



mpi max planck institut
informatik

SIC Saarland Informatics
Campus

High Level Computer Vision

Backpropagation & Convolutional Neural Networks @ April 24, 2019

Bernt Schiele & Mario Fritz

www.mpi-inf.mpg.de/hlcv/

**Max Planck Institute for Informatics & Saarland University,
Saarland Informatics Campus Saarbrücken**

Overview Today's Lecture

- Backpropagation - Gradient Descent
 - ▶ illustrated using computational graphs
 - ▶ chain rule - upstream and local gradients
 - ▶ modularization simple
- What is Deep Learning
 - ▶ intuition why deep learning can help
 - ▶ integrated learning of features and classifier
- Convolutional Neural Networks (CNNs)
 - ▶ one of the (few) highly successful NNs

Where we are

$$s = f(x; W) = Wx$$

scores function

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

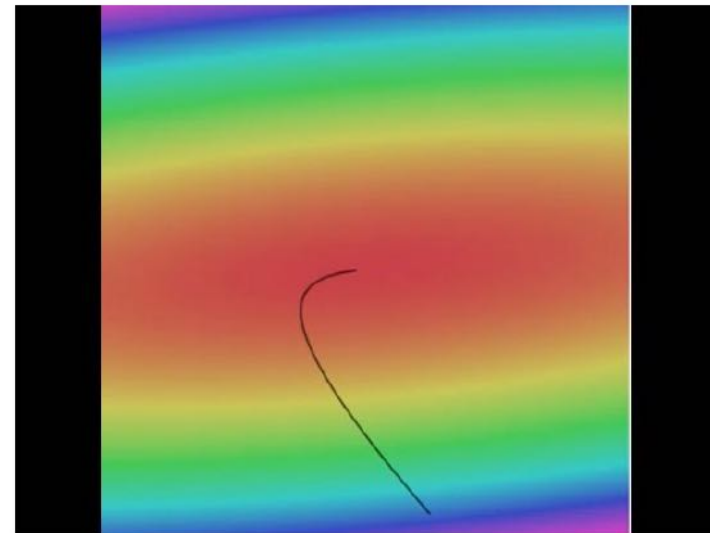
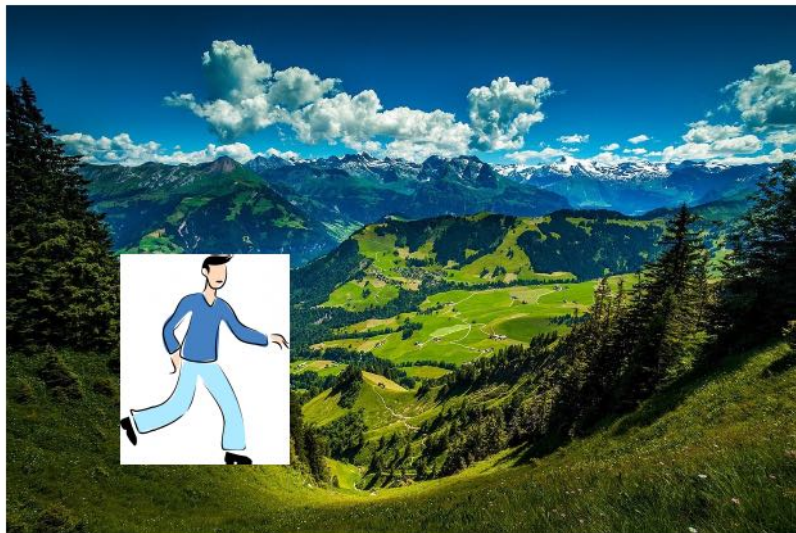
SVM loss

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \sum_k W_k^2$$

data loss + regularization

want $\nabla_W L$

Optimization



```
# Vanilla Gradient Descent  
  
while True:  
    weights_grad = evaluate_gradient(loss_fun, data, weights)  
    weights += - step_size * weights_grad # perform parameter update
```

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[Walking man image](#) is [CC0 1.0](#) public domain

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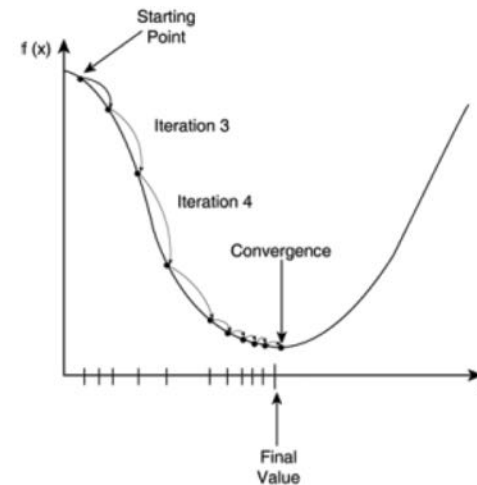
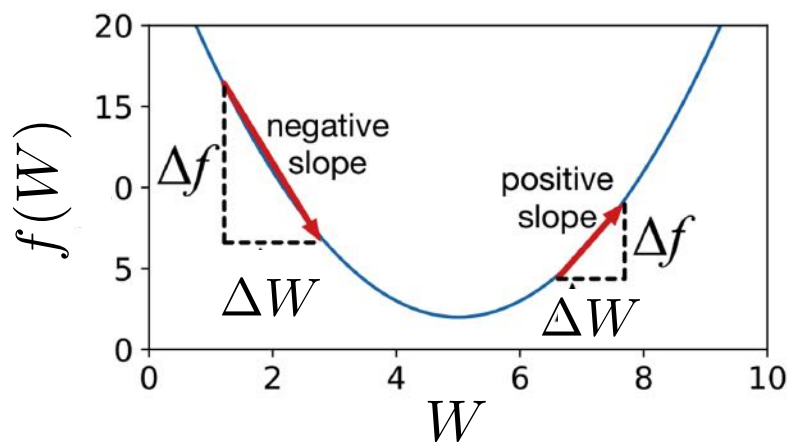
Gradient Descent

General gradient descent: Start with initial point W_0

$$\text{Sequence: } W_{t+1} = W_t + \alpha_t d_t$$

Steepest Descent:

