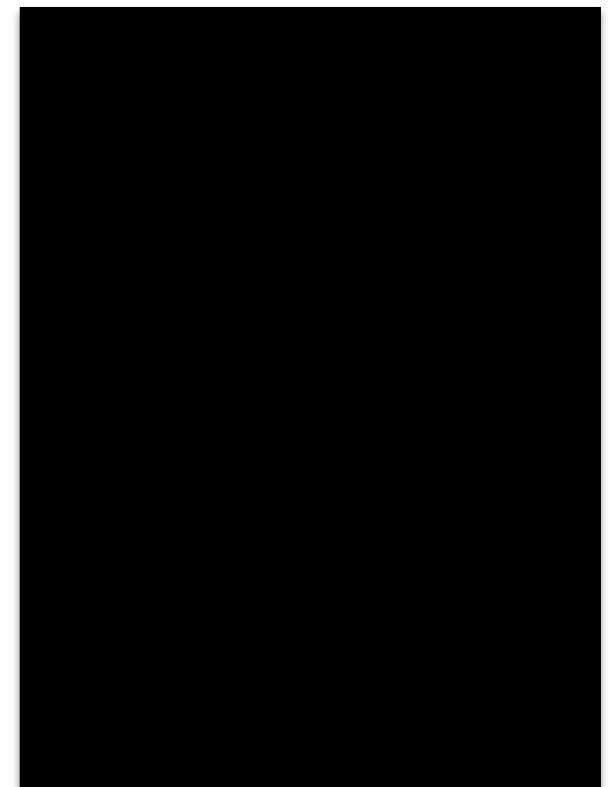
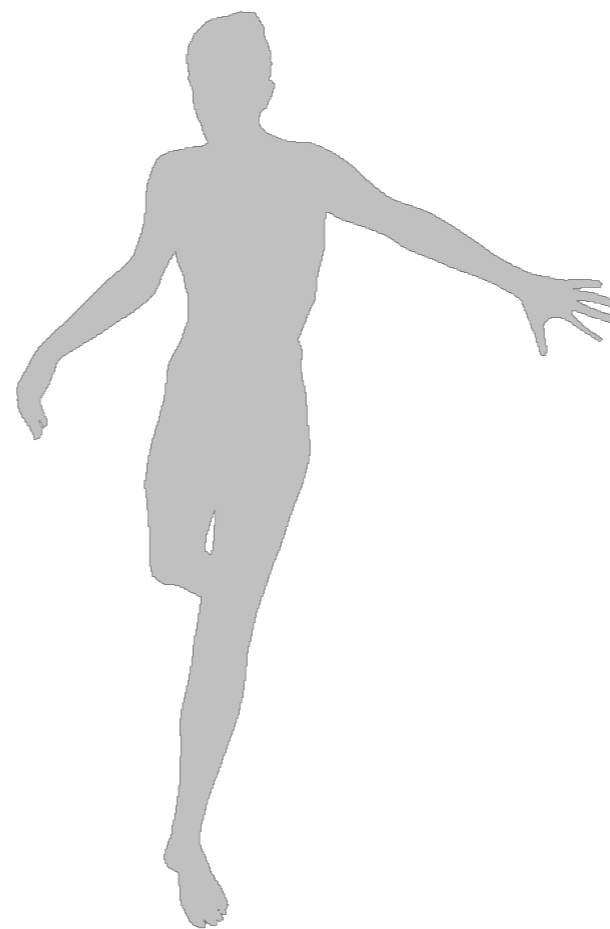
shape



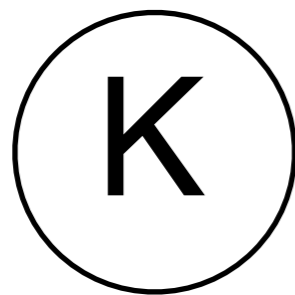
pose



image

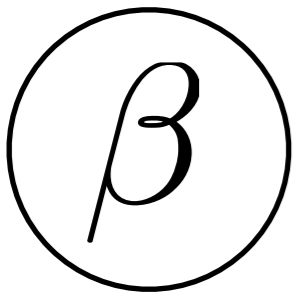


camera

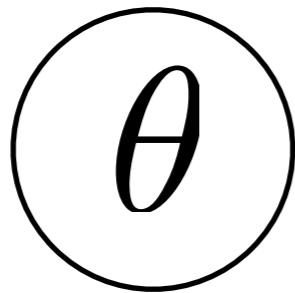


# Generating an image

shape



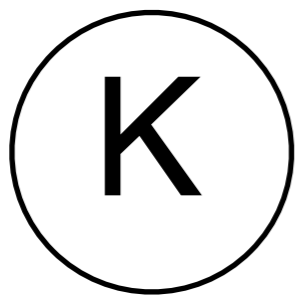
pose



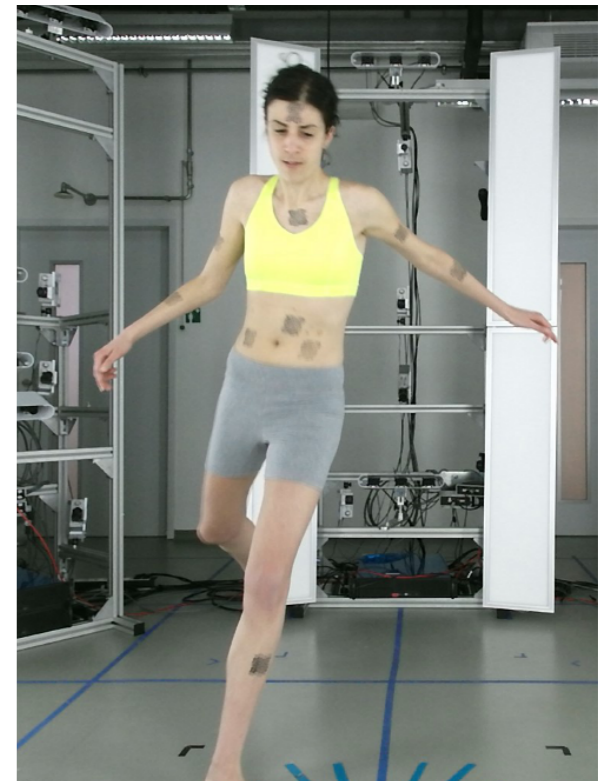
lighting



camera



image



# Albedo and shading

Albedo is constant: depends on physical properties of the surface  
Shading is transient: given by the interplay between surface reflectance and lighting