$d_t = -\nabla f(W_t)$ (we move in the opposite direction of the gradient).



```
# Vanilla Gradient Descent
```

```
while True:
```

```
    weights_grad = evaluate_gradient(loss_fun, data, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```

Gradient Descent - Variants...

- Assume Loss to be:
 - ▶ with n the number of training samples
 - ▶ L_i the loss for training sample x_i

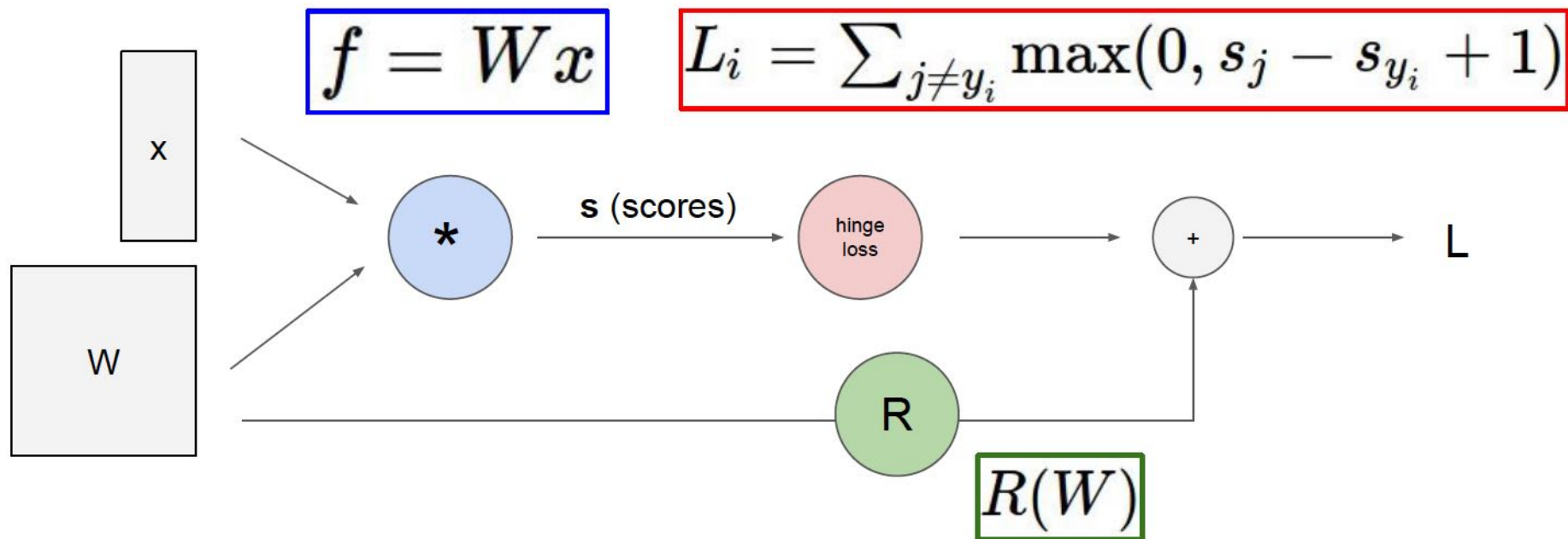
$$L(W) = \frac{1}{n} \sum_{i=1}^n L_i(W)$$

- **Stochastic Gradient Descent:**
 - ▶ randomly choose one training sample x_i
 - ▶ update weights based on loss $L_i(W)$

- **Mini-batch training:**
 - ▶ process a subset of training samples $M \subset \{1, \dots, n\}$
 - ▶ update weights based on $L_M(W) = \frac{1}{|M|} \sum_{i \in M} L_i(W)$

- **Batch training:**
 - ▶ process all training samples
 - ▶ update weights based on $L(W) = \frac{1}{n} \sum_{i=1}^n L_i(W)$

Computational Graphs



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Neural Network Example...

Convolutional network (AlexNet)

input image

weights

loss

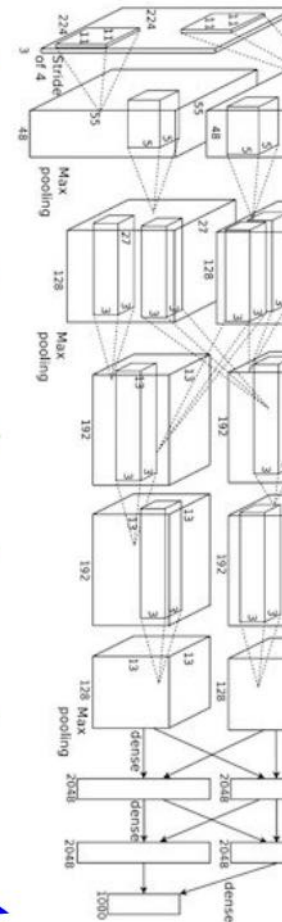


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Neural Network Example...

Neural Turing Machine

input image

loss

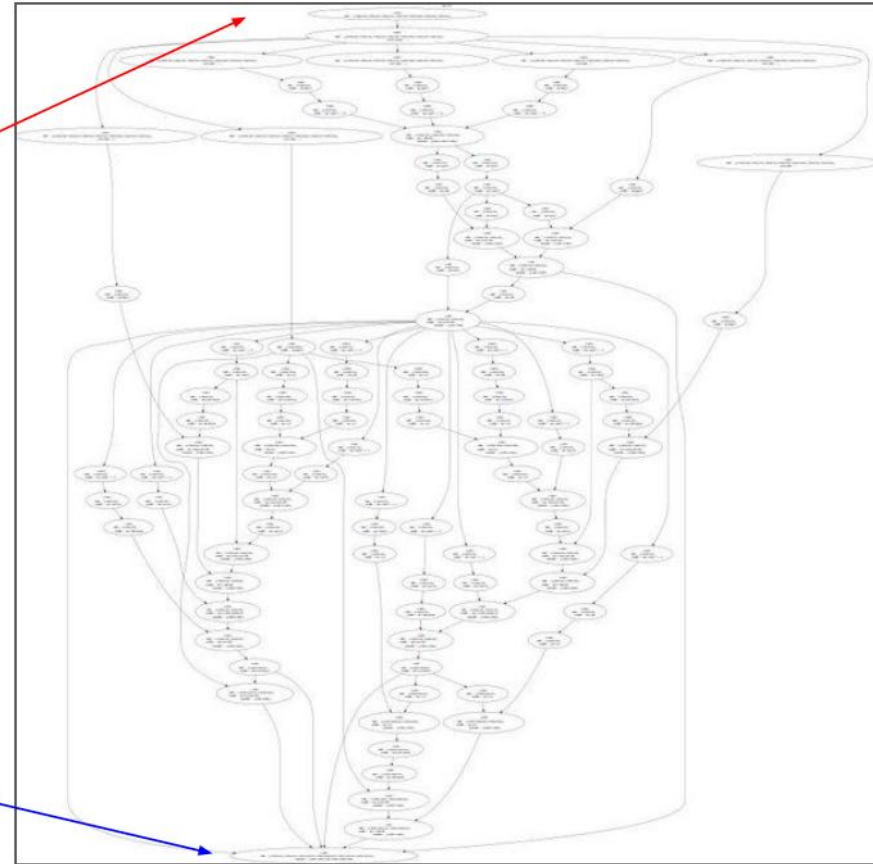


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Backpropagation - A Simple Example

Backpropagation: a simple example

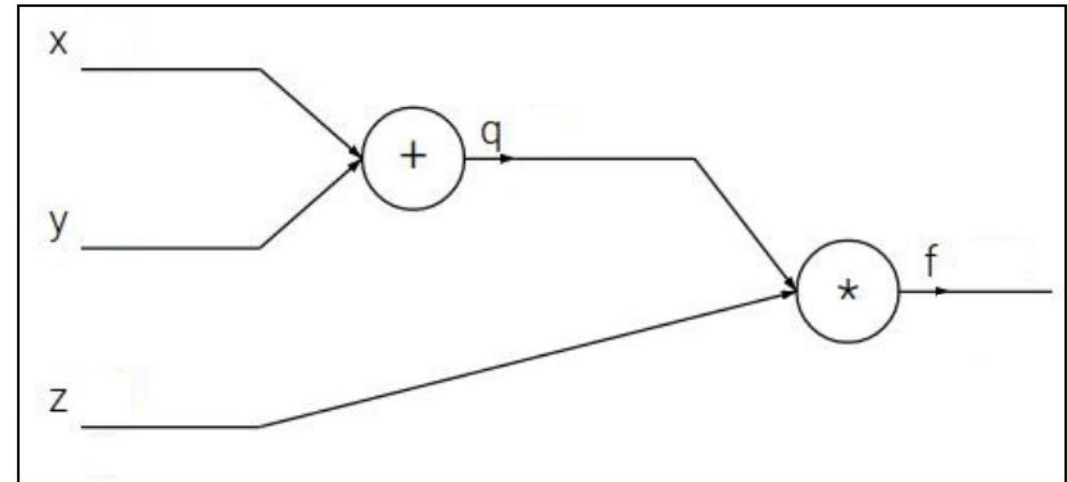
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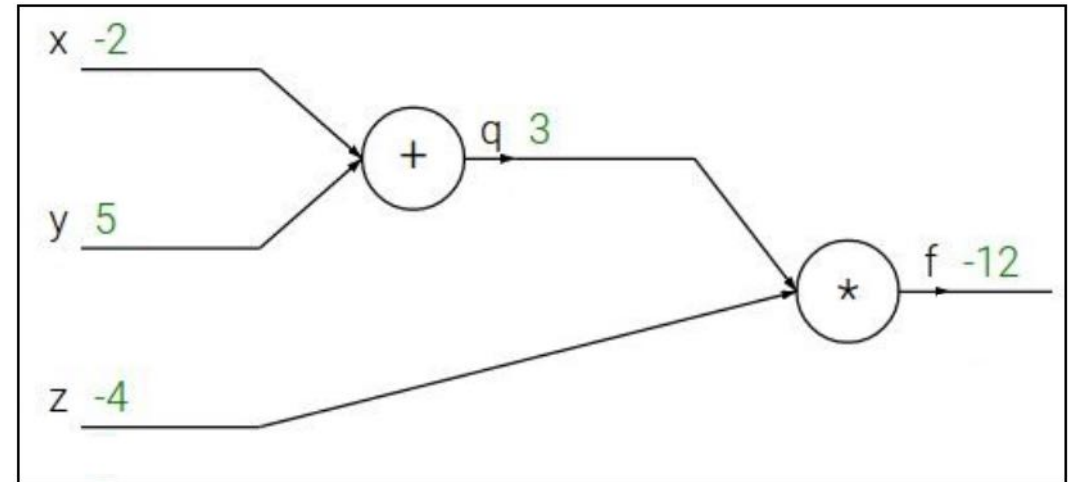
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e.g. $x = -2, y = 5, z = -4$