real image



albedo



shading

# Reflectance models

## Lambertian reflectance

$$i_x = (\mathbf{n}_x \cdot \mathbf{l}_x) a_x l$$

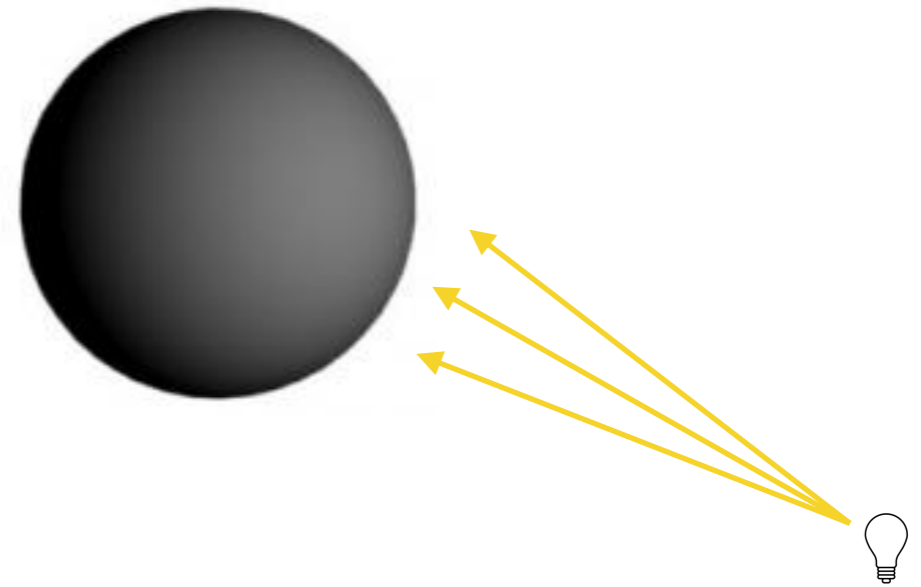
Diagram illustrating the components of the Lambertian reflectance equation:

- $i_x$ : surface color
- $\mathbf{n}_x$ : surface normal
- $\mathbf{l}_x$ : direction from  $x$  to light source
- $a_x$ : albedo
- $l$ : light intensity



# Lighting models

Point light sources

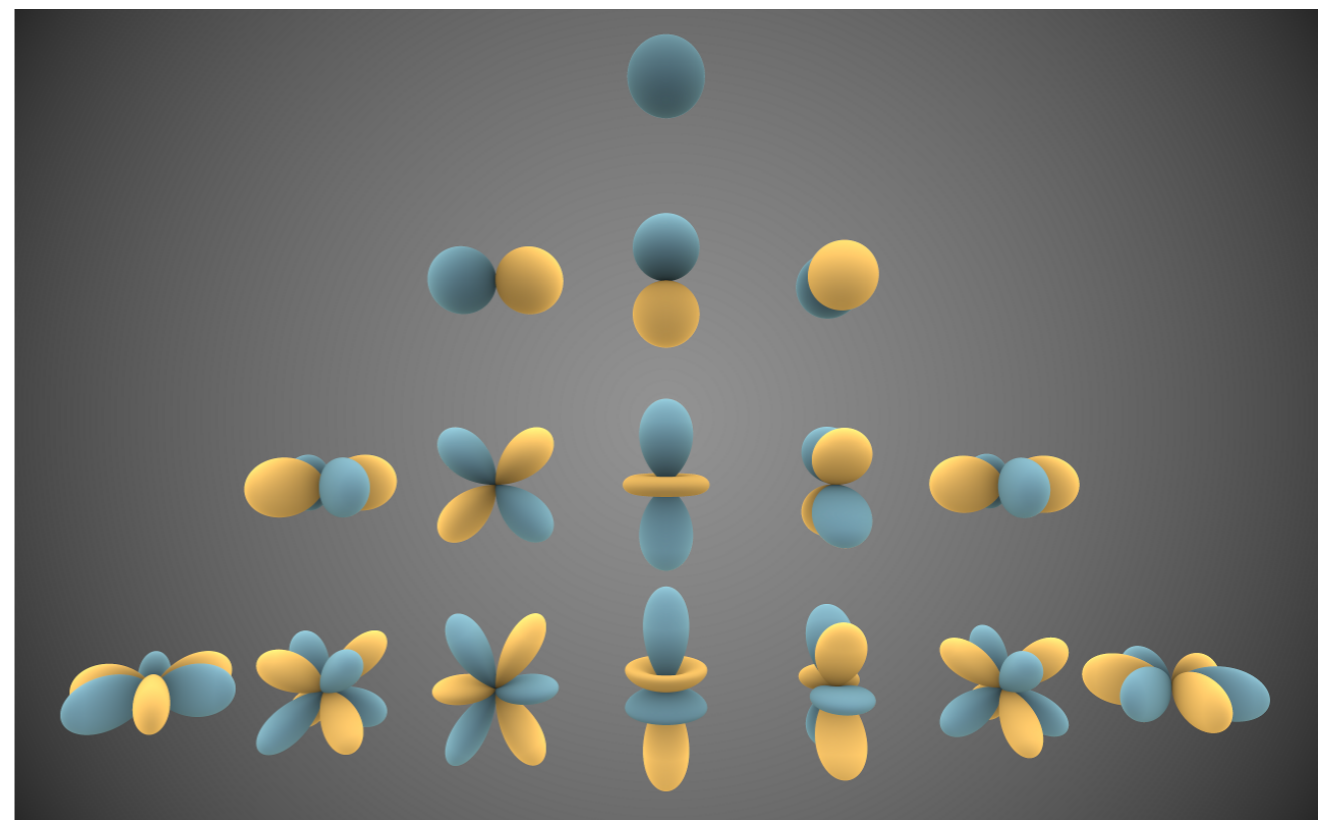


# Lighting models

## Spherical Harmonics (SH)

Lighting as a function over the sphere, projected onto a low-order SH basis

Simple and efficient for diffuse environments



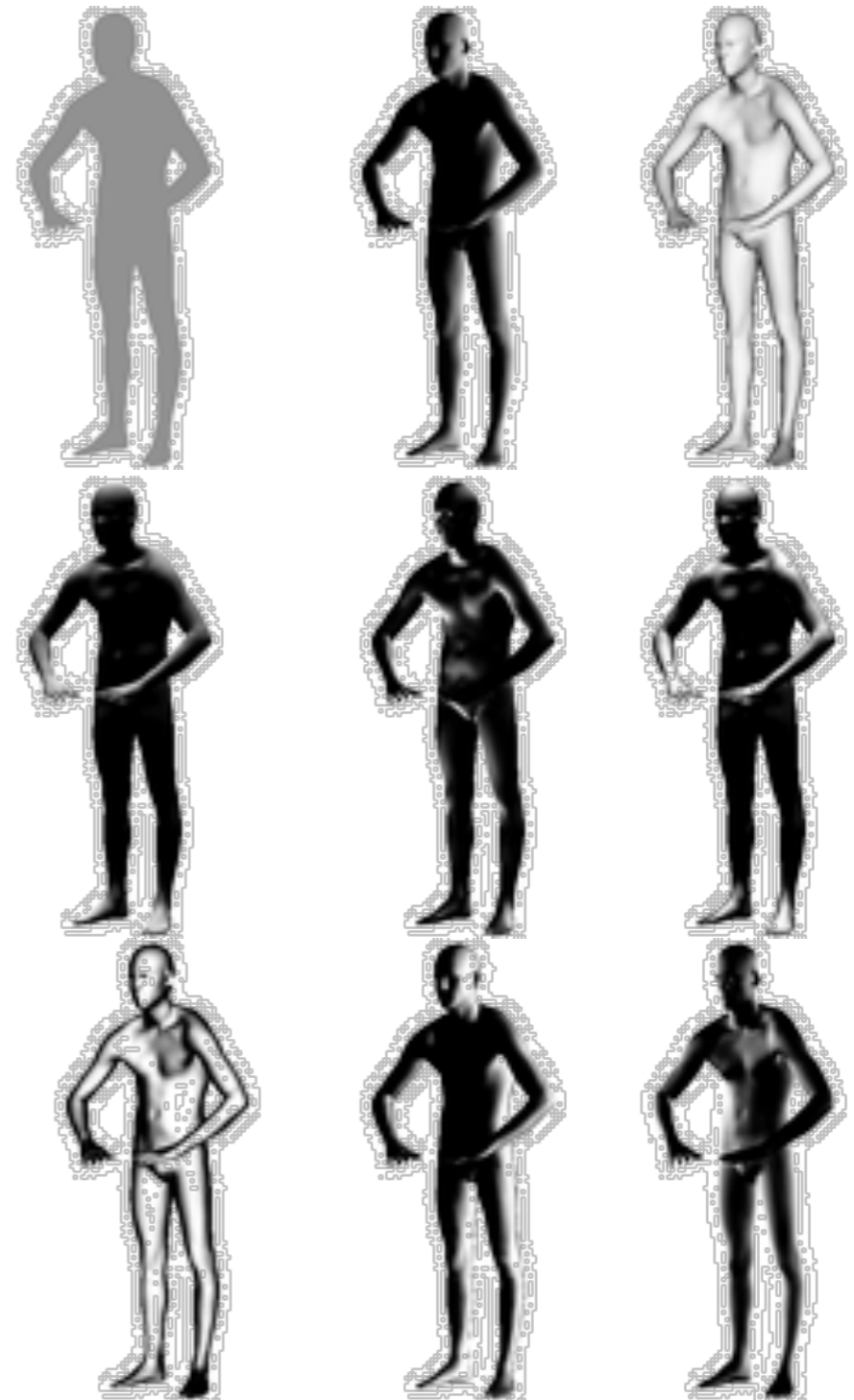


# Lighting models

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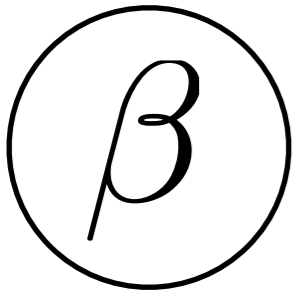
Simple and efficient for diffuse environments



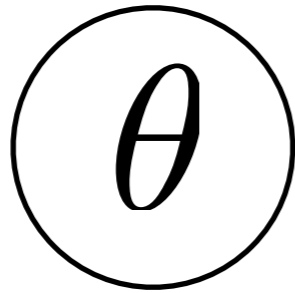
Sloan et al., SIGGRAPH 2002.  
Basri et al., IEEE TPAMI, 2003.