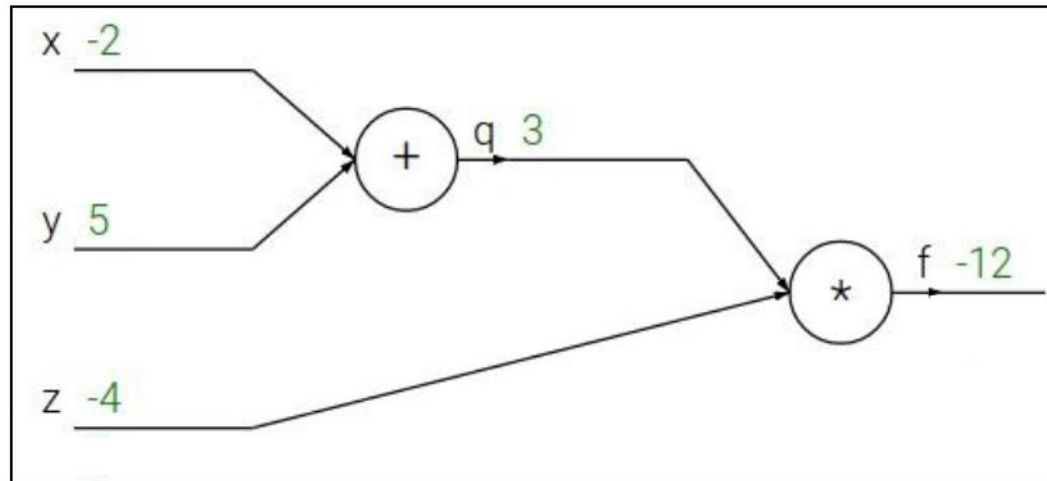
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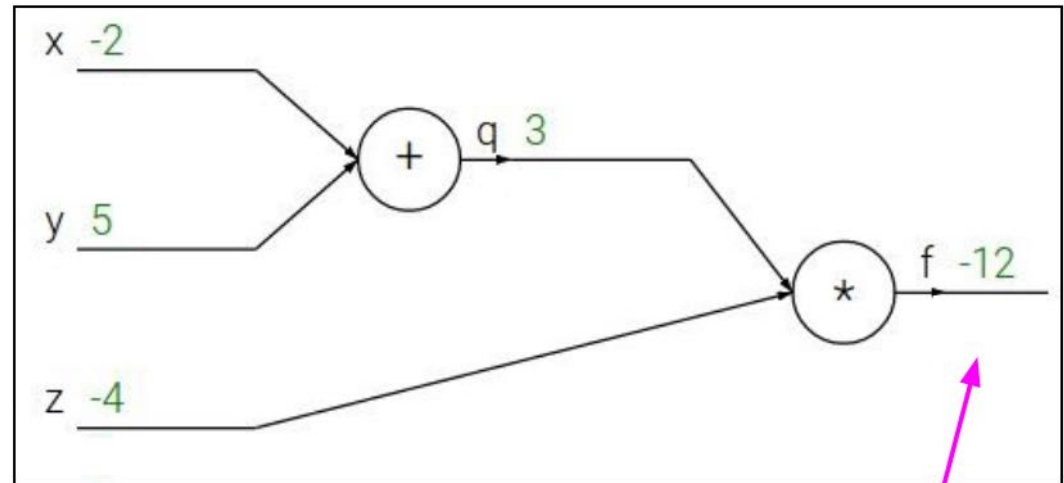
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$$\frac{\partial f}{\partial f}$$

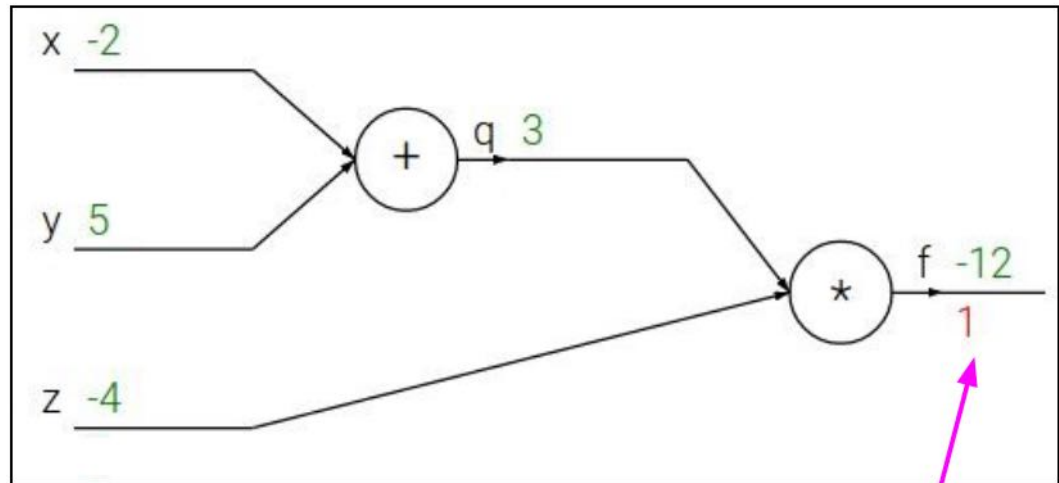
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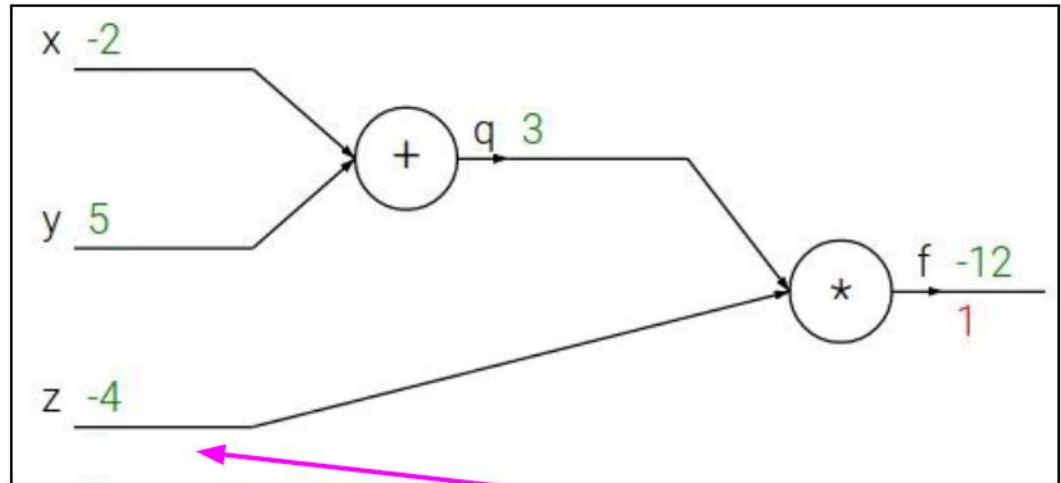
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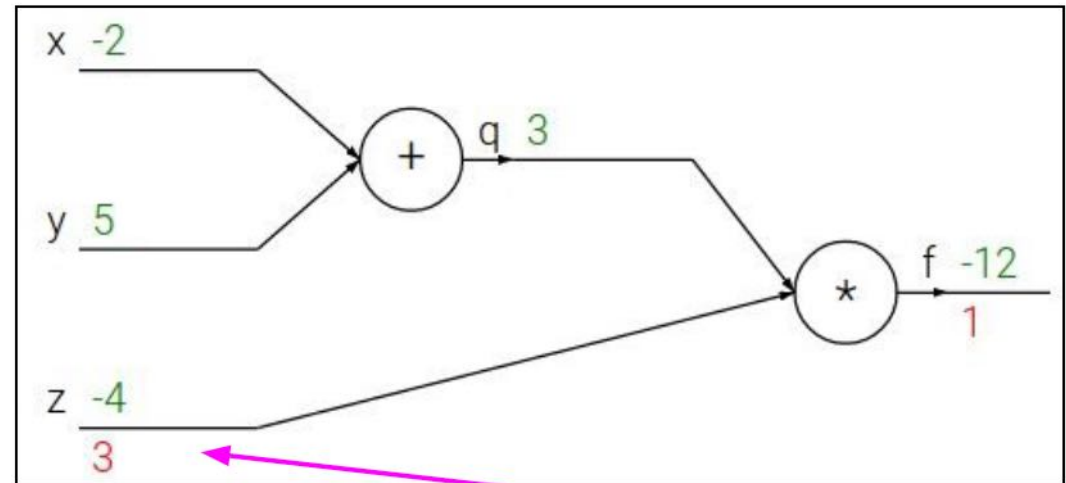
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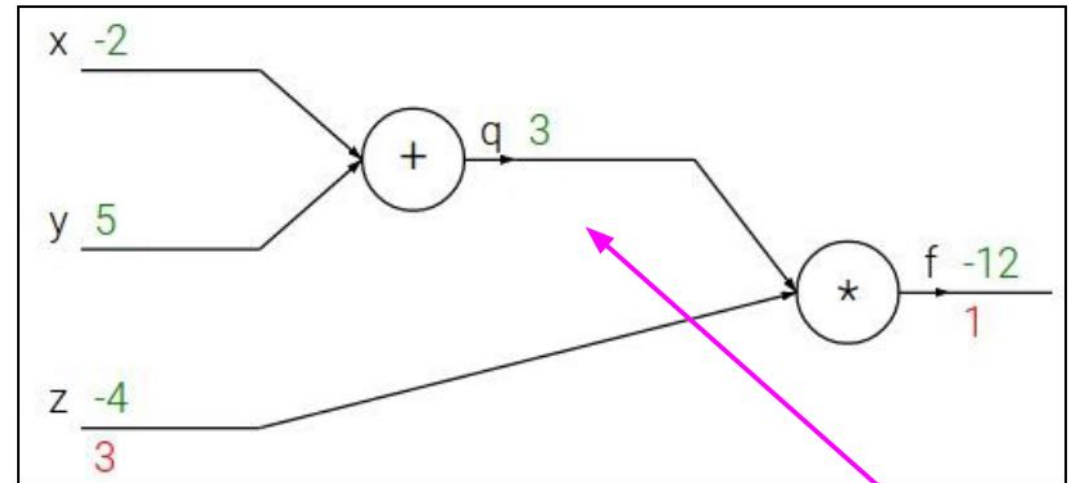
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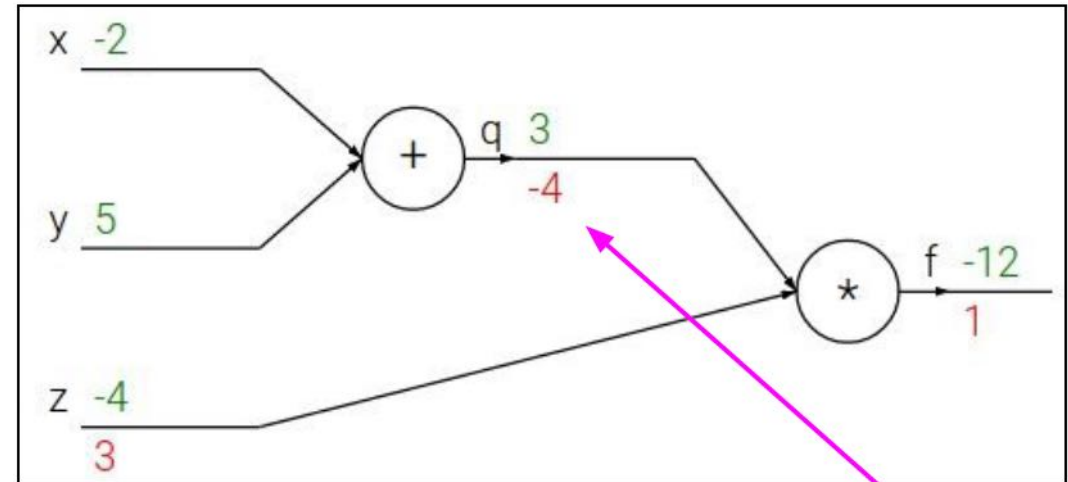
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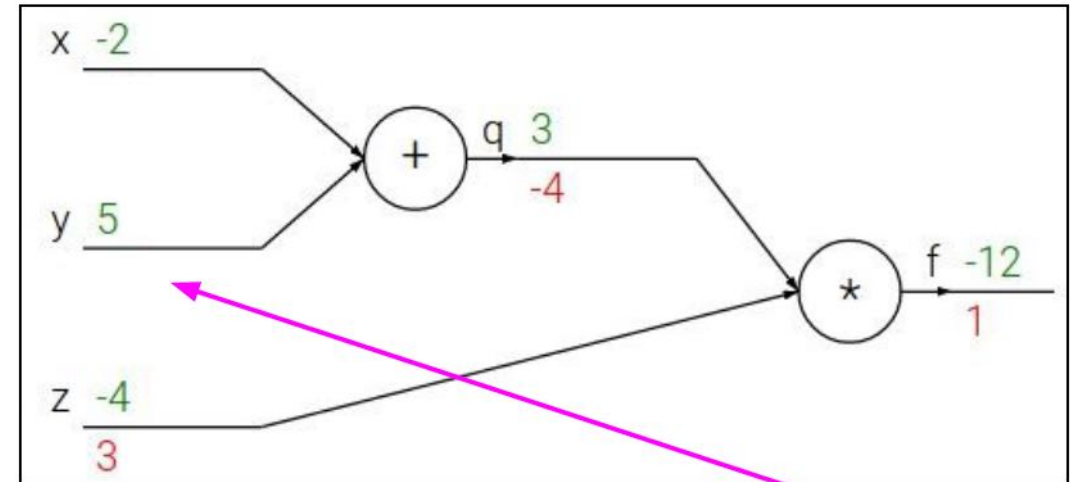
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$$\frac{\partial f}{\partial y}$$

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

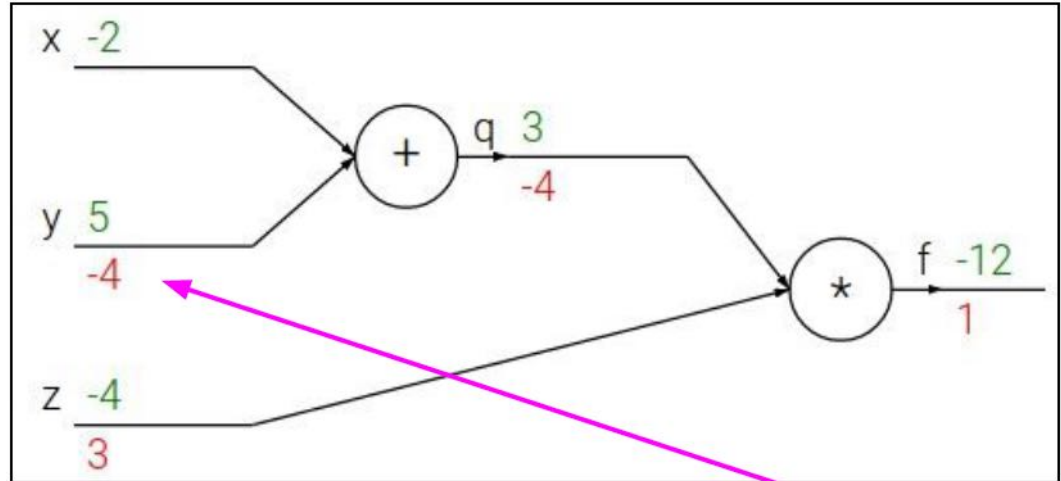
Upstream gradient Local gradient

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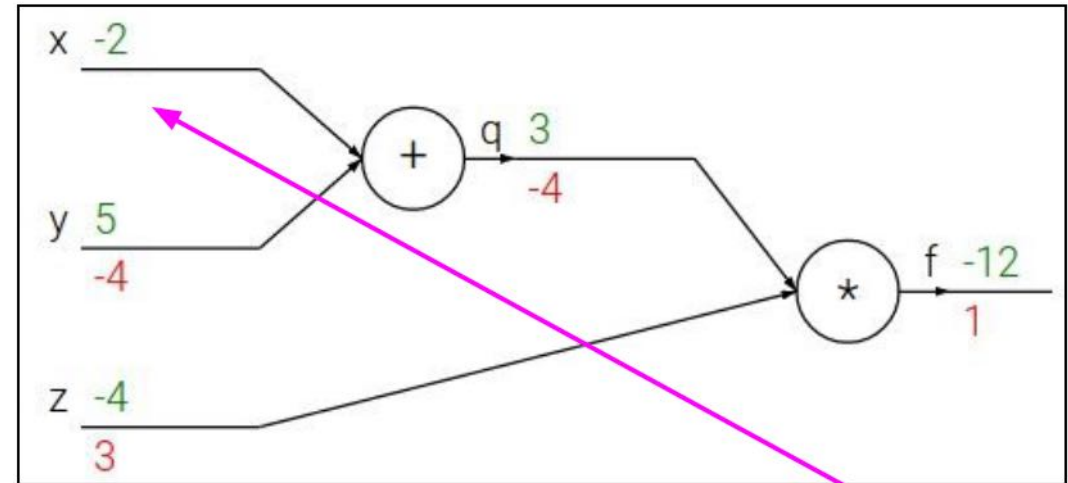
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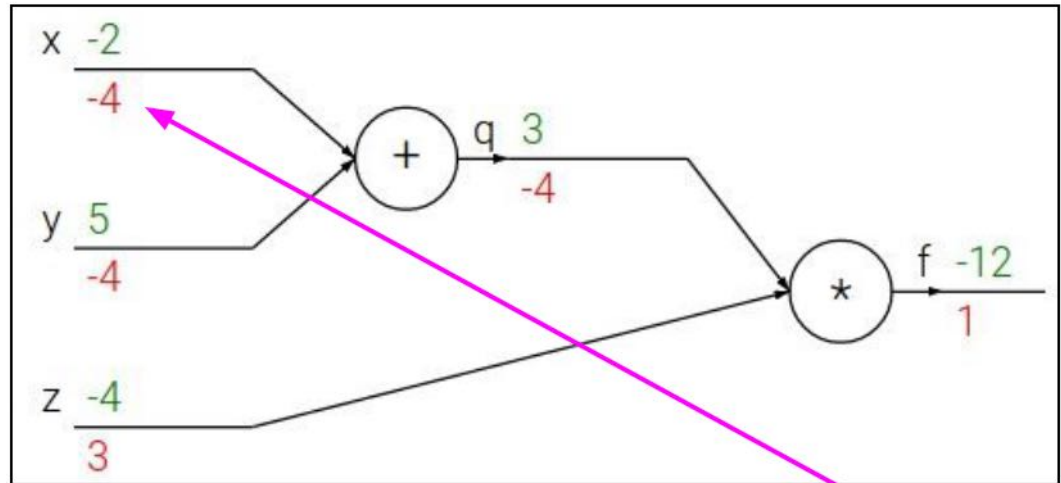
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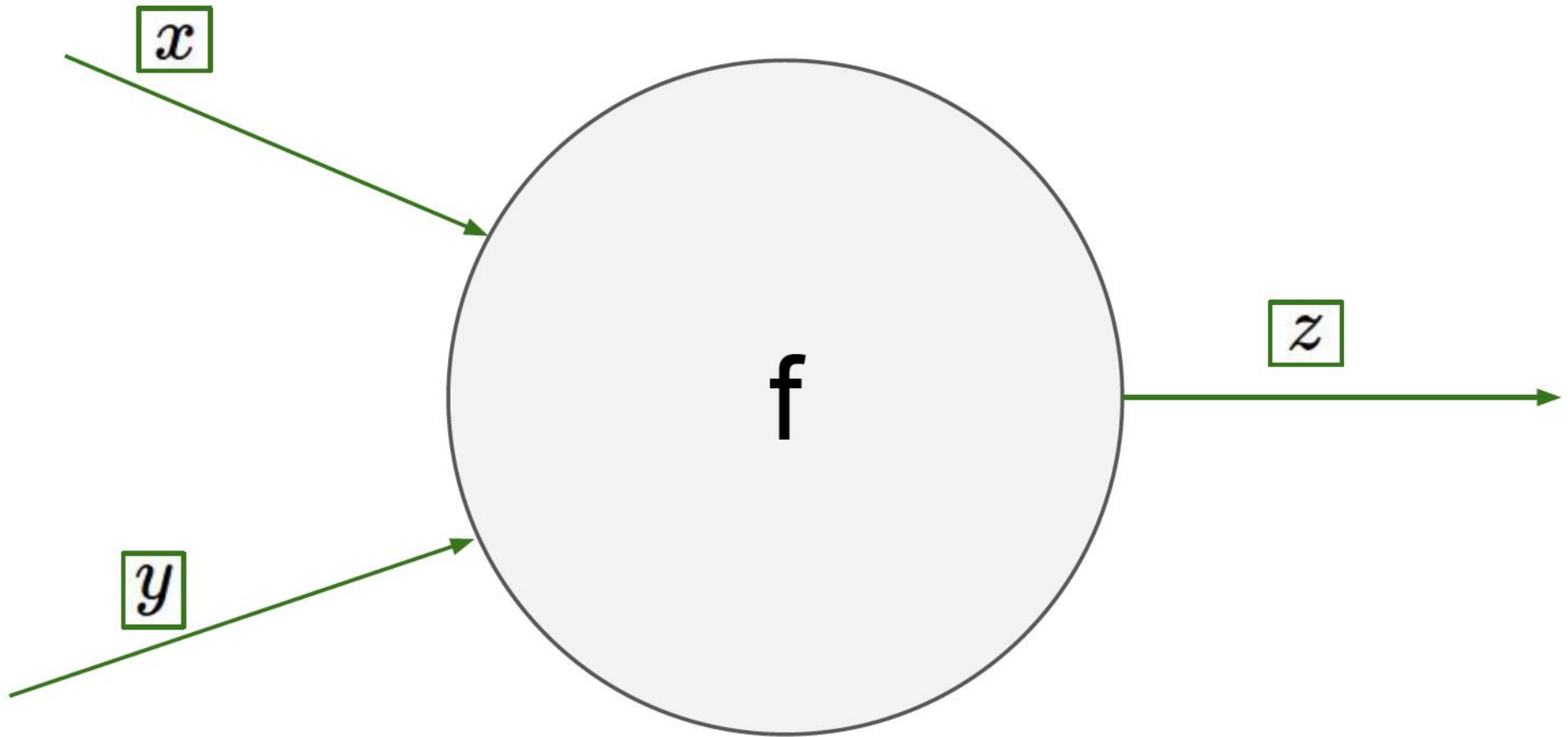
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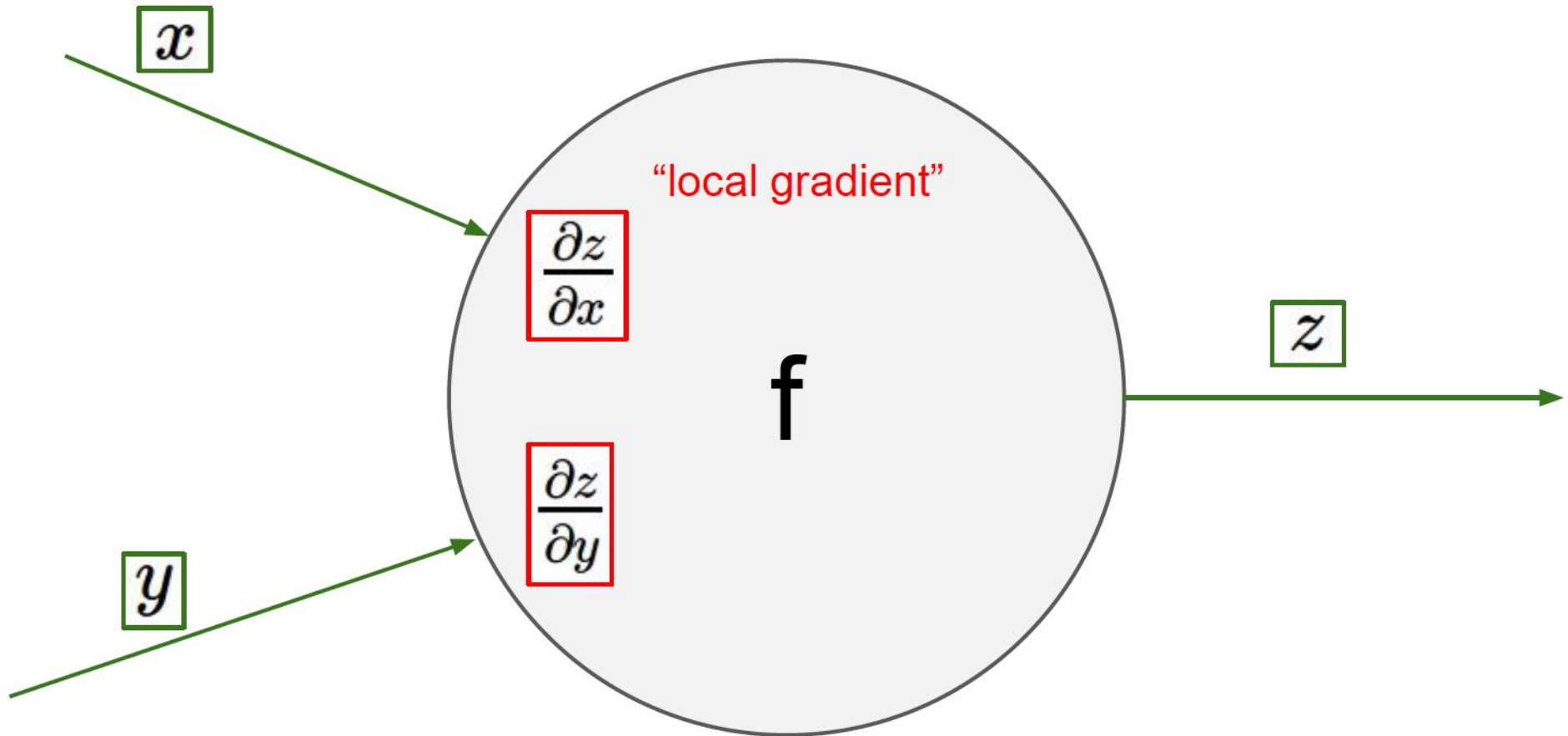
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Backpropagation - Local View



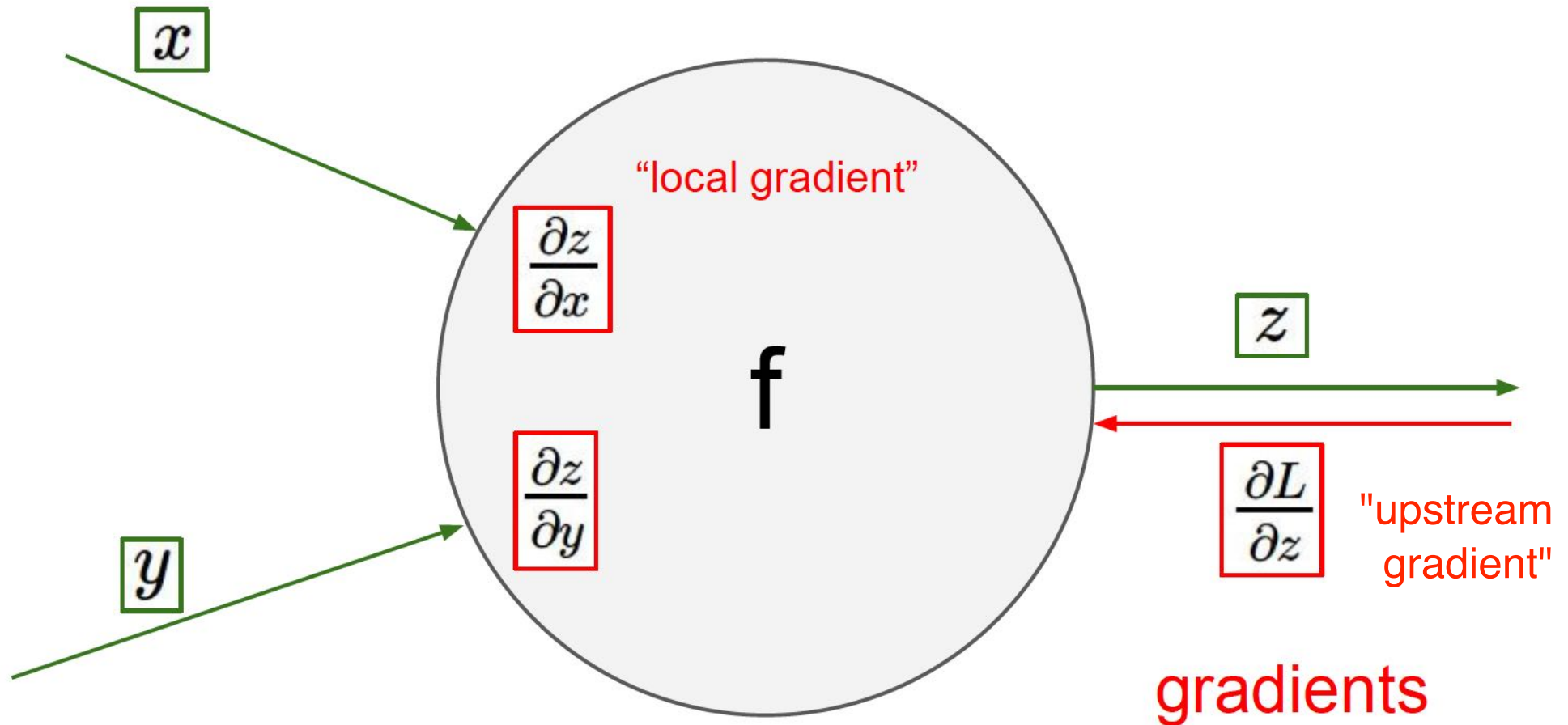
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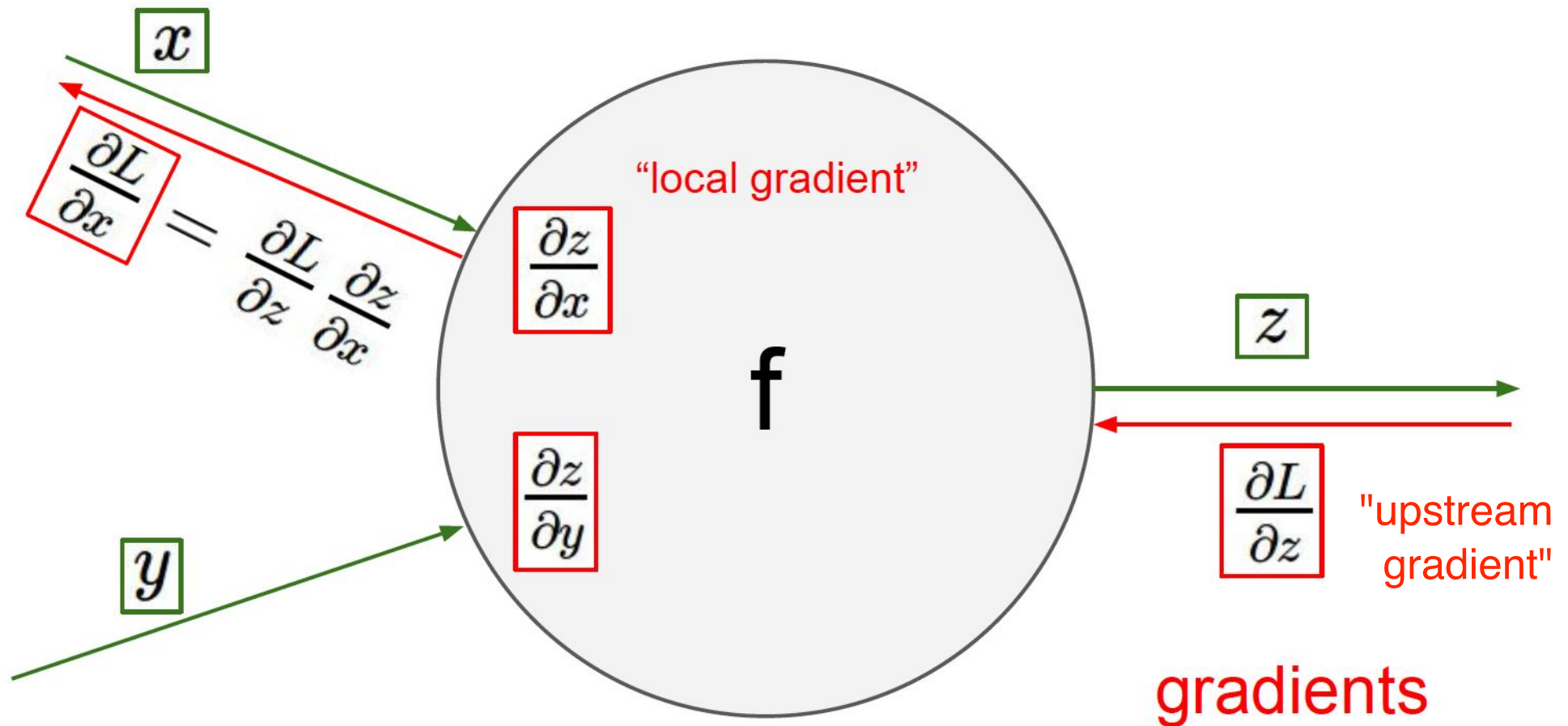
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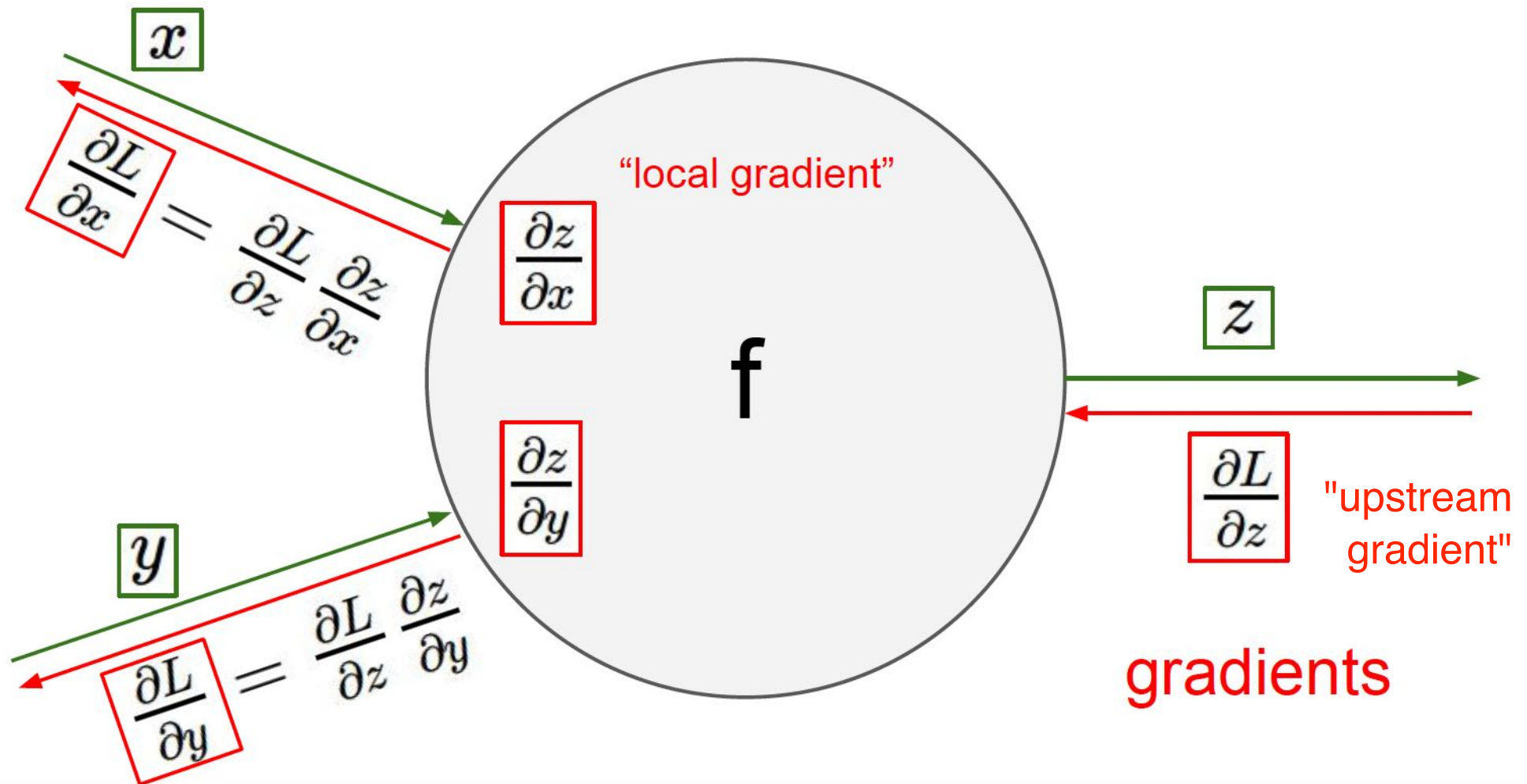