# Modeling all together

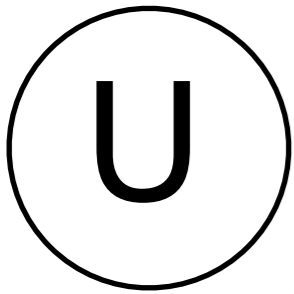
shape



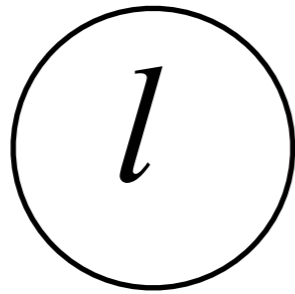
pose



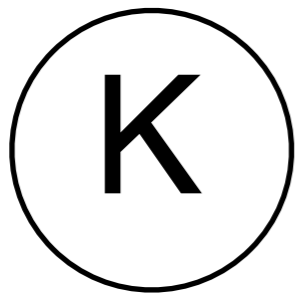
UV map



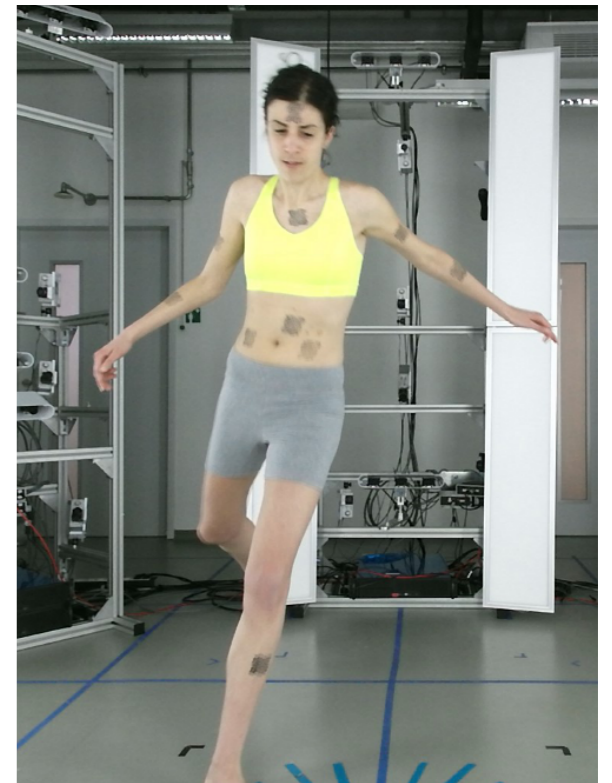
lighting



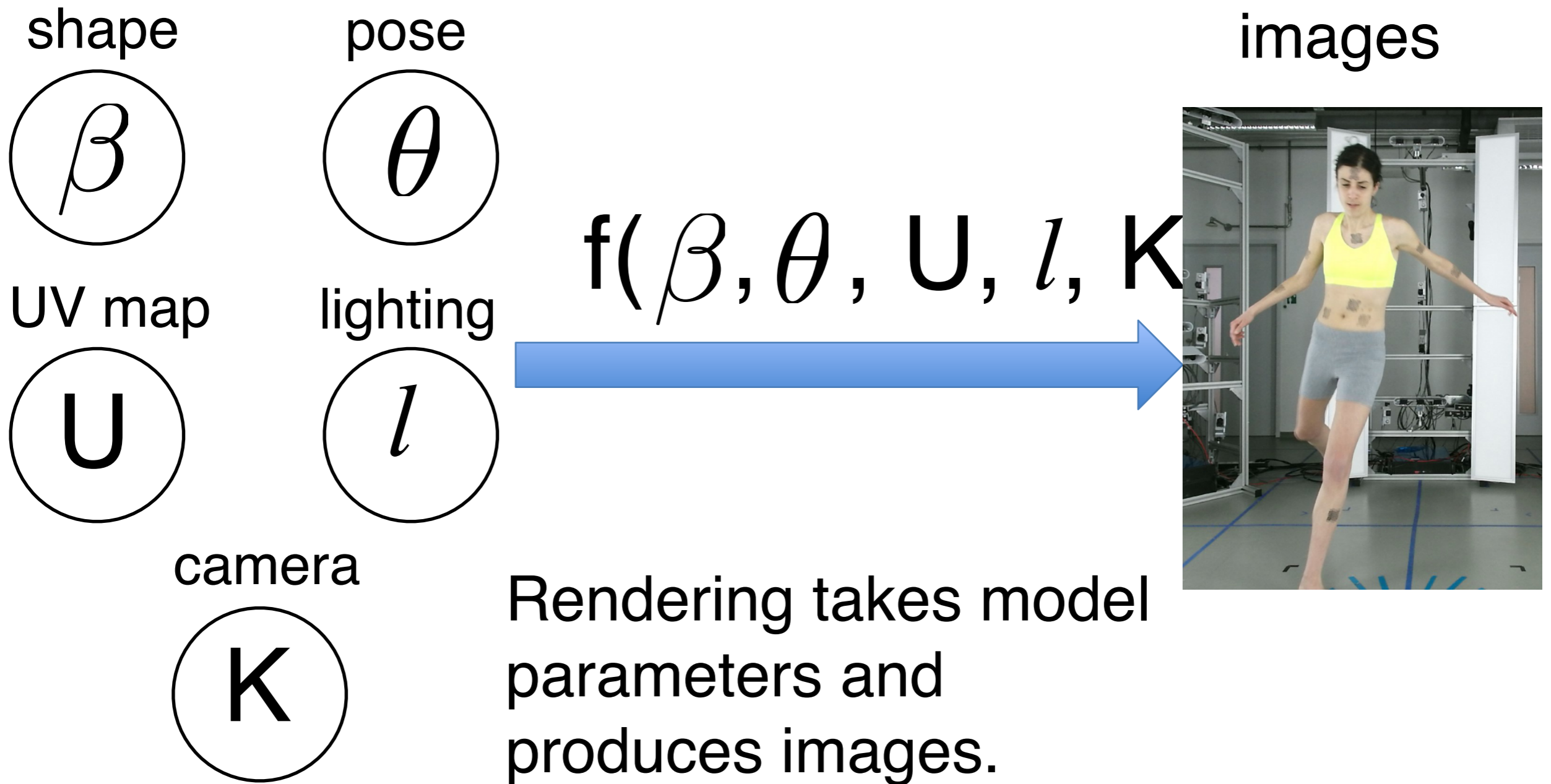
camera



images



# Forward rendering process



# Gradient-based optimization?

- We want to exploit images to obtain better registrations
- We saw that we can optimise a function given its derivatives
- Most of the functions involved in the rendering are linear operators
- Anybody wants to write the jacobians by hand?

# OpenDR

An open source differentiable rendering framework for:

- approximating a rendering process
- differentiating this approximation
- finding parameter estimates



*<http://open-dr.org>*

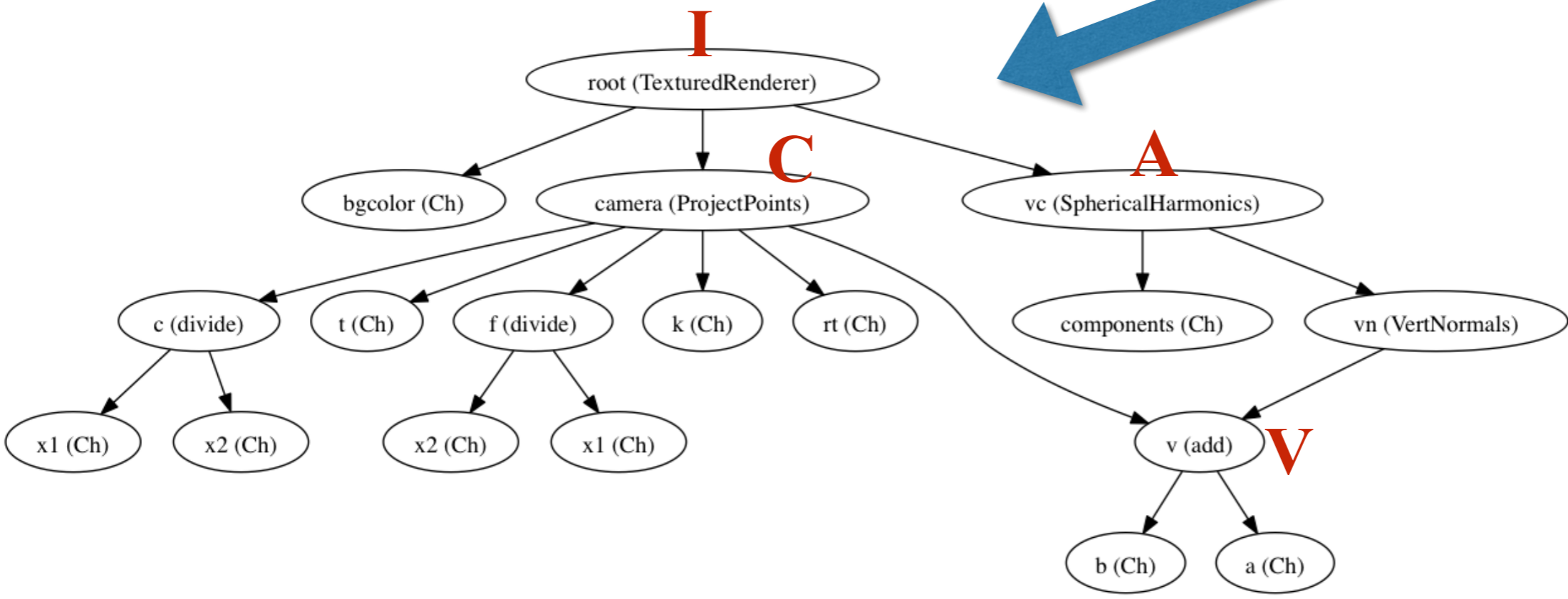
Loper and Black, ECCV 2014.

# OpenDR

```
import chumpy as ch
from opendr.everything import *

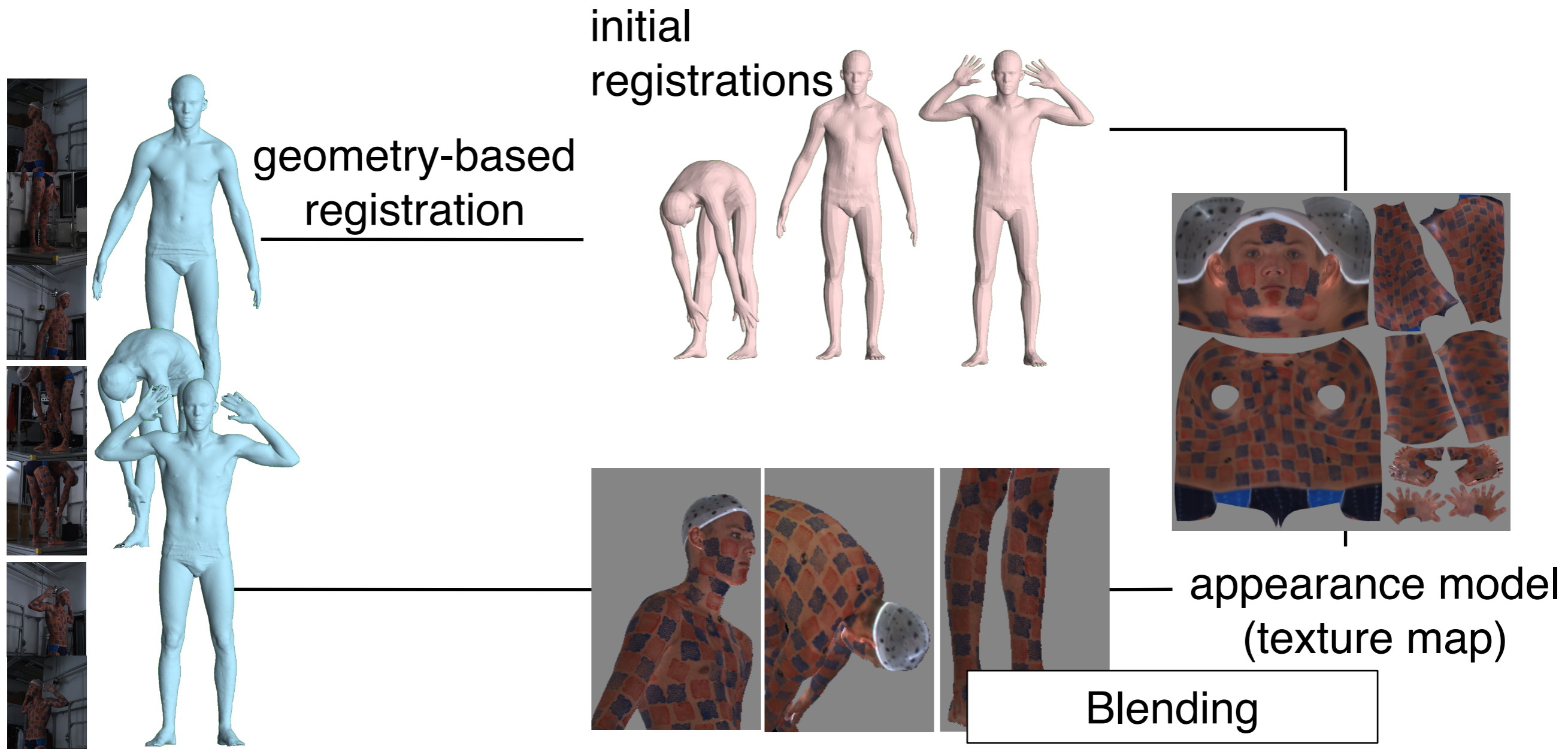
# Load mesh
m = load_mesh('/Users/matt/geist/OpenDR/test_dr/nasa_earth.obj')
m.v += ch.array([0,0,4])
w, h = (320, 240)
trans = ch.array([[0,0,0]])

# Construct renderer
rn = TexturedRenderer()
```