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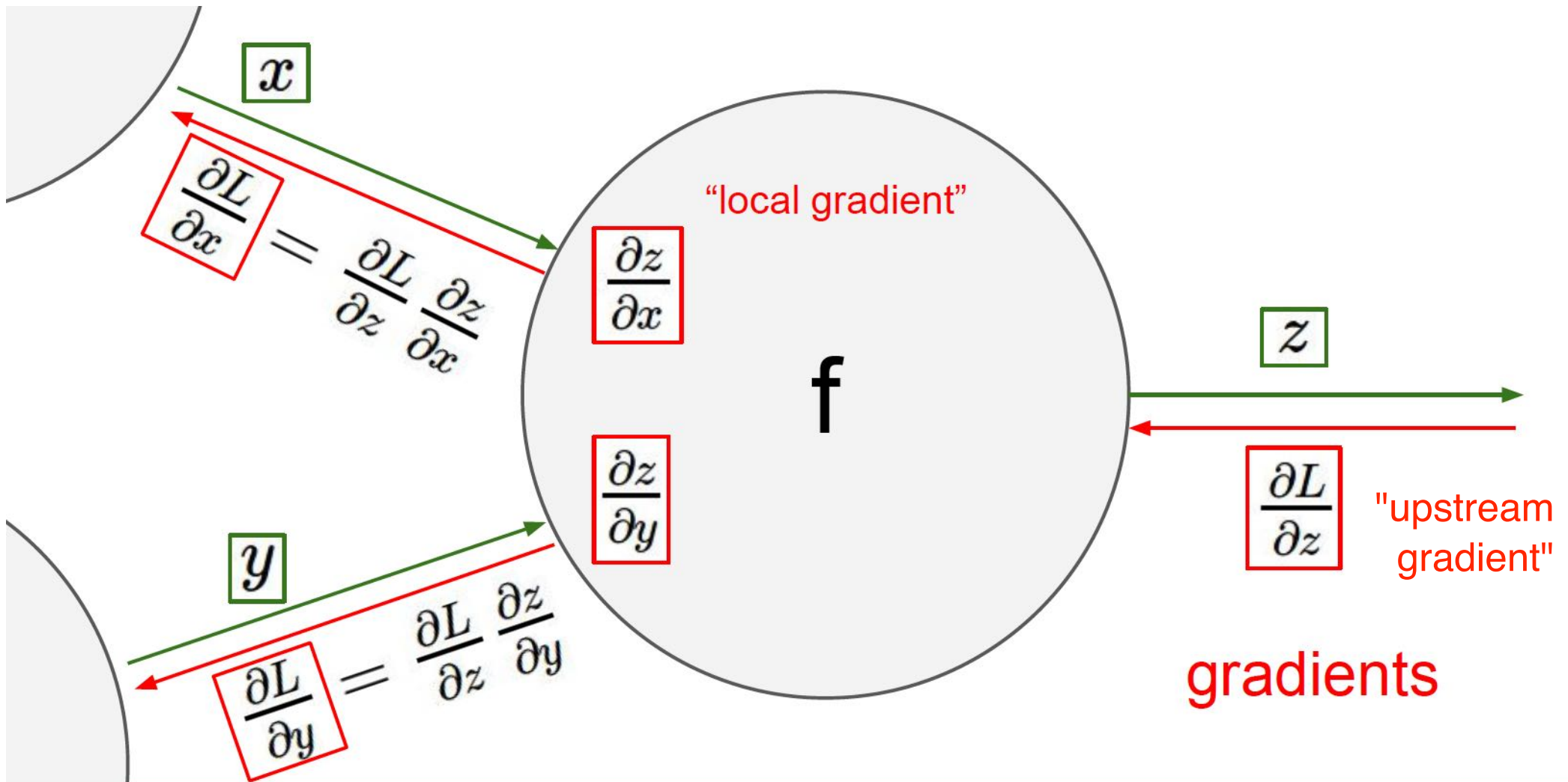
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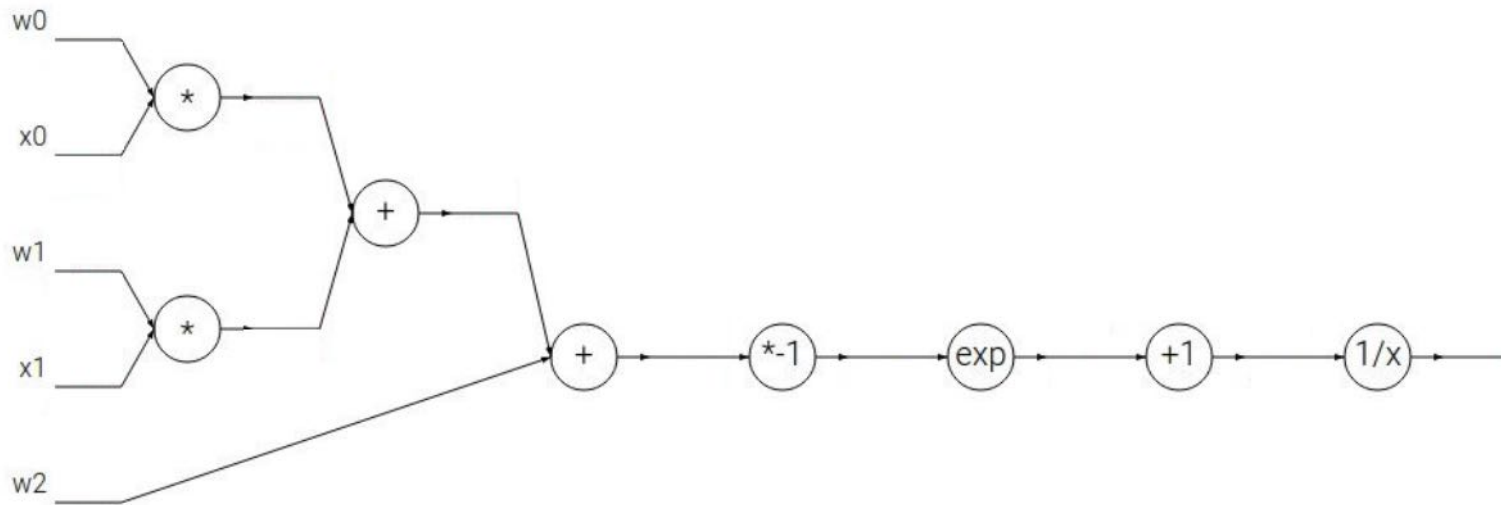
Backpropagation - Local View



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Backpropagation

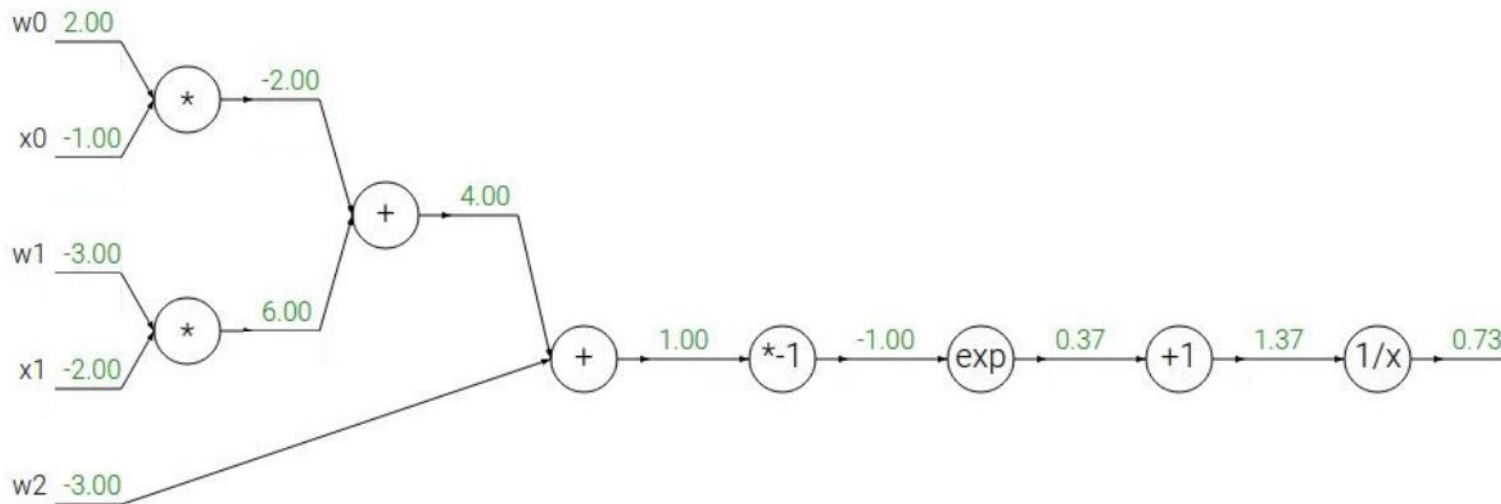
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



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Backpropagation

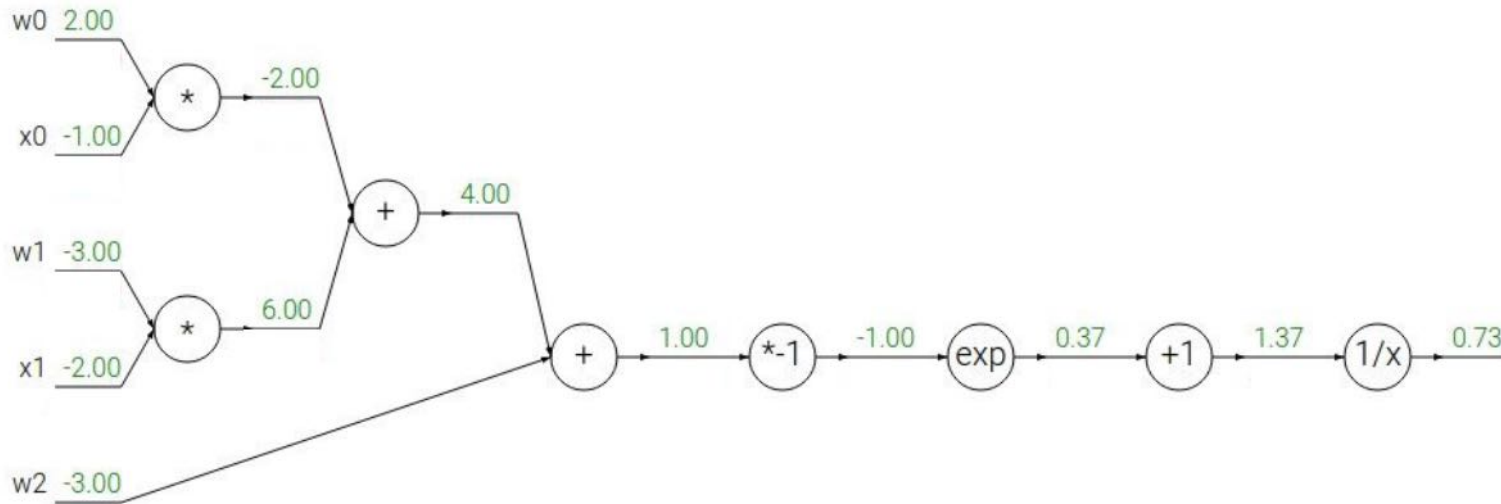
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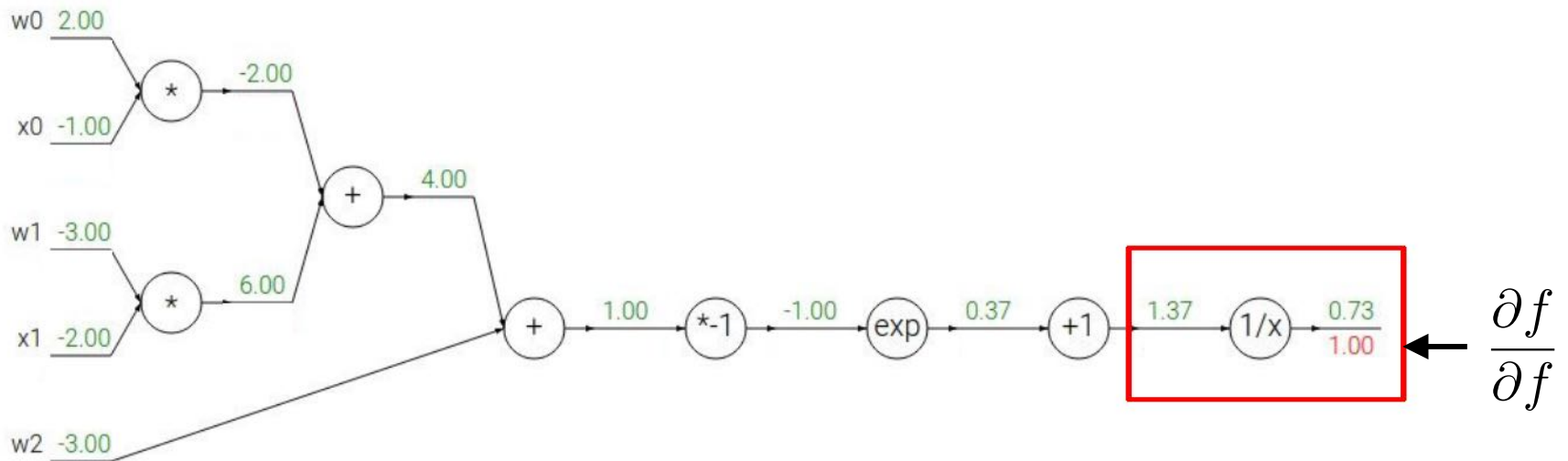


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$f_a(x) = ax$	→	$\frac{df}{dx} = a$		$f_c(x) = c + x$	→	$\frac{df}{dx} = 1$

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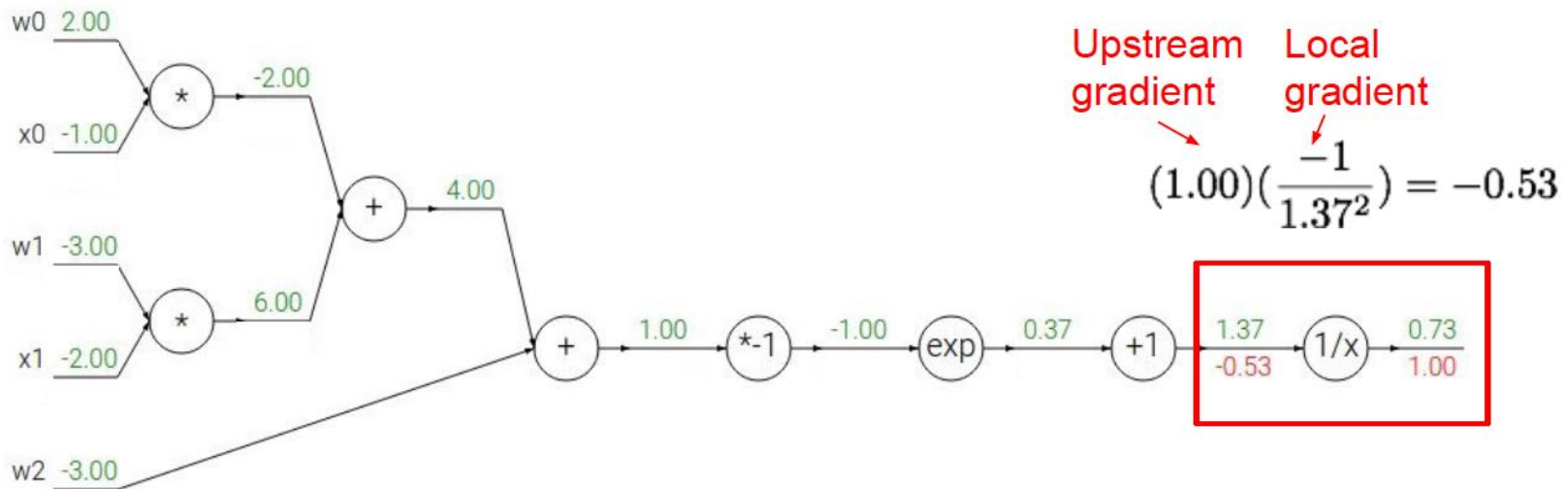
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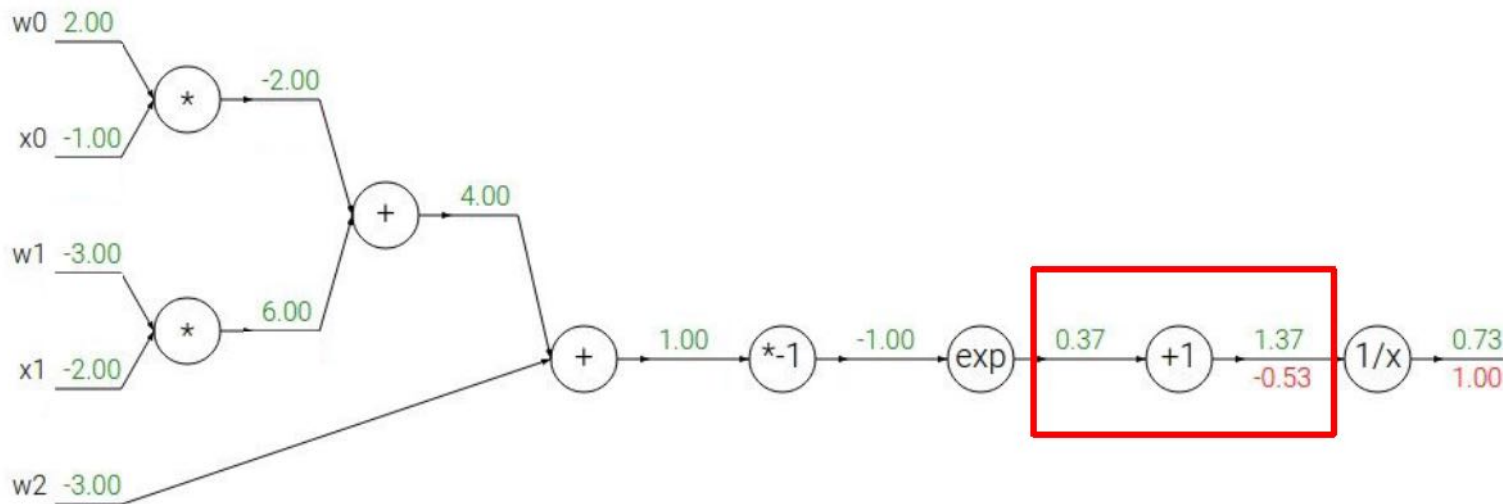


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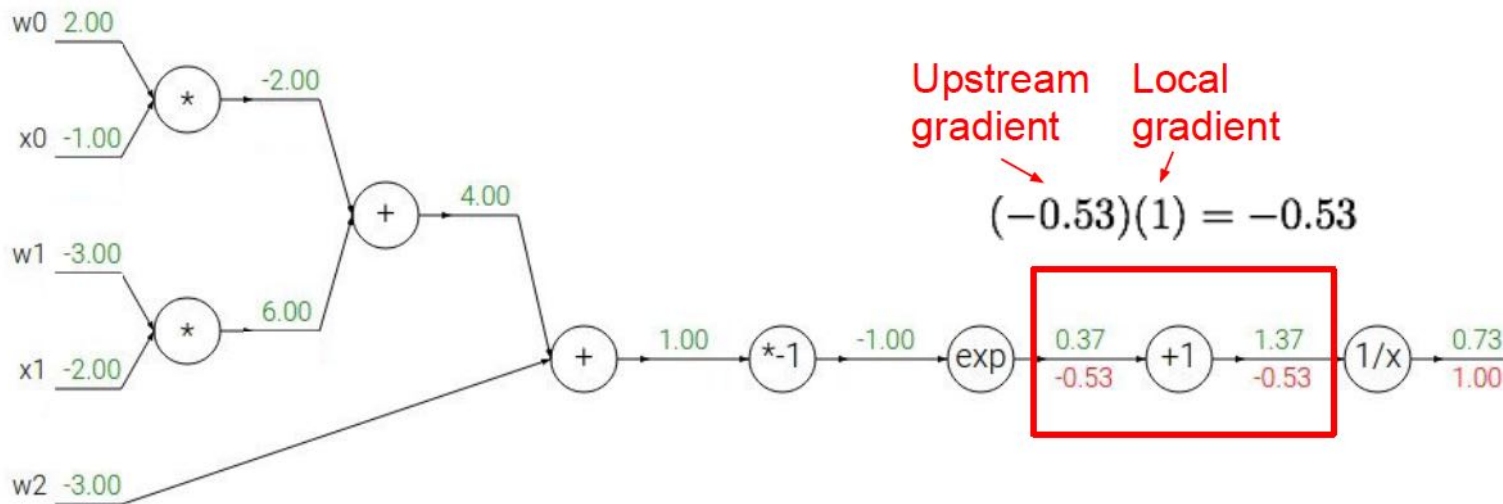


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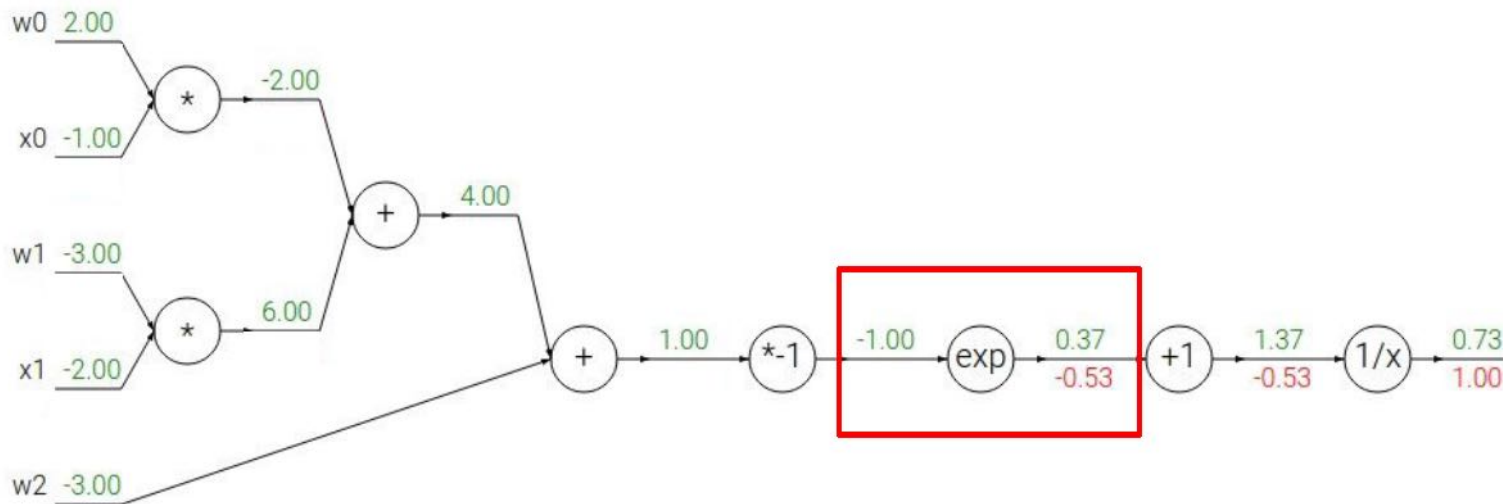


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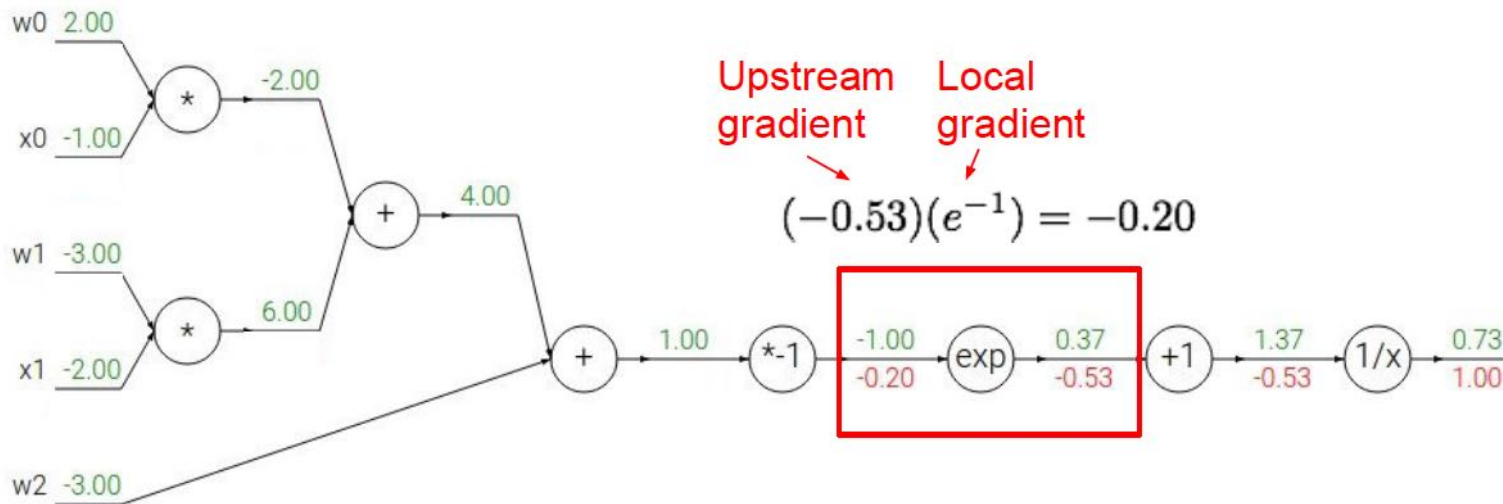
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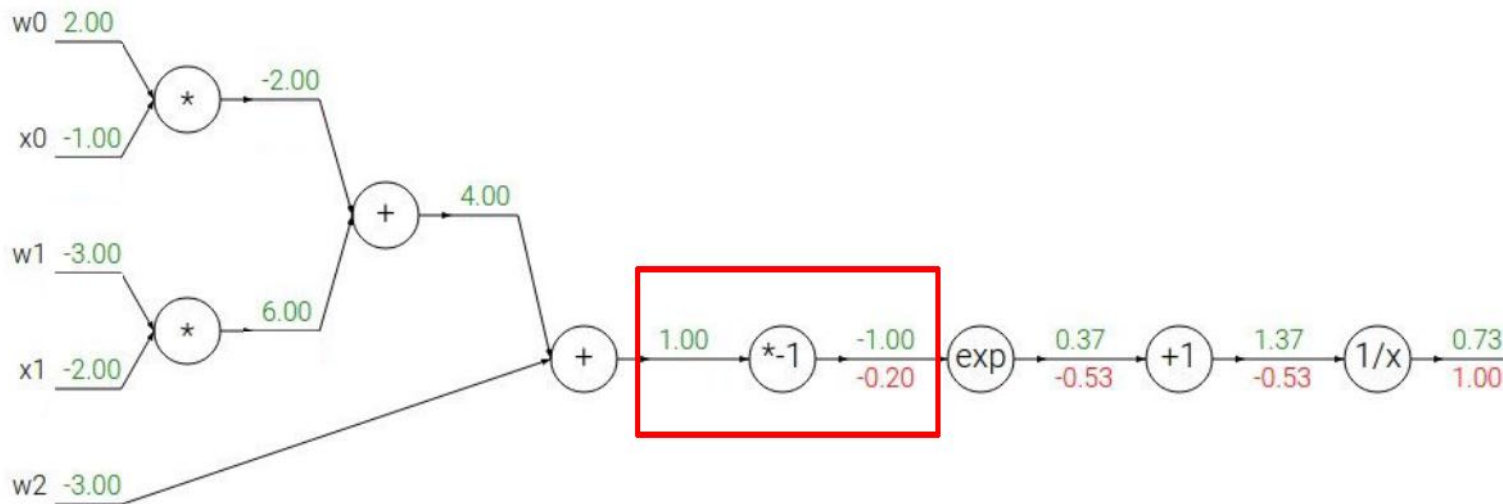
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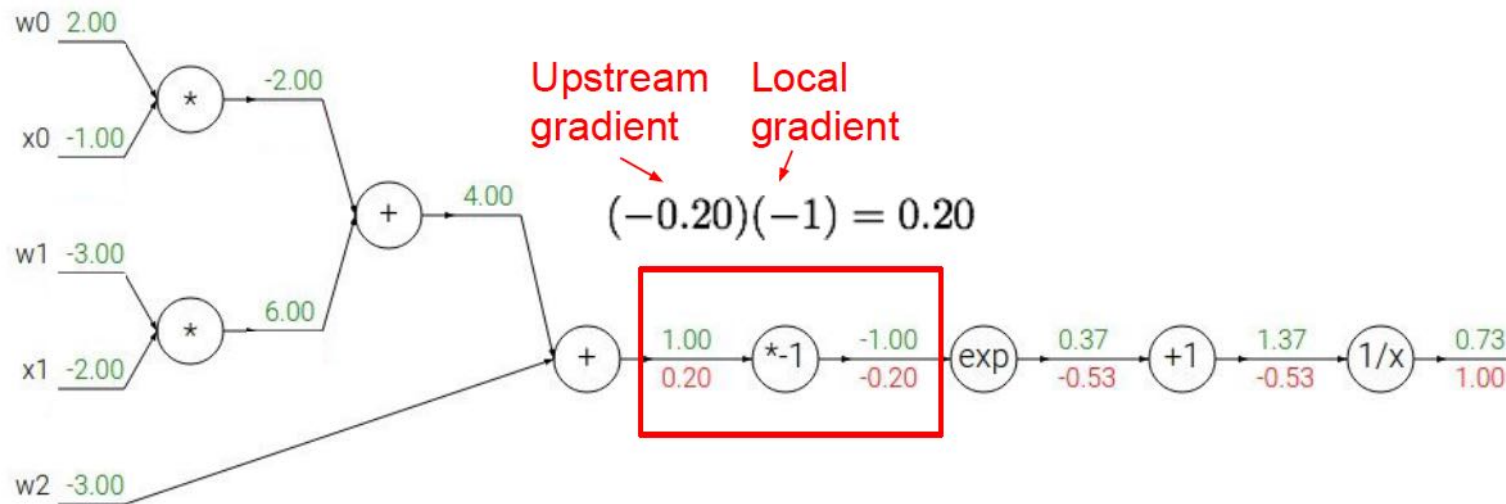


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slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$

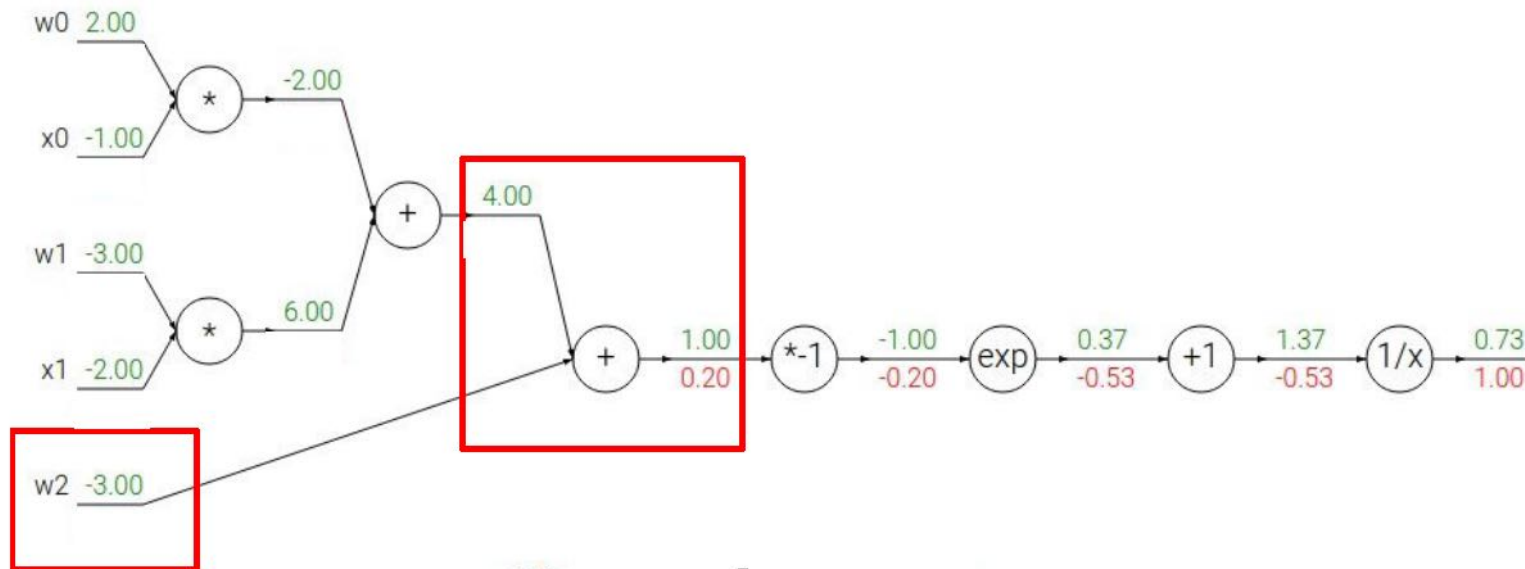