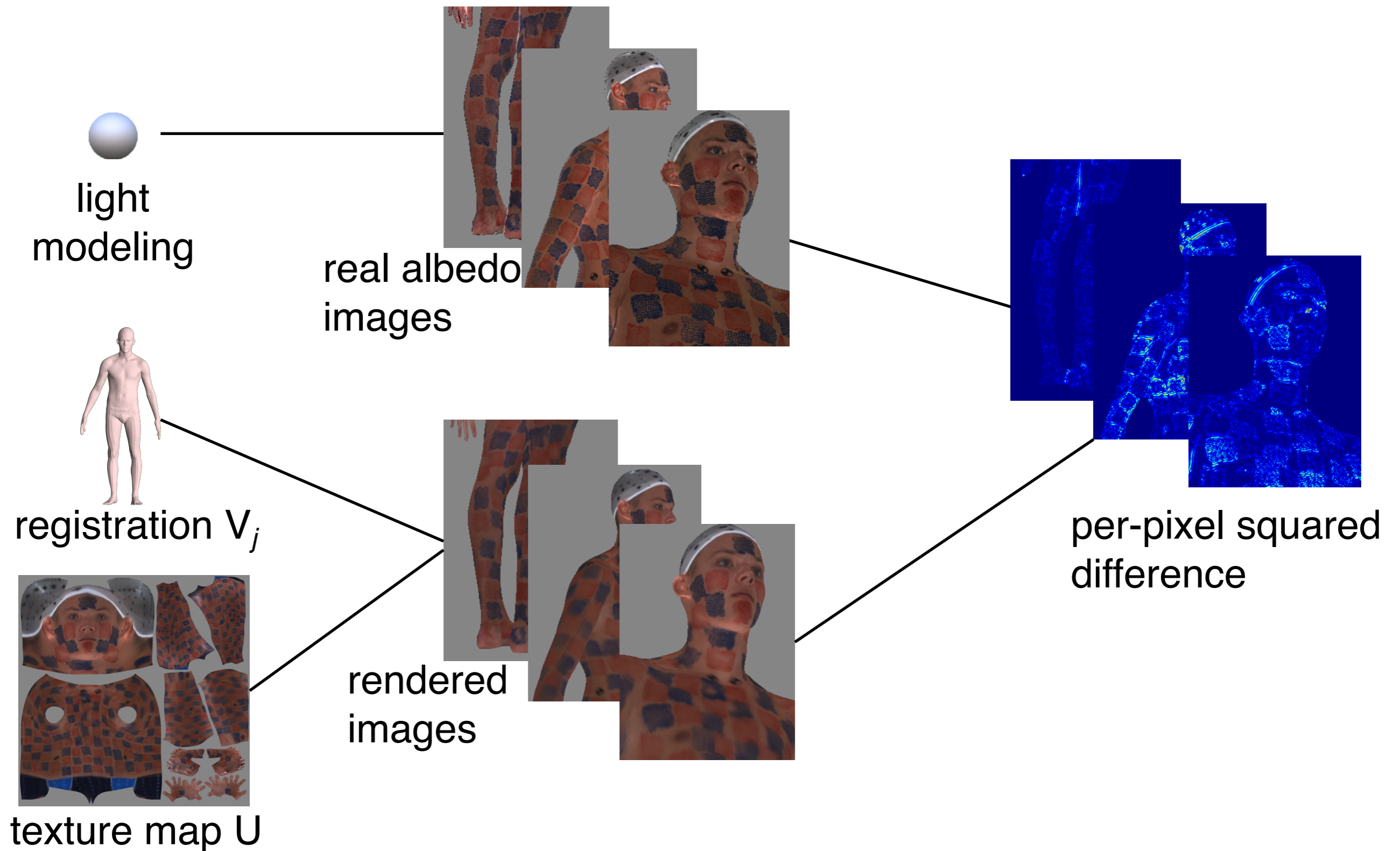
# Appearance-based registration

# Building an appearance model





# Appearance-based error term



# New registration objective

$$\begin{aligned}\vec{\theta}, \vec{\beta} &= \arg \min_{\vec{\theta}, \vec{\beta}} \|M(\vec{\theta}, \vec{\beta}) - \mathbf{V}\|^2 \\ &+ E_{\theta}(\vec{\theta}) \\ &+ E_{\beta}(\vec{\beta}) \\ &+ E_U(\mathbf{I}, \mathbf{K}, \mathbf{U}, M(\vec{\theta}, \vec{\beta})) \\ E_U &\equiv \sum_i \|\mathbf{I}_i - r(M(\vec{\theta}, \vec{\beta}), \mathbf{U}, \mathbf{K}_i)\|^2\end{aligned}$$

# With OpenDR...

```
import chumpy as ch
import cv2
from opendr.camera import ProjectPoints
from opendr.renderers import TexturedRenderer

# Load meshes, create other objectives...
# ...

# Construct renderer
rn = TexturedRenderer()
rn.camera = ProjectPoints(v=m.v, vc=m.vc, rt=ch.zeros(3), t=ch.zeros(3),
                        f=ch.array([w,w])/2., c = ch.array([w,h])/2., k=ch.zeros(5))
rn.frustum = {'near': 1., 'far': 10., 'width': w, 'height': h}
rn.set(f=m.f, texture_image=m.texture_img, ft=m.ft, vt=m.vt, bgcolor=ch.zeros(3))

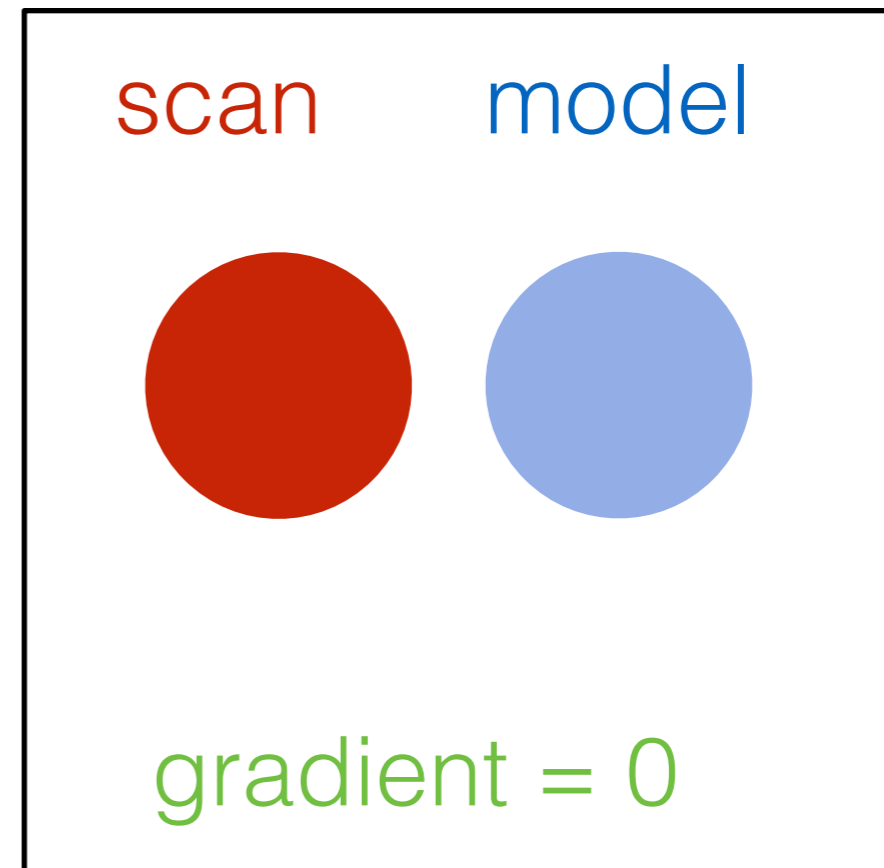
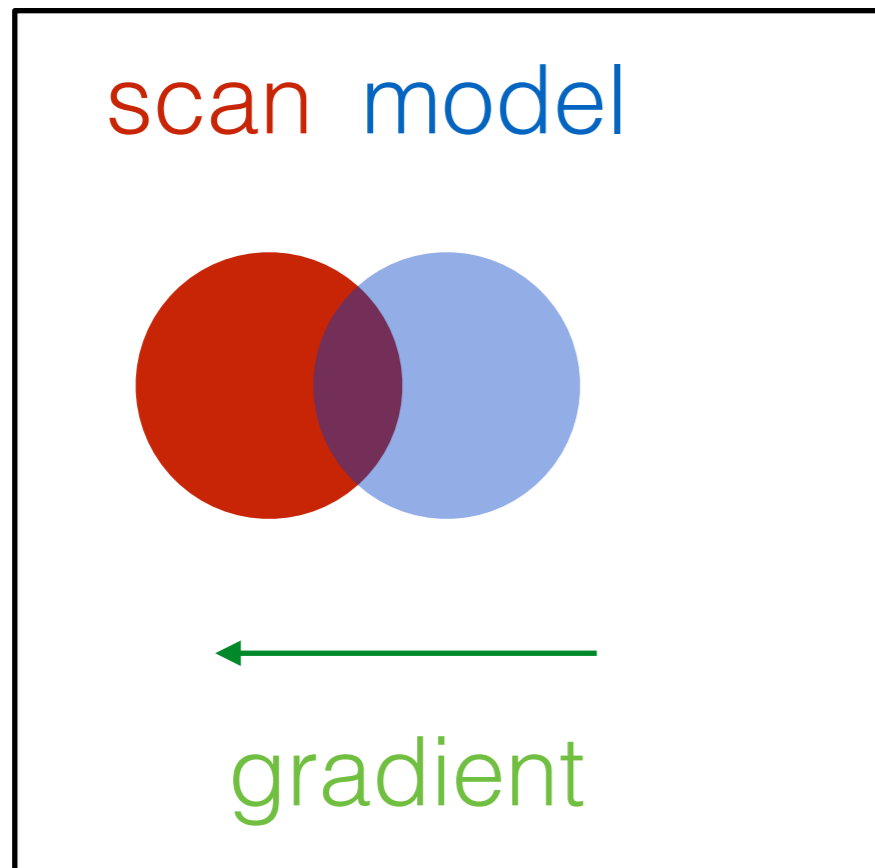
# Define the error term
obj = rn - cv2.imread(real_img_path)

# Minimize
ch.minimize(obj, x0=[m.v], method='dogleg')
```