$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	\rightarrow	$\frac{df}{dx} = a$		$f_c(x) = c + x$	\rightarrow	$\frac{df}{dx} = 1$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

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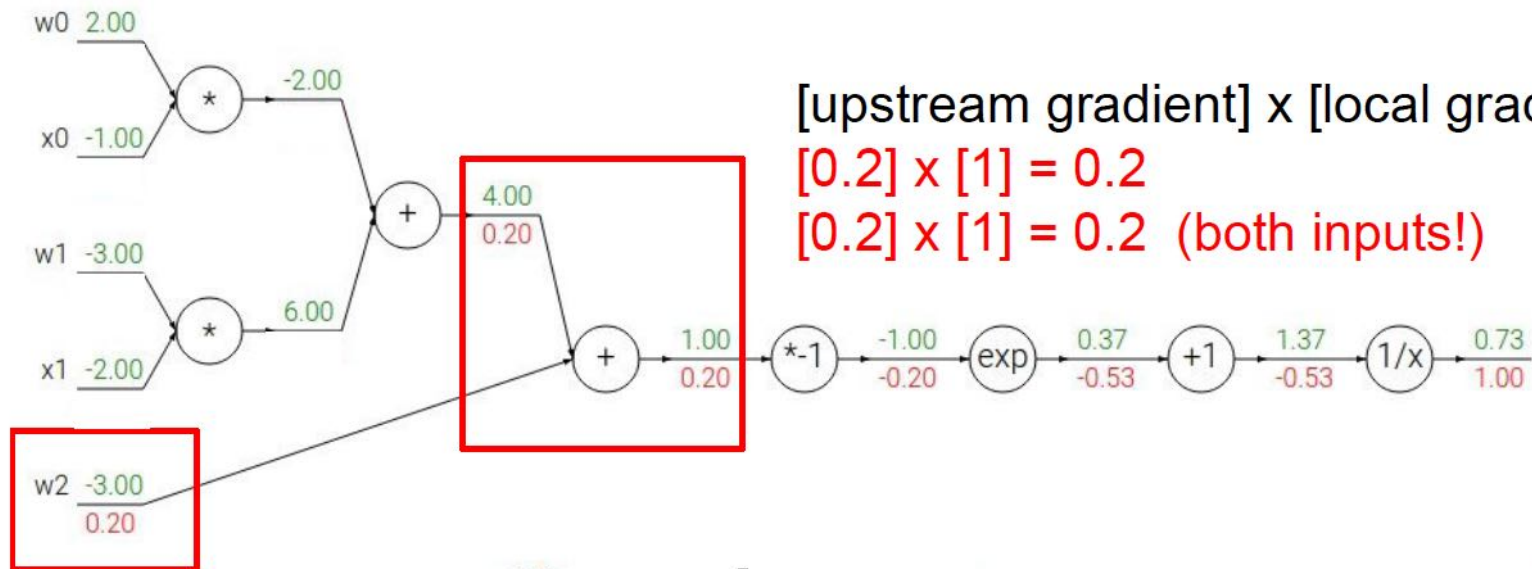


$f(x) = e^x$	→	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	→	$\frac{df}{dx} = -1/x^2$
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Backpropagation

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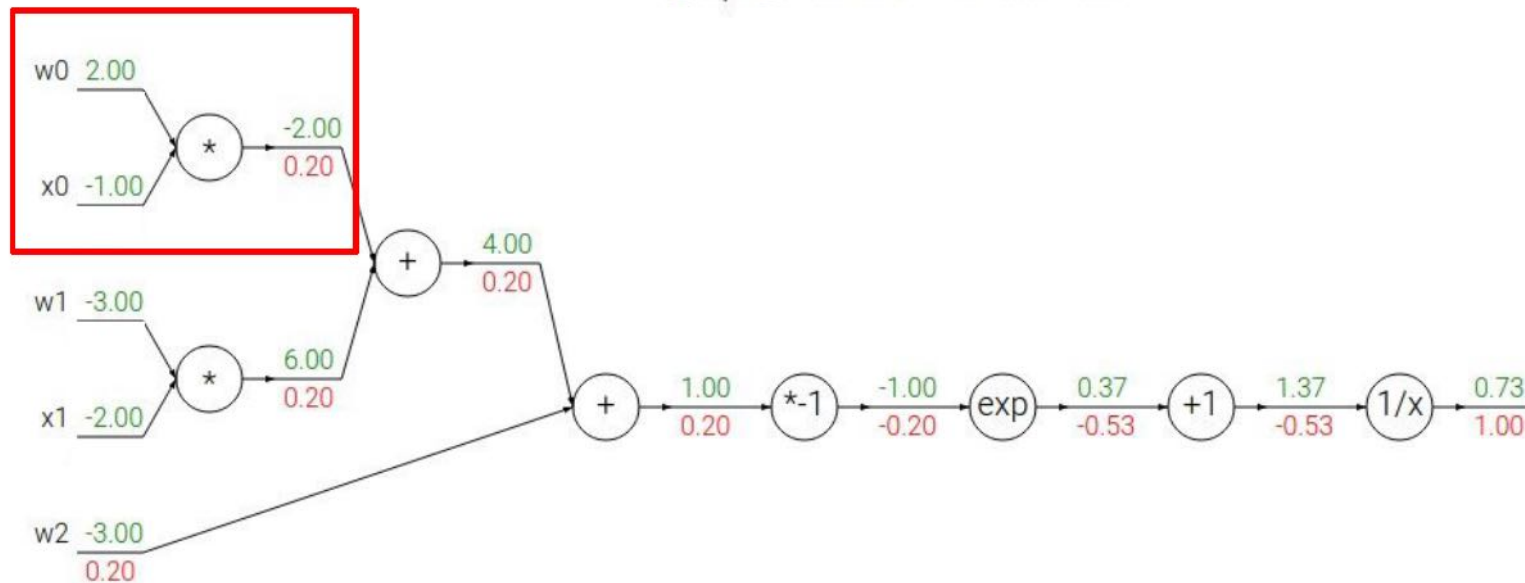
[upstream gradient] x [local gradient]
 $[0.2] \times [1] = 0.2$
 $[0.2] \times [1] = 0.2$ (both inputs!)

$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
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Backpropagation

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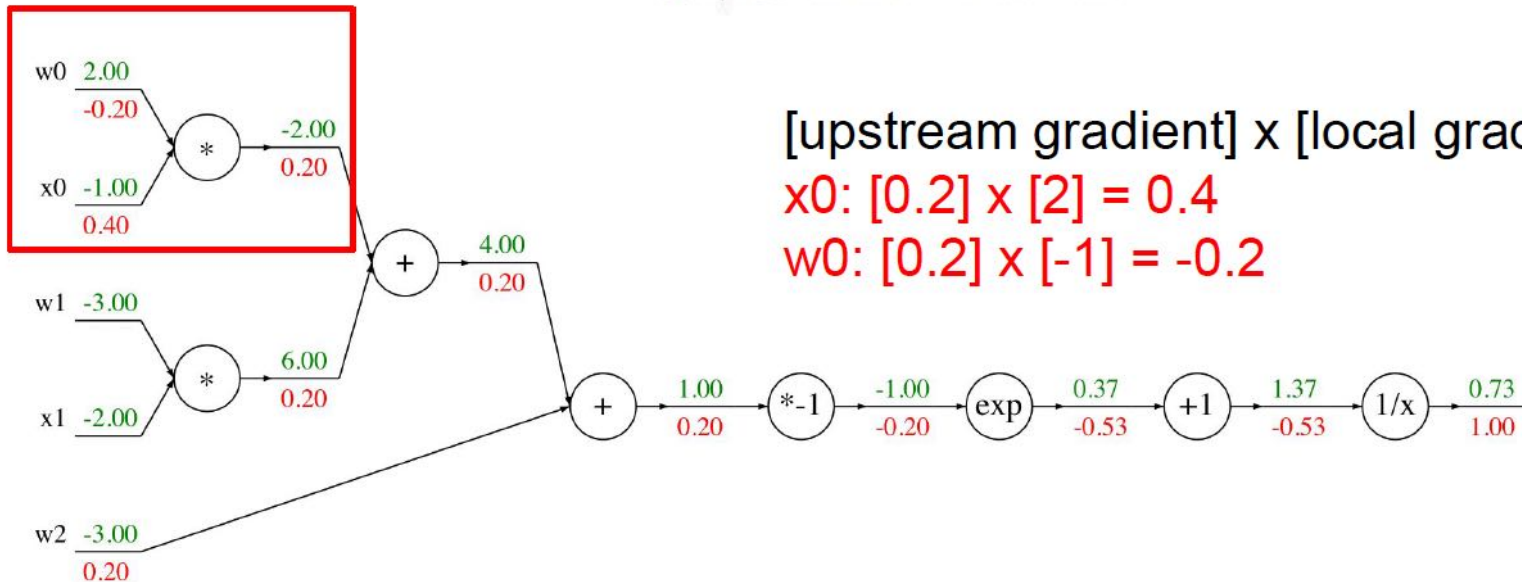
$$\begin{array}{lcl}
 f(x) = e^x & \rightarrow & \frac{df}{dx} = e^x \\
 f_a(x) = ax & \rightarrow & \frac{df}{dx} = a
 \end{array}
 \quad \Bigg| \quad
 \begin{array}{lcl}
 f(x) = \frac{1}{x} & \rightarrow & \frac{df}{dx} = -1/x^2 \\
 f_c(x) = c + x & \rightarrow & \frac{df}{dx} = 1
 \end{array}$$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



[upstream gradient] x [local gradient]

$$x_0: [0.2] \times [2] = 0.4$$

$$w_0: [0.2] \times [-1] = -0.2$$

$f(x) = e^x$	→	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	→	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	→	$\frac{df}{dx} = a$		$f_c(x) = c + x$	→	$\frac{df}{dx} = 1$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

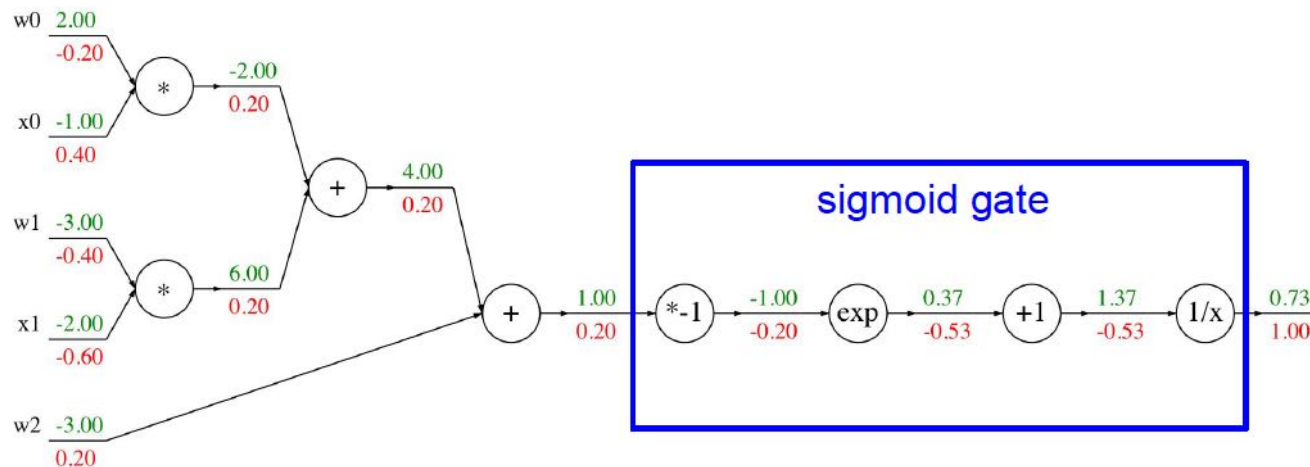
$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