lighting encoded in vc  
appearance encoded in

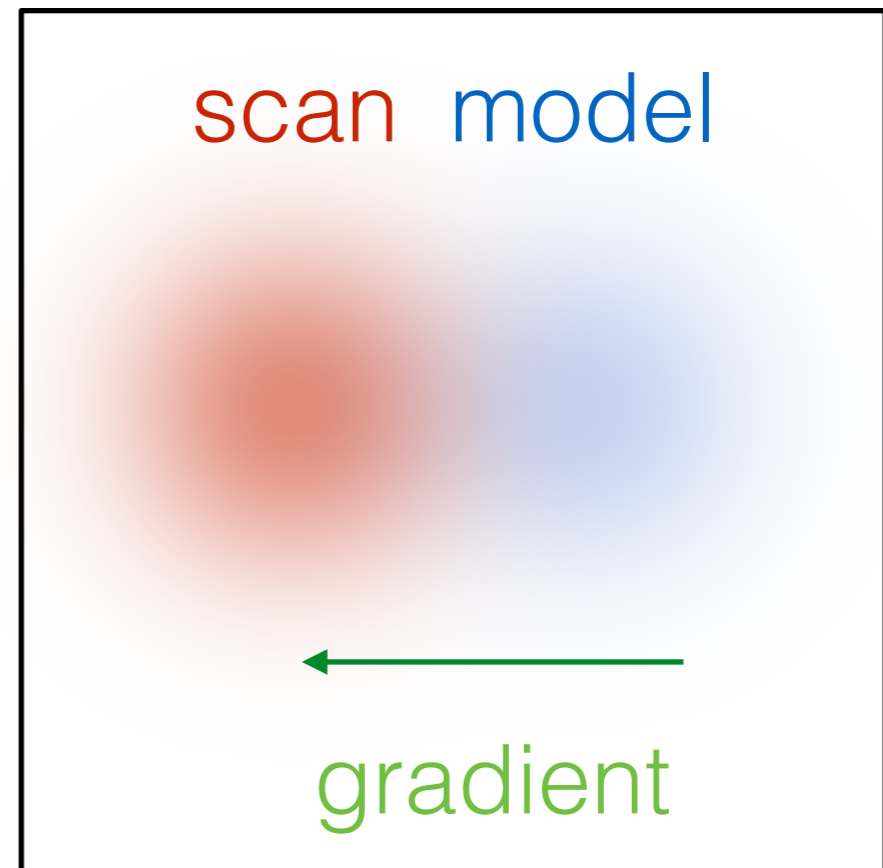
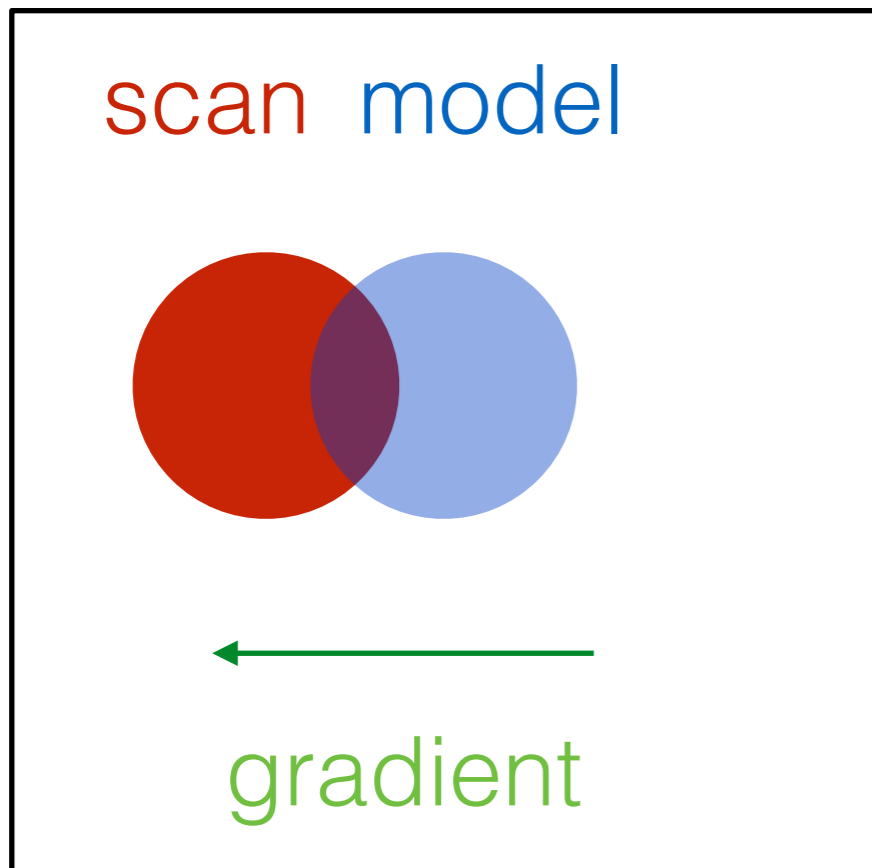
# Texture-based registration

- The appearance objective function has MANY local minima



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  - Optimisation will be slower

# Texture-based registration

- The appearance objective function has MANY local minima
  - Pyramids of blurred images help
- The dimensionality of this objective is much bigger than the geometric one
  - Optimisation will be slower
- Open problems: Lighting optimisation? Occlusions?

# Take-home message

- Optimising SMPL pose and shape with chumpy is easy
  - But the devil is in the details: point2surface, regularisers
- We can add color to our model either with per-vertex colors, or texture maps
- Apart from making the model match the scan geometrically, we can make it match in terms of COLOR
- OpenDR differentiates the rendering process for us