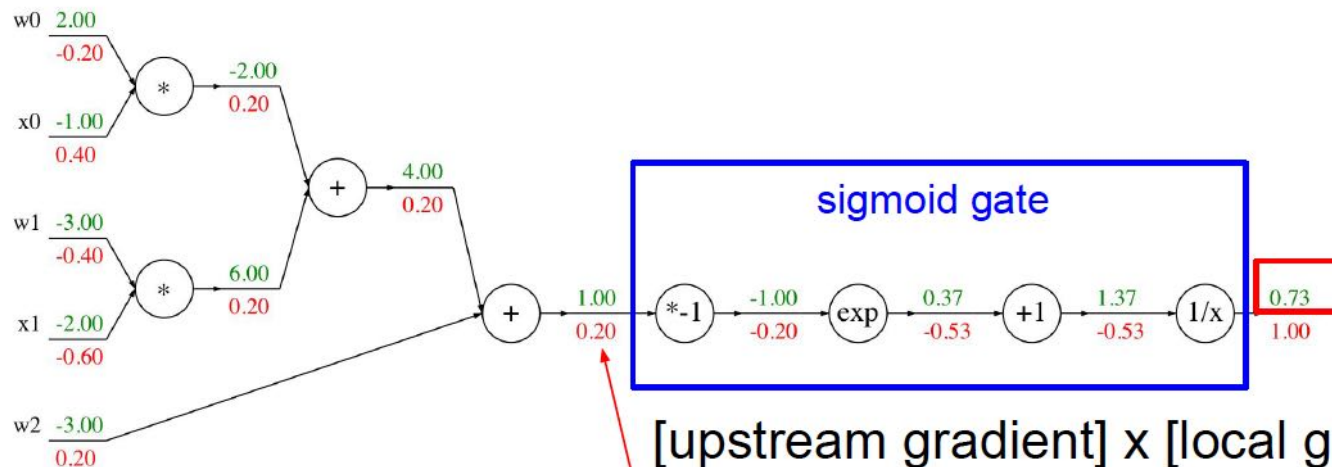
$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

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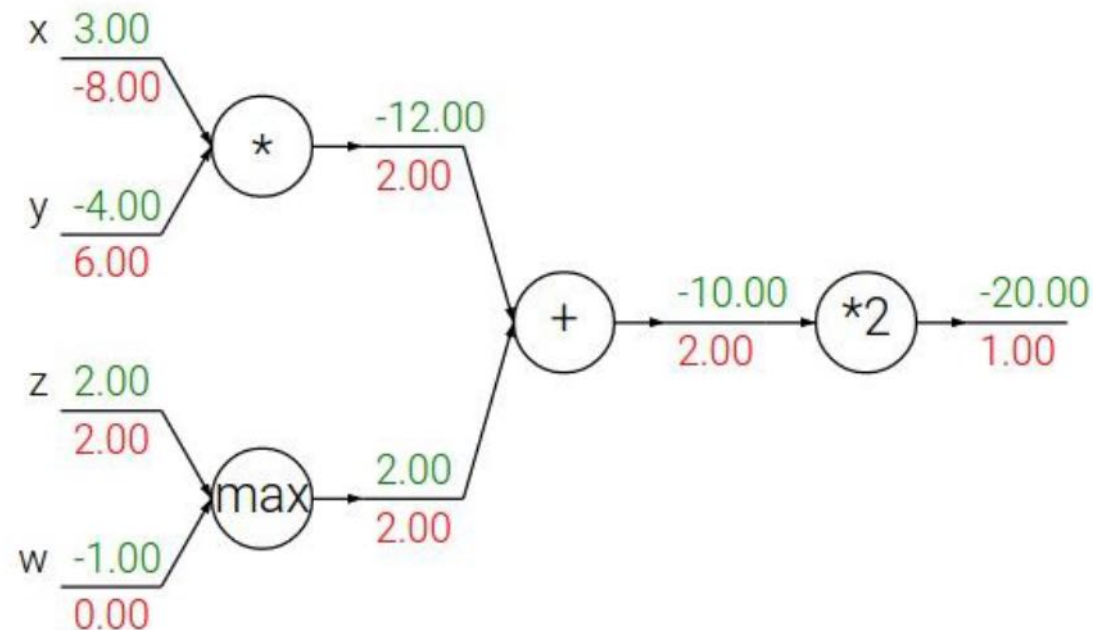
[upstream gradient] x [local gradient]
 $[1.00] \times [(1 - 0.73) (0.73)] = 0.2$

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Patterns in backward flow

add gate: gradient distributor



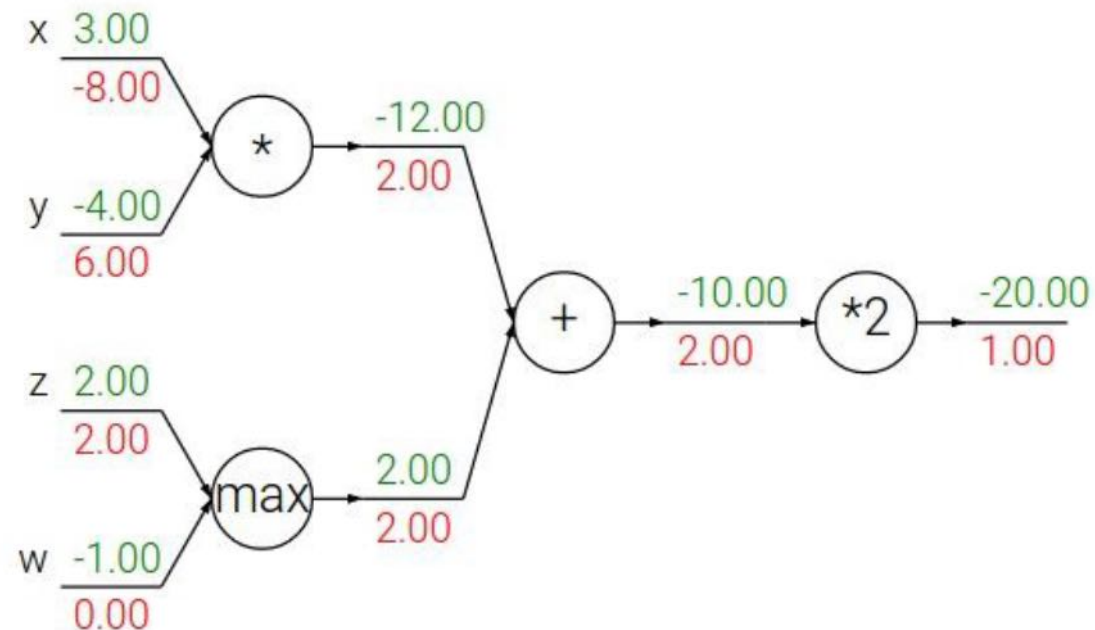
slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Patterns in backward flow

add gate: gradient distributor

Q: What is a **max** gate?



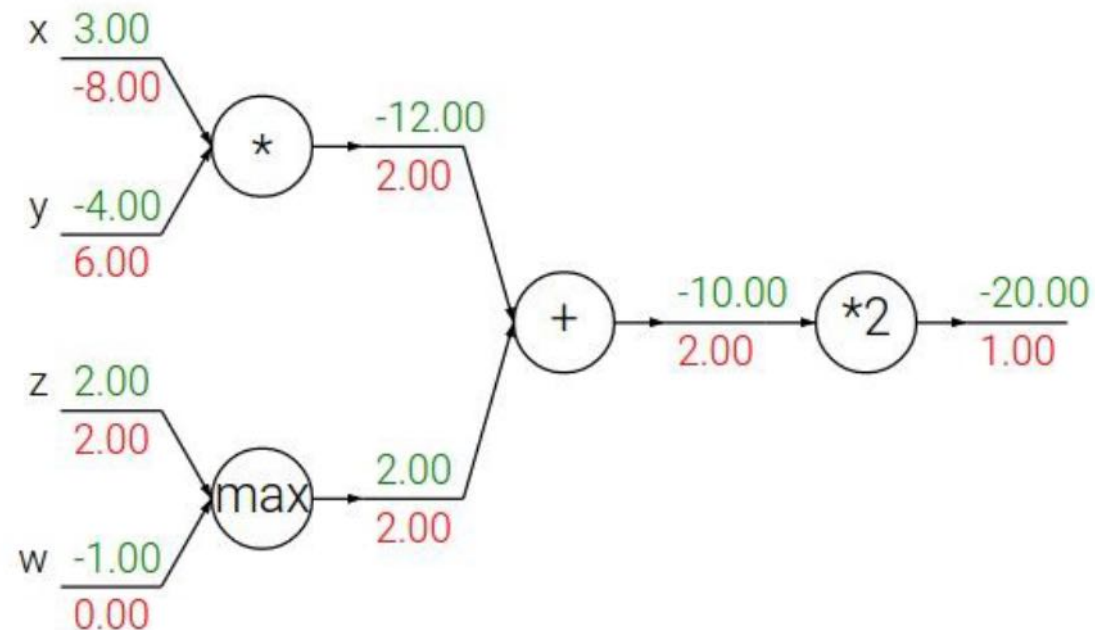
slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

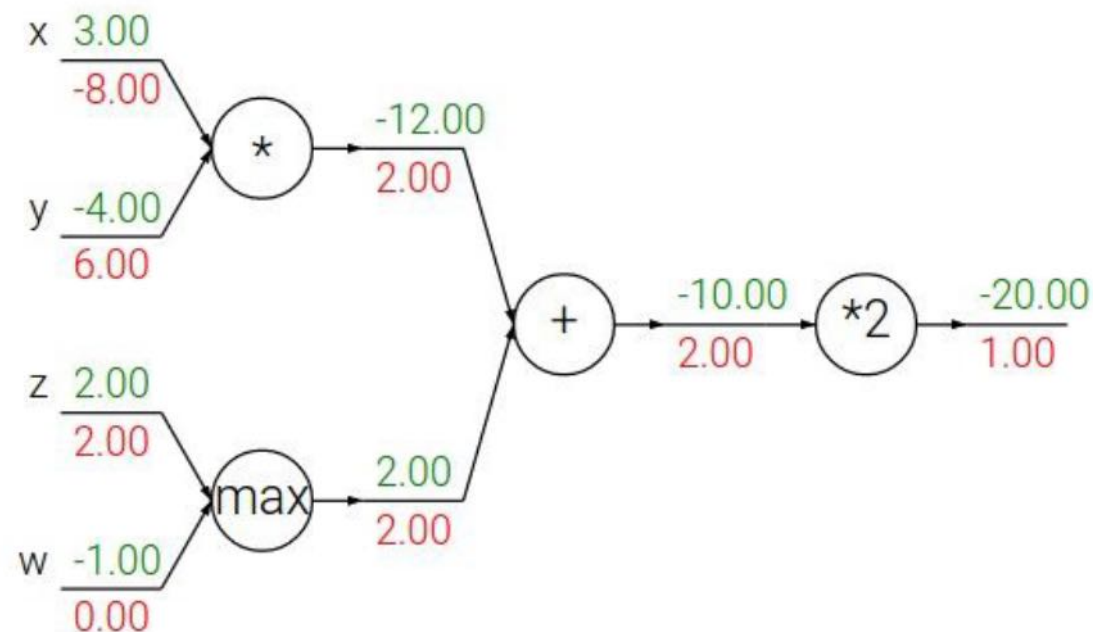
Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router

Q: What is a **mul** gate?



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

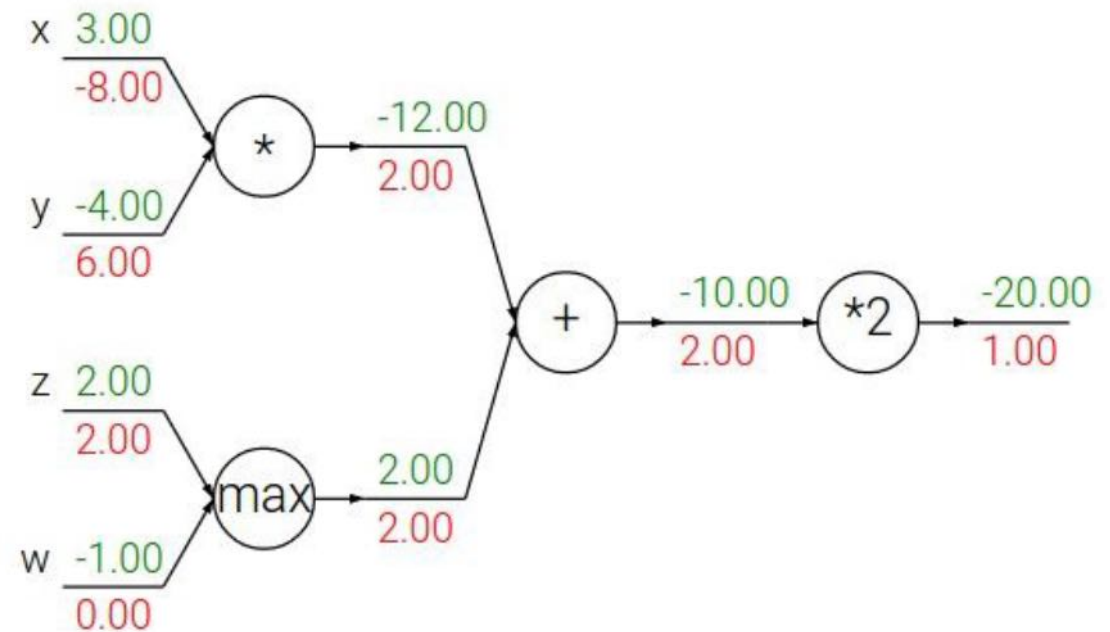
Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router

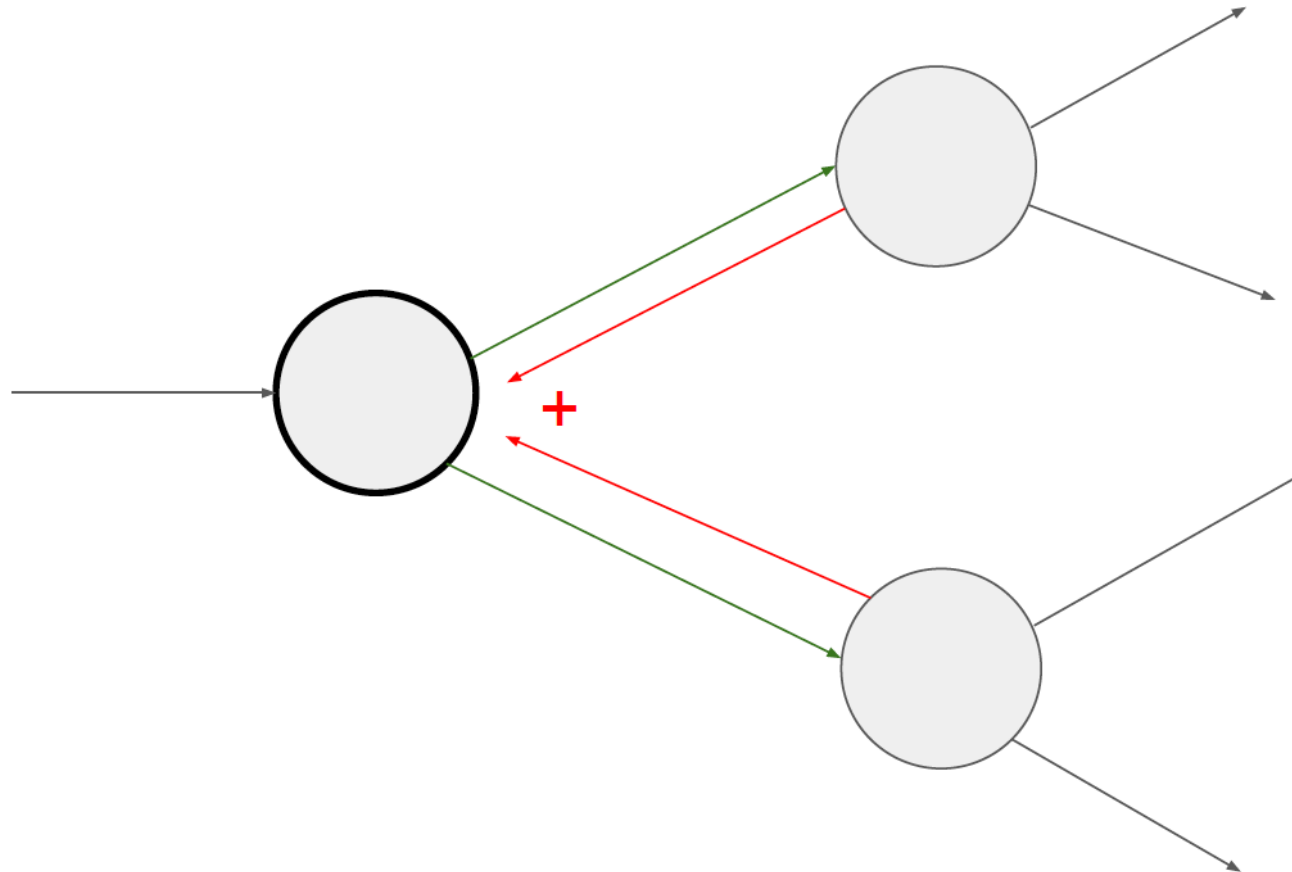
mul gate: gradient switcher



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Gradients add at branches



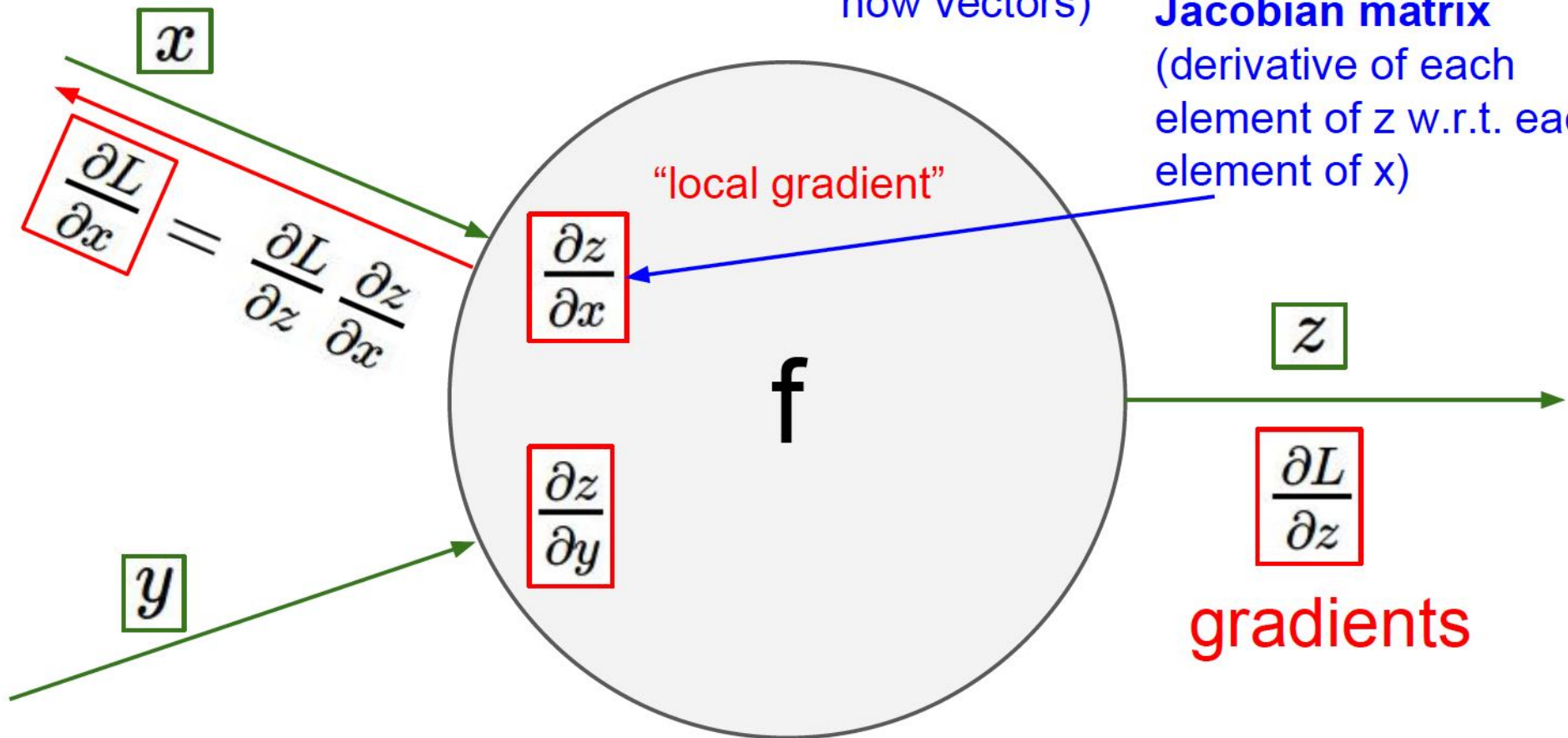
slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Gradients for vectorized code

(x, y, z are now vectors)

This is now the **Jacobian matrix** (derivative of each element of z w.r.t. each element of x)

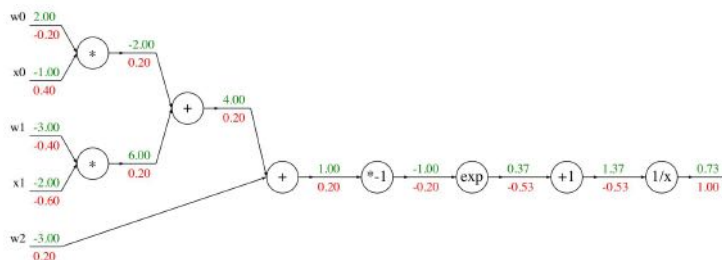


slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Modularized implementation: forward / backward API

Graph (or Net) object (*rough pseudo code*)

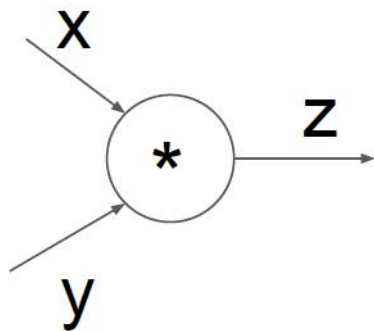


```
class ComputationalGraph(object):  
    #...  
    def forward(inputs):  
        # 1. [pass inputs to input gates...]  
        # 2. forward the computational graph:  
        for gate in self.graph.nodes_topologically_sorted():  
            gate.forward()  
        return loss # the final gate in the graph outputs the loss  
    def backward():  
        for gate in reversed(self.graph.nodes_topologically_sorted()):  
            gate.backward() # little piece of backprop (chain rule applied)  
        return inputs_gradients
```

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Backpropagation

Modularized implementation: forward / backward API



(x,y,z are scalars)

```
class MultiplyGate(object):  
    def forward(x,y):  
        z = x*y  
        self.x = x # must keep these around!  
        self.y = y  
        return z  
    def backward(dz):  
        dx = self.y * dz # [dz/dx * dL/dz]  
        dy = self.x * dz # [dz/dy * dL/dz]  
        return [dx, dy]
```

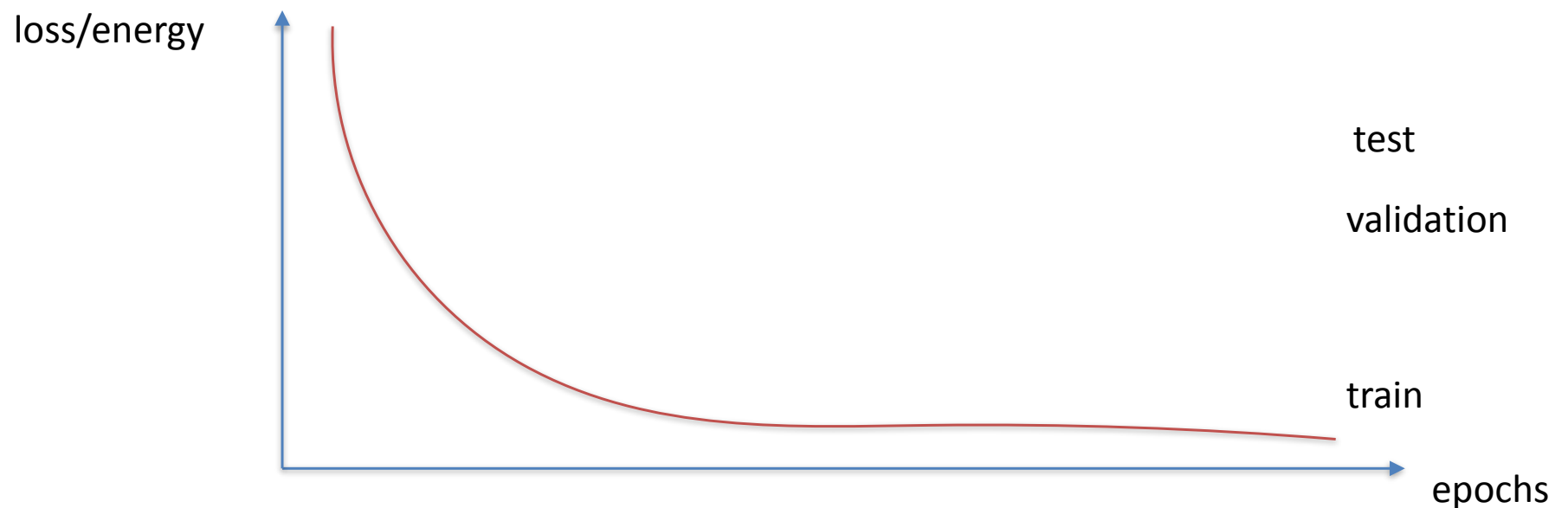
Local gradient

Upstream gradient variable

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

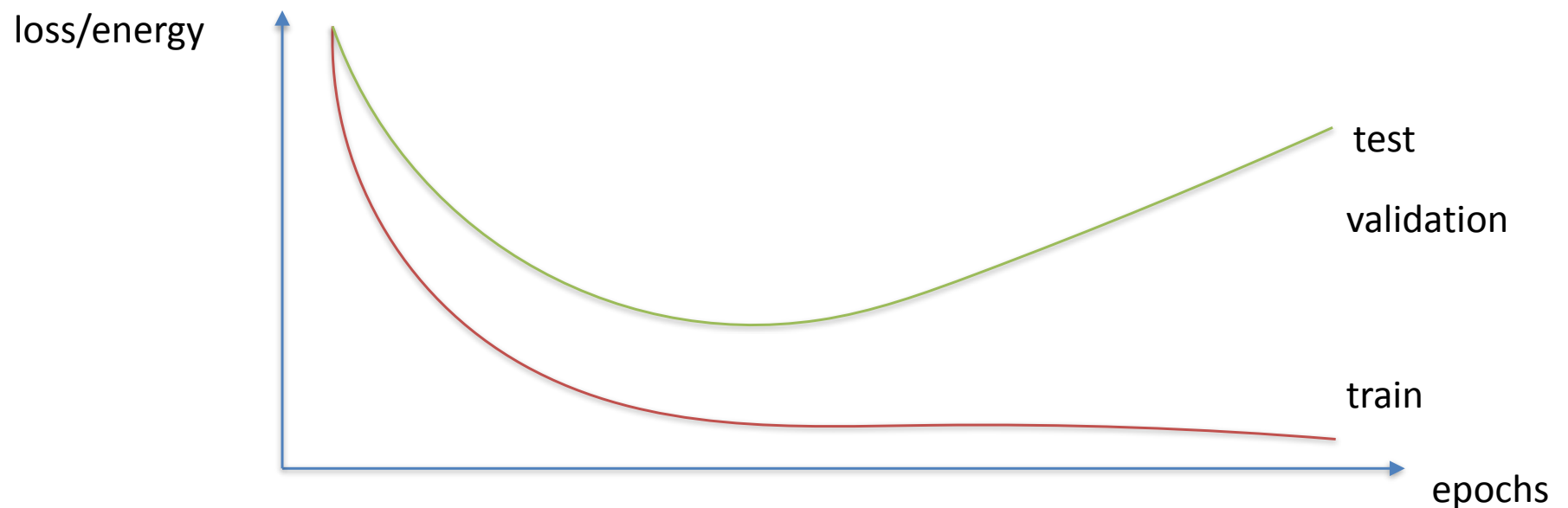
Gradient & Training

- Only two things need to be implemented to define new layer:
 - ▶ forward pass
 - ▶ error back propagation
- Watch overfitting



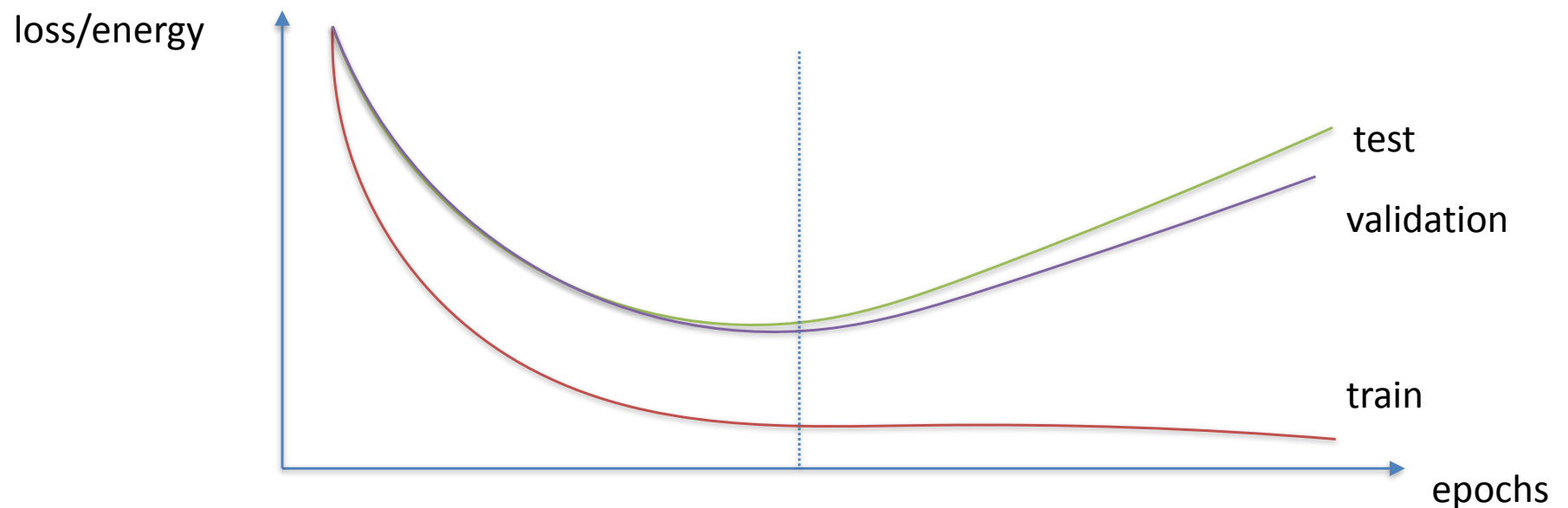
Gradient & Training

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Gradient & Training

- Only two things need to be implemented to define new layer:
 - ▶ forward pass
 - ▶ error back propagation
- Watch overfitting



Summary so far...

- neural nets will be very large: impractical to write down gradient formula by hand for all parameters
- **backpropagation** = recursive application of the chain rule along a computational graph to compute the gradients of all inputs/parameters/intermediates
- implementations maintain a graph structure, where the nodes implement the **forward()** / **backward()** API
- **forward**: compute result of an operation and save any intermediates needed for gradient computation in memory
- **backward**: apply the chain rule to compute the gradient of the loss function with respect to the inputs

slide credit: Fei-Fei, Justin Johnson, Serena Yeung



Overview Today's Lecture

- Backpropagation - Gradient Descent
 - ▶ illustrated using computational graphs
 - ▶ chain rule - upstream and local gradients
 - ▶ modularization simple
- **What is Deep Learning**
 - ▶ intuition why deep learning can help
 - ▶ integrated learning of features and classifier
- Convolutional Neural Networks (CNNs)
 - ▶ one of the (few) highly successful NNs

Simple Neural Networks

(**Before**) Linear score function: $f = Wx$

Simple Neural Networks

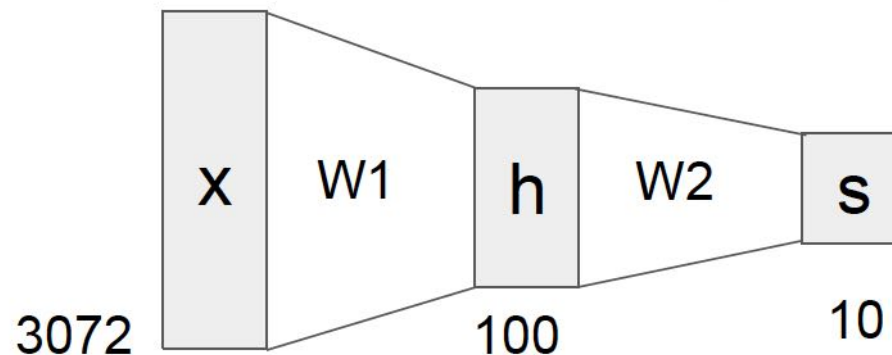
(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

Simple Neural Networks

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

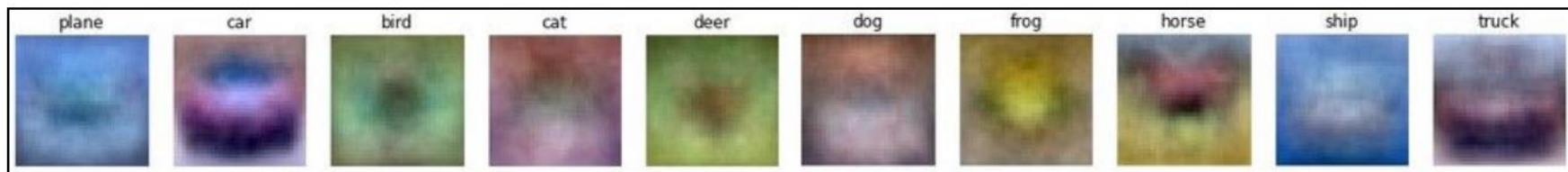
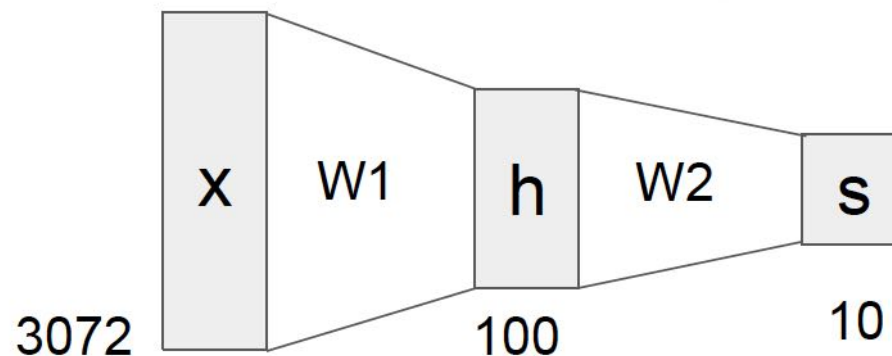


slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Simple Neural Networks

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

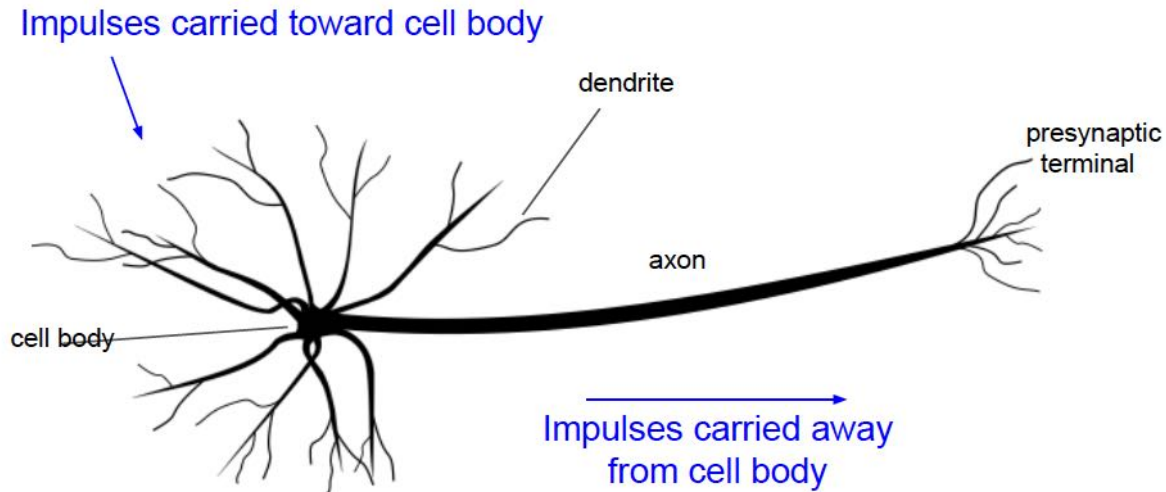
Simple Neural Networks

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network
or 3-layer Neural Network $f = W_2 \max(0, W_1 x)$

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

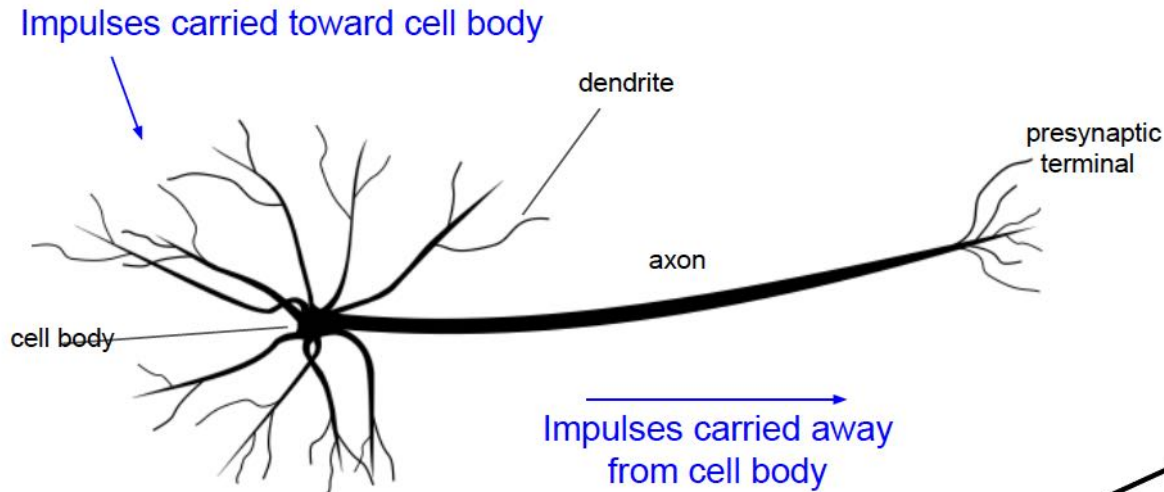
Some Inspiration from the (Human) Brain



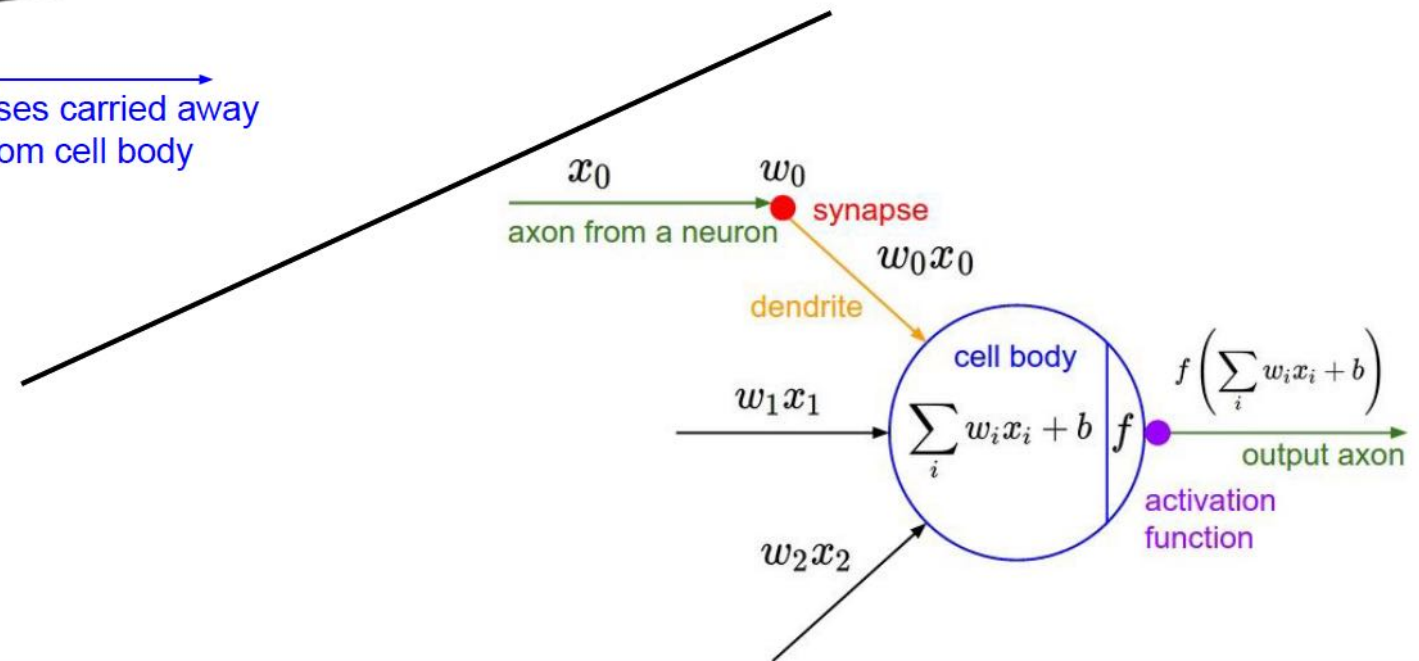
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slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Some Inspiration from the (Human) Brain

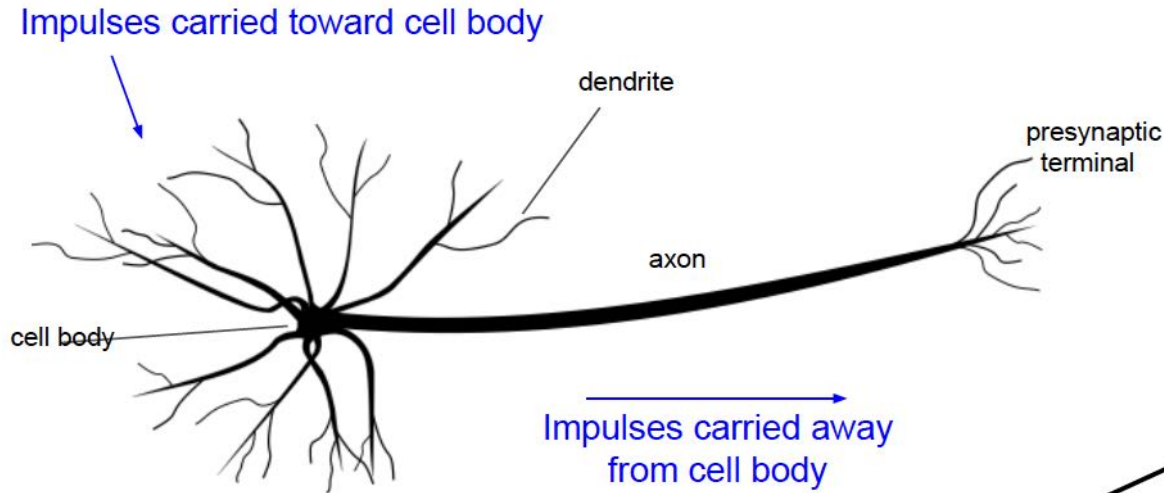


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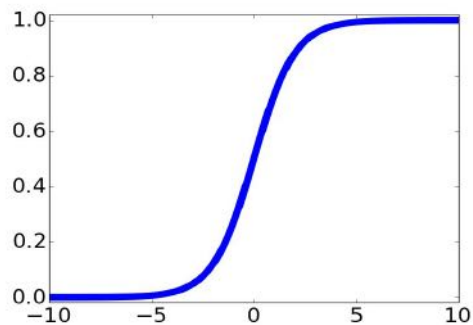


slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Some Inspiration from the (Human) Brain

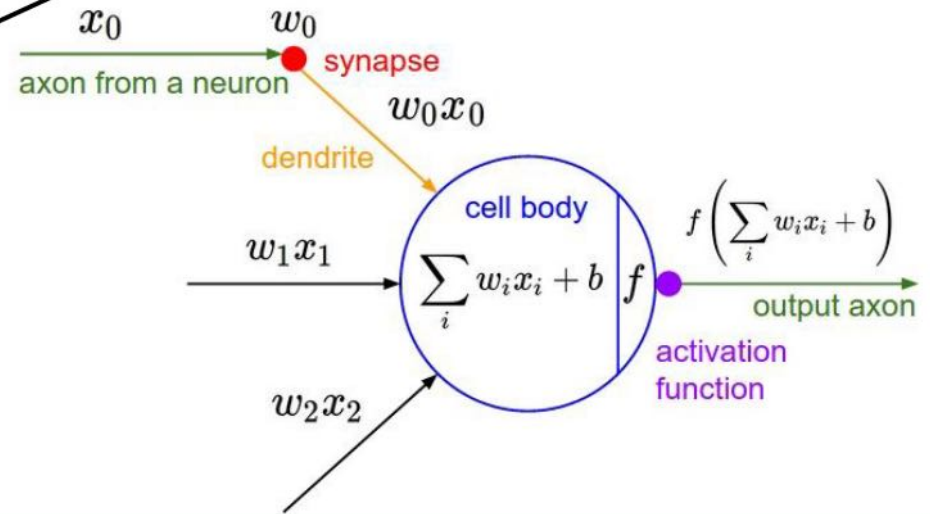


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sigmoid activation function

$$\frac{1}{1 + e^{-x}}$$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Some Inspiration from the (Human) Brain

Be very careful with your brain analogies!

Biological Neurons:

- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system
- Rate code may not be adequate

[Dendritic Computation. London and Hausser]

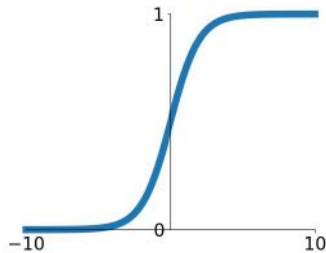
slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Activation Functions

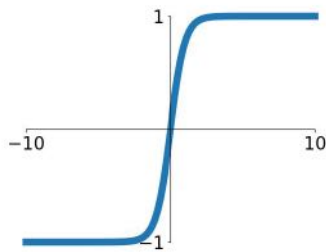
Activation functions

Sigmoid

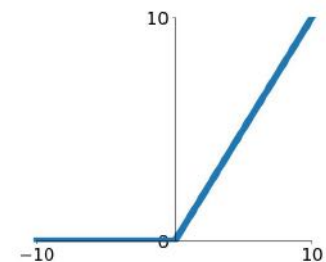
$$\sigma(x) = \frac{1}{1+e^{-x}}$$



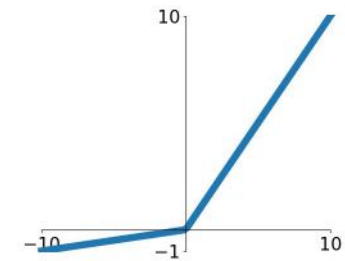
tanh

$$\tanh(x)$$


ReLU

$$\max(0, x)$$


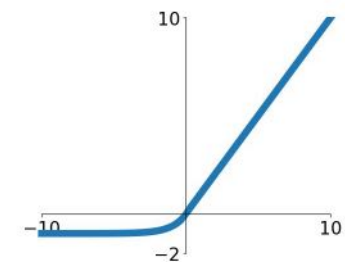
Leaky ReLU

$$\max(0.1x, x)$$


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

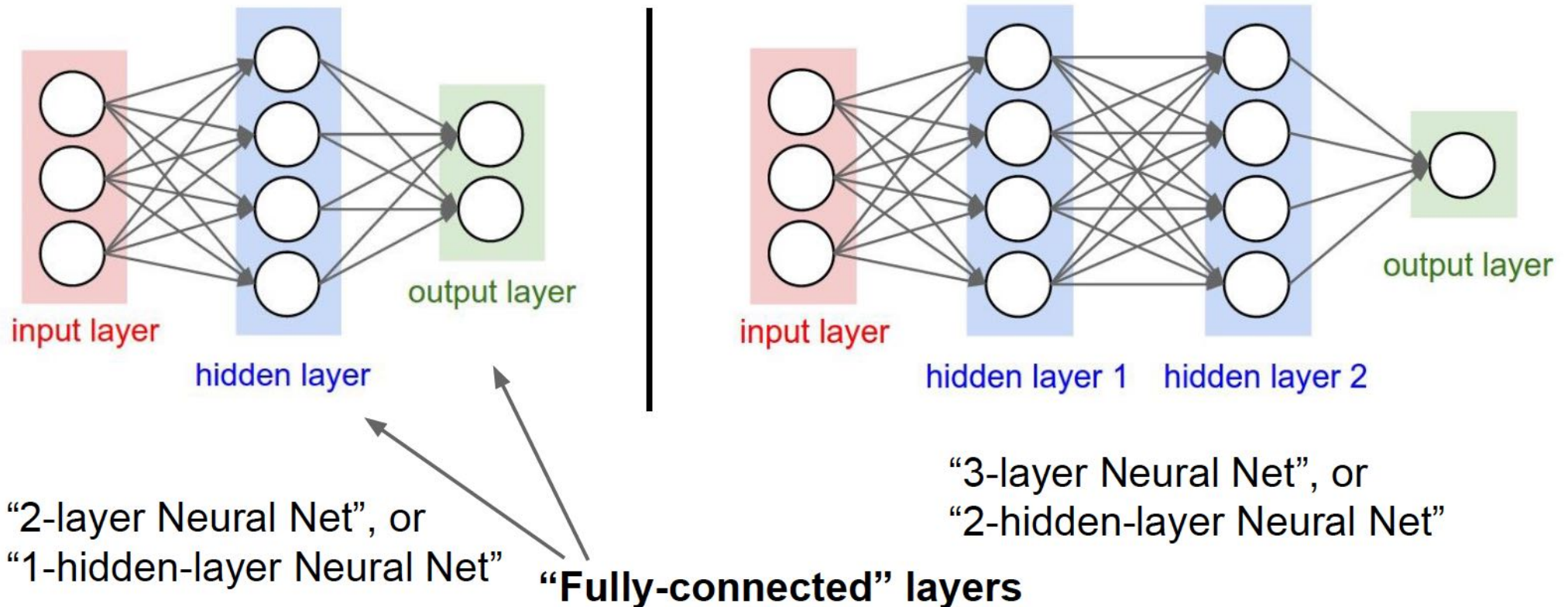
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

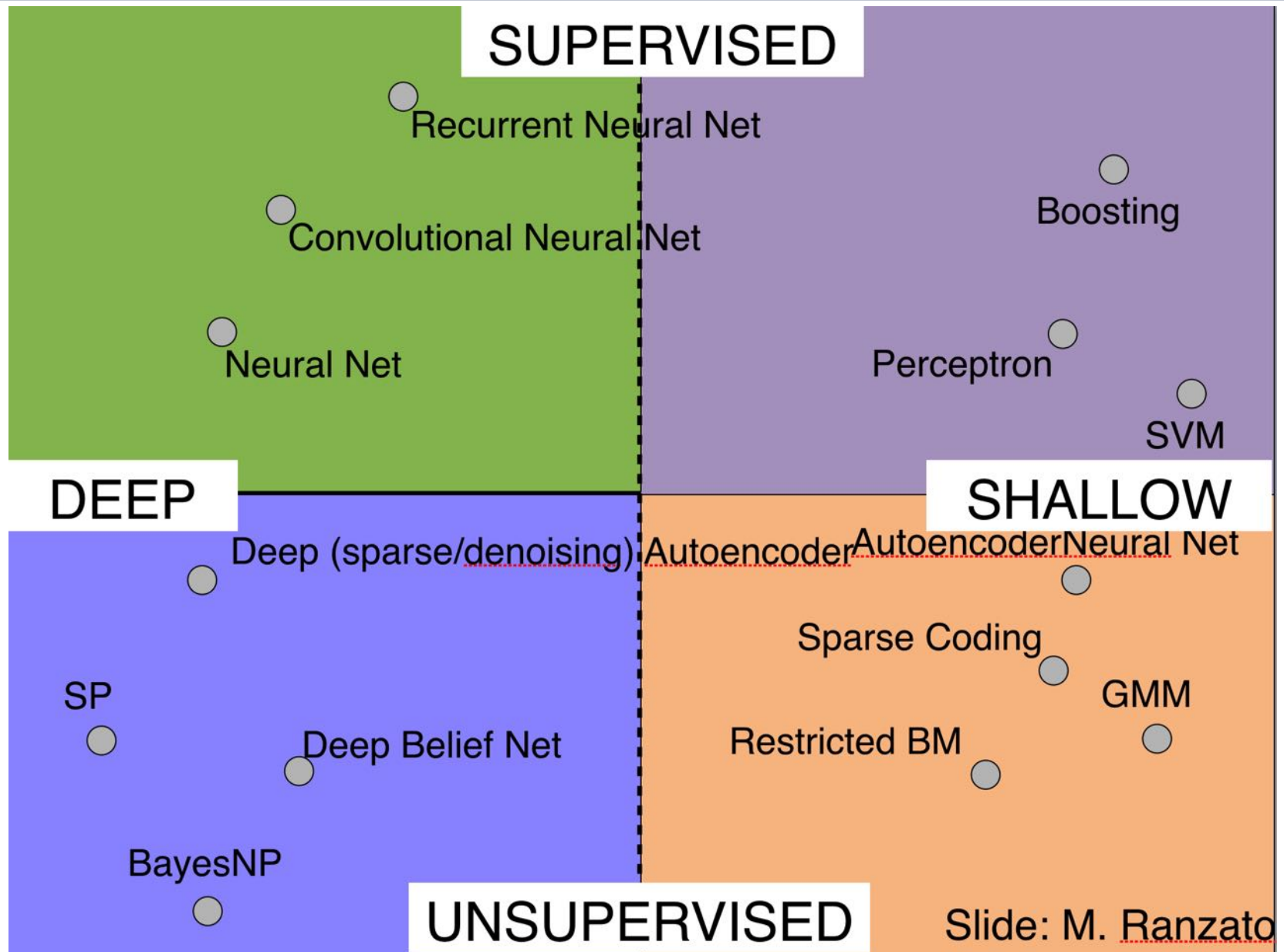
Neural Networks - Classic Architectures

- (Multi-Layer) Perceptron



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

Overview of Neural Network Architectures

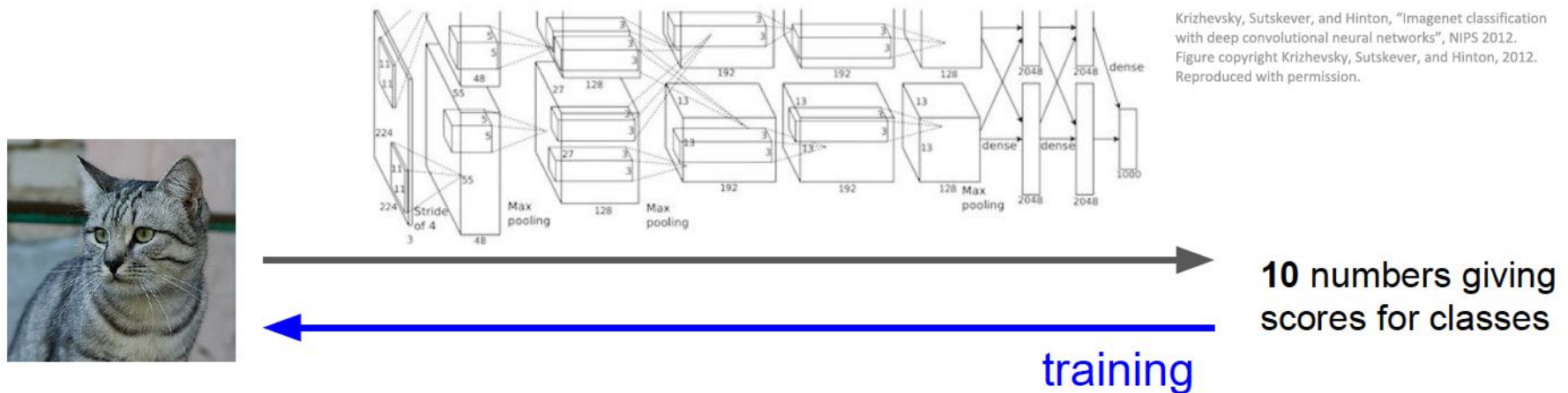
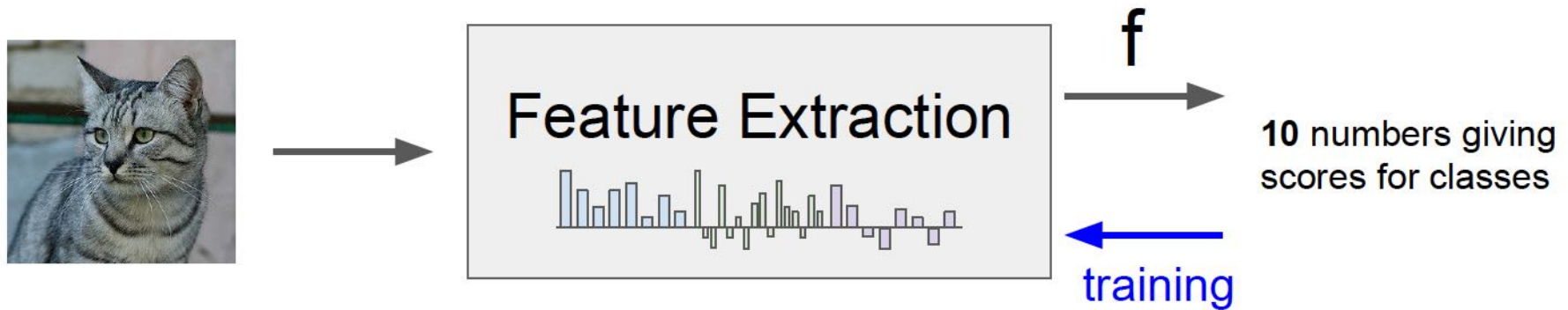


Deep Learning Ingredients

- Deep Learning is based on
 - ▶ Availability of large datasets
 - ▶ Massive parallel compute power
 - ▶ Advances in machine learning over the years
- Strong improvements due to
 - ▶ Internet (availability of large-scale data)
 - ▶ GPUs (availability of parallel compute power)
 - ▶ Deep / hierarchical models with end-to-end learning

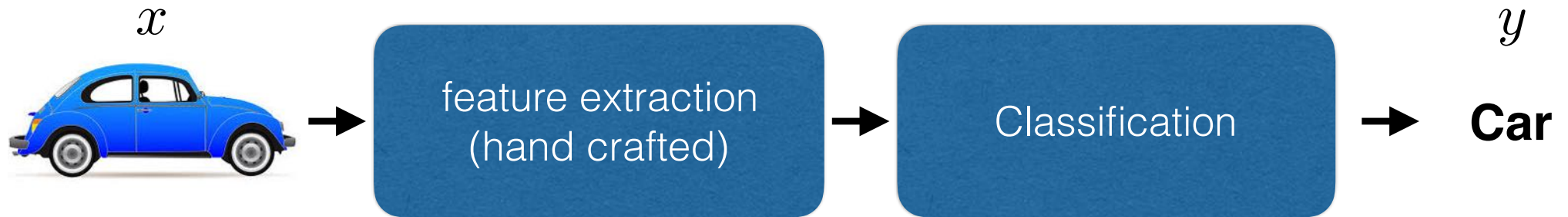


Image Features vs. Deep Learning



slide credit: Fei-Fei, Justin Johnson, Serena Yeung

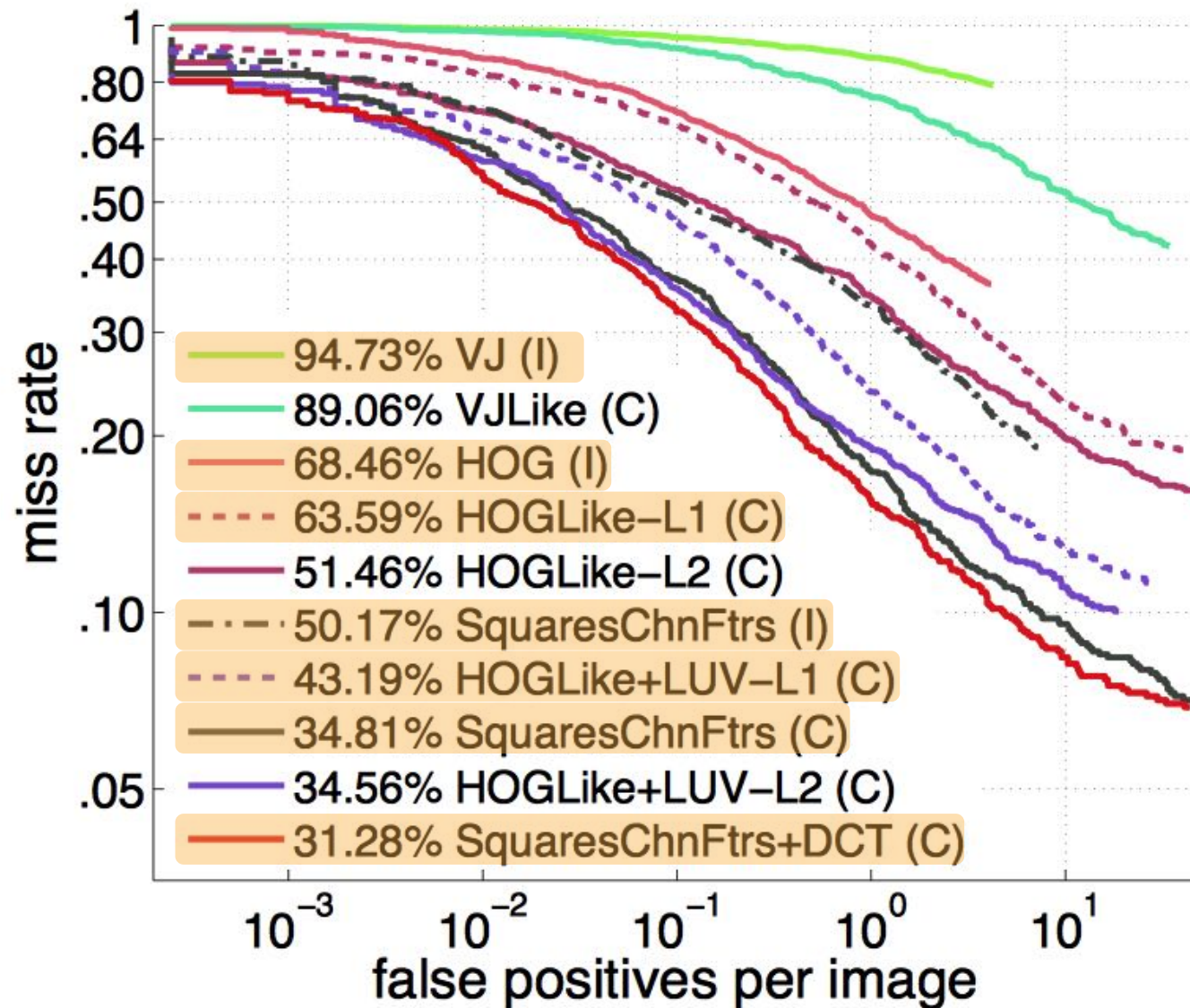
Traditional Approach



- Feature extraction
 - ▶ often hand crafted and fixed
 - ▶ might be too general (not task-specific enough)
 - ▶ might be too specific (does not generalize to other tasks)
- How to achieve best classification performance
 - ▶ more complex classifier (e.g. multi-feature, non-linear)?
 - ▶ how specialized for the task?

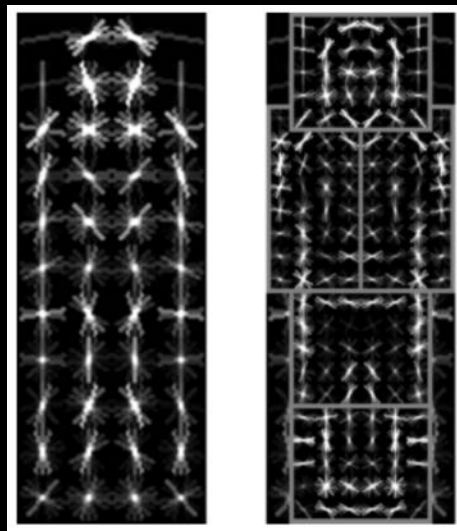
Features Alone Can Explain (Almost) 10 Years of Progress

[Benenson, Omran, Hosang, Schiele@ECCV workshop'14]

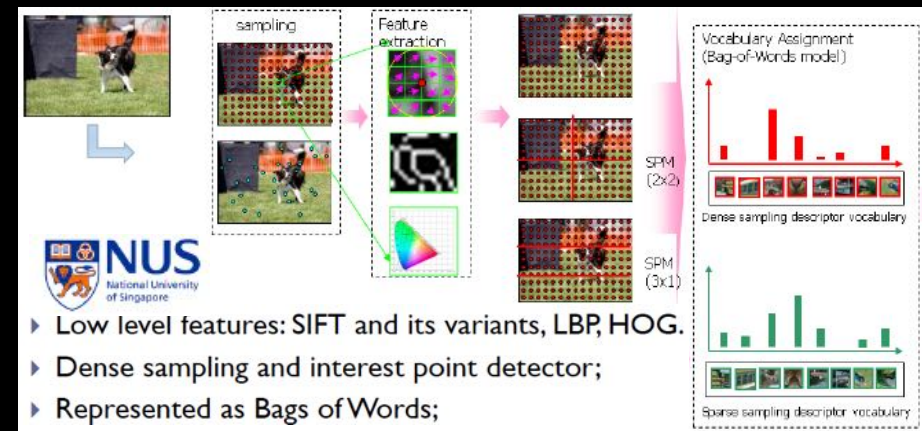


Motivation

- Features are key to recent progress in recognition
- Multitude of hand-designed features currently in use
 - SIFT, HOG, LBP, MSER, Color-SIFT.....
- Where next? Better classifiers? Or keep building more features?



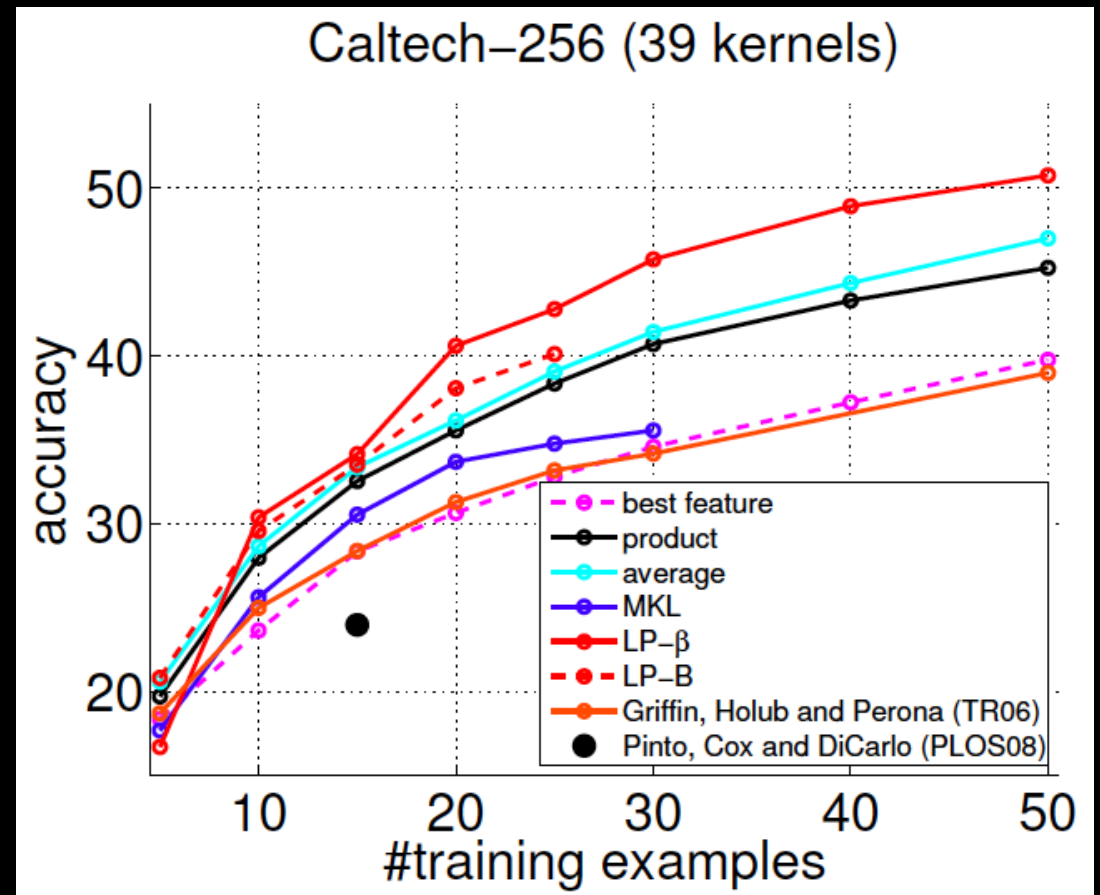
Felzenszwalb, Girshick,
McAllester and Ramanan, PAMI 2007



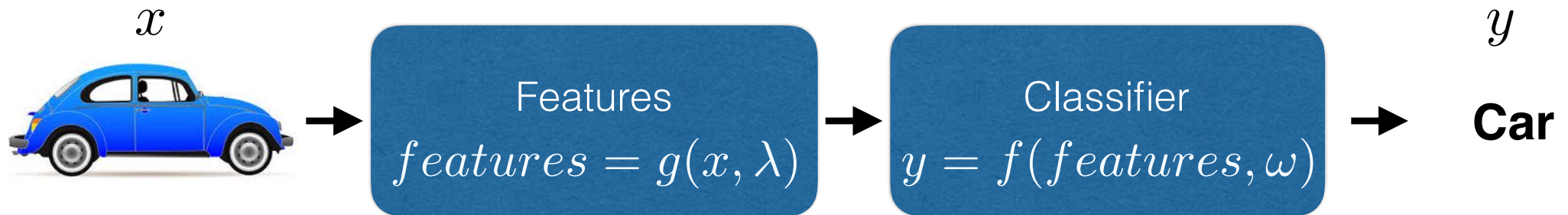
Yan & Huang
(Winner of PASCAL 2010 classification competition)

Hand-Crafted Features

- LP- β Multiple Kernel Learning (MKL)
 - Gehler and Nowozin, On Feature Combination for Multiclass Object Classification, ICCV'09
- 39 different kernels
 - PHOG, SIFT, V1S+, Region Cov. Etc.
- MKL only gets few % gain over averaging features
 - Features are doing the work

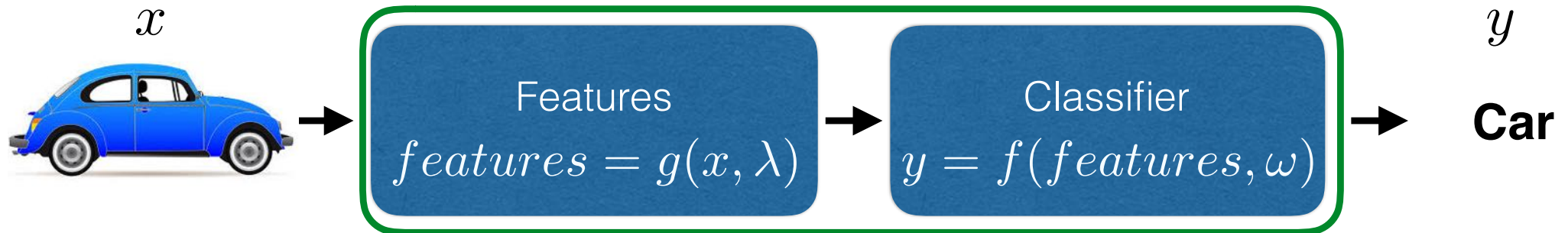


Deep Learning: Trainable features



- Parameterized feature extraction
- Features should be
 - ▶ efficient to compute
 - ▶ efficient to train (differentiable)

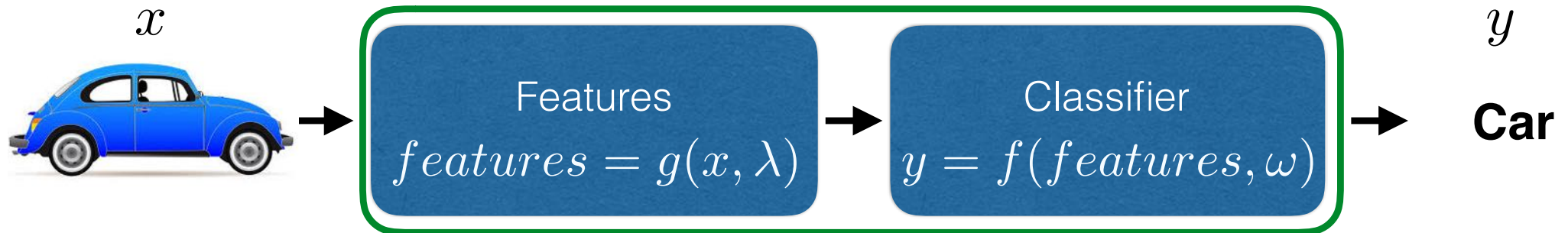
Deep Learning: Joint Training of all Parameters



“End-to-End” System

- Parameterized feature extraction
- Features should be
 - ▶ efficient to compute
 - ▶ efficient to train (differentiable)
- **Joint** training of **feature** extraction and **classification**
- Feature extraction and classification merge into one pipeline

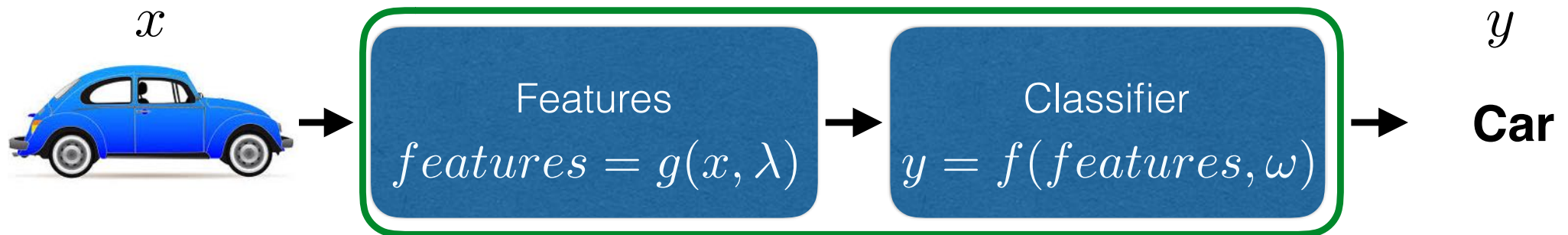
Deep Learning: Joint Training of all Parameters



“End-to-End” System

- All parts are adaptive
- No differentiation between feature extraction and classification
- Non linear transformation from input to desired output

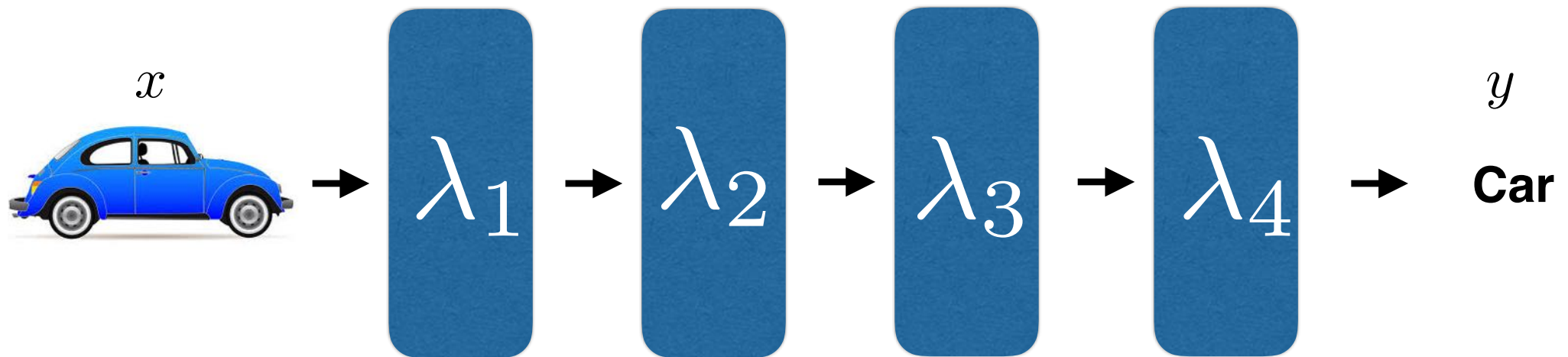
Deep Learning: Complex Functions by Composition



“End-to-End” System

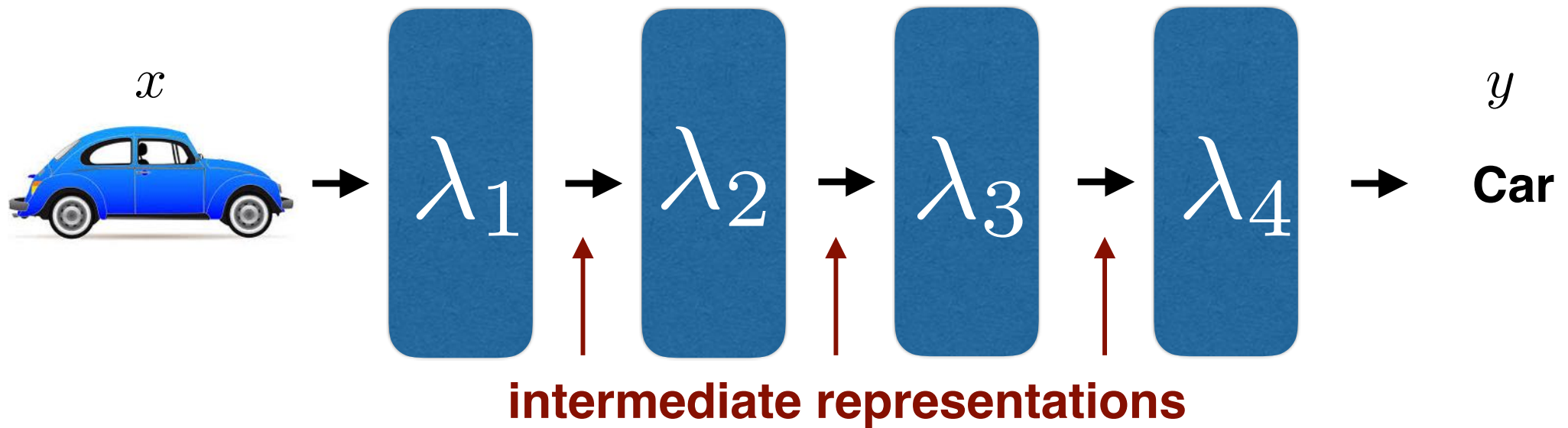
- How can we build such systems?
- What is the parameterization (hypothesis)?
- Composition of simple building blocks can lead to complex systems (e.g. neurons - brain)

Deep Learning: Complex Functions by Composition



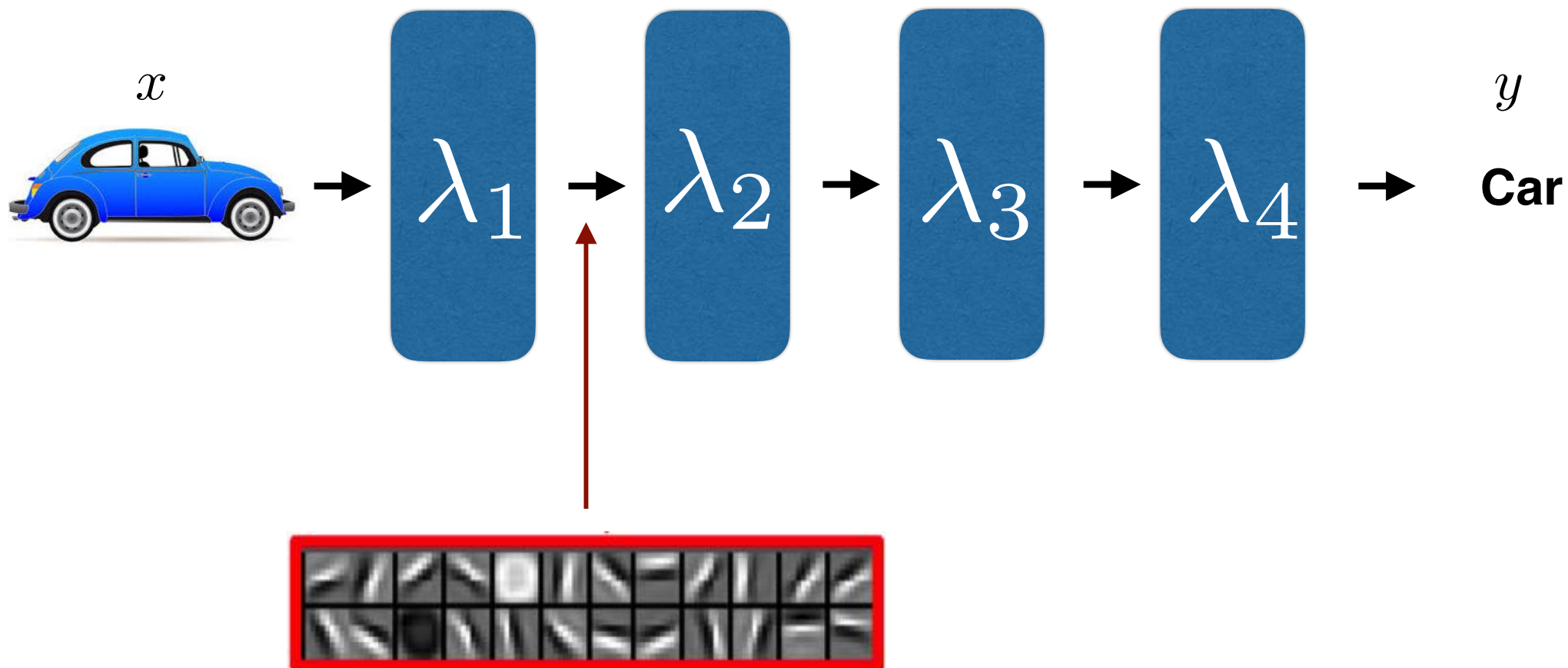
- How can we build such systems?
- What is the parameterization (hypothesis)?
- Composition of simple building blocks can lead to complex systems (e.g. neurons - brain) each block has trainable parameters
- Each block has trainable parameters λ_i

Deep Learning: Complex Functions by Composition



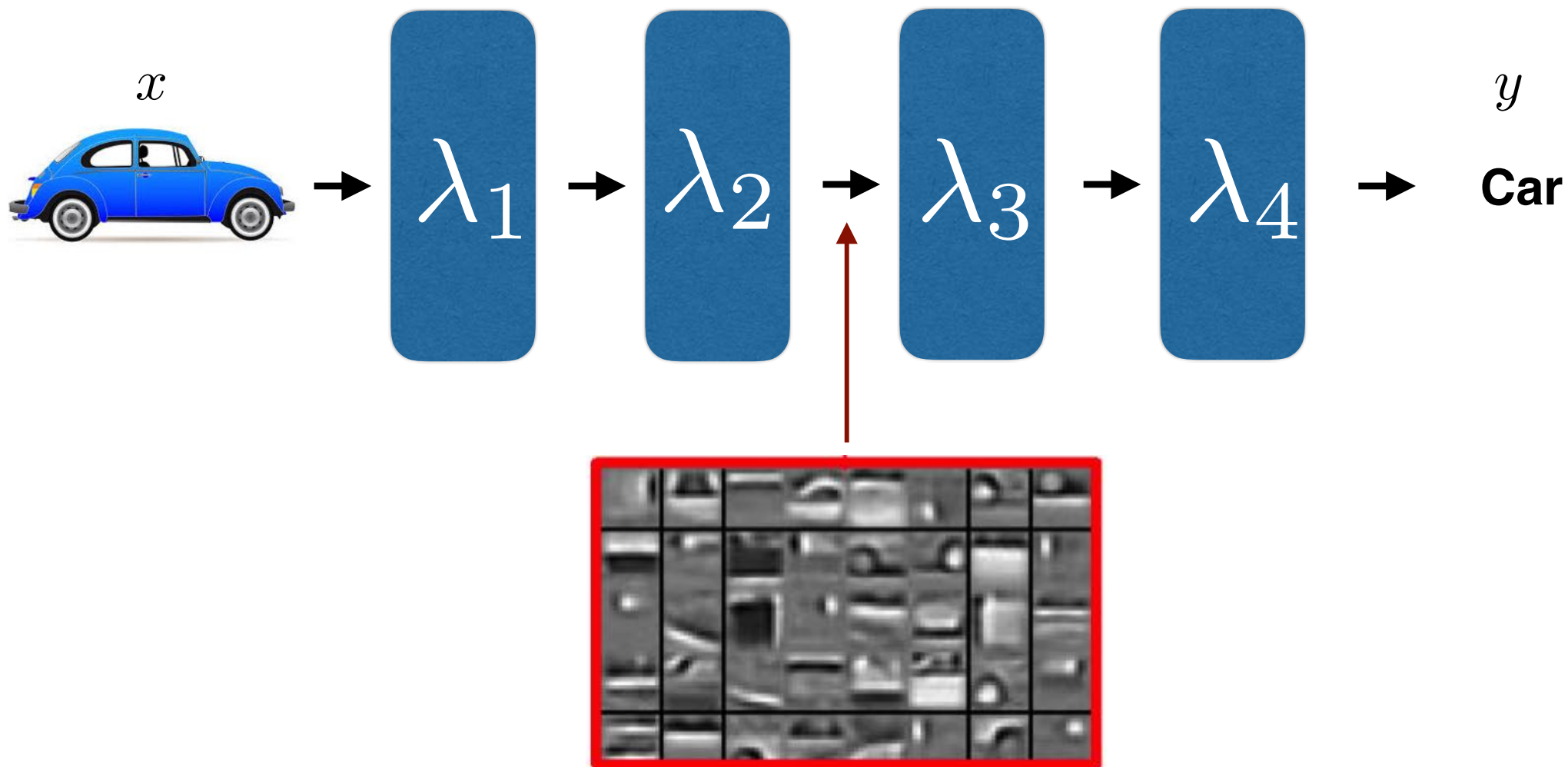
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Deep Learning: Complex Functions by Composition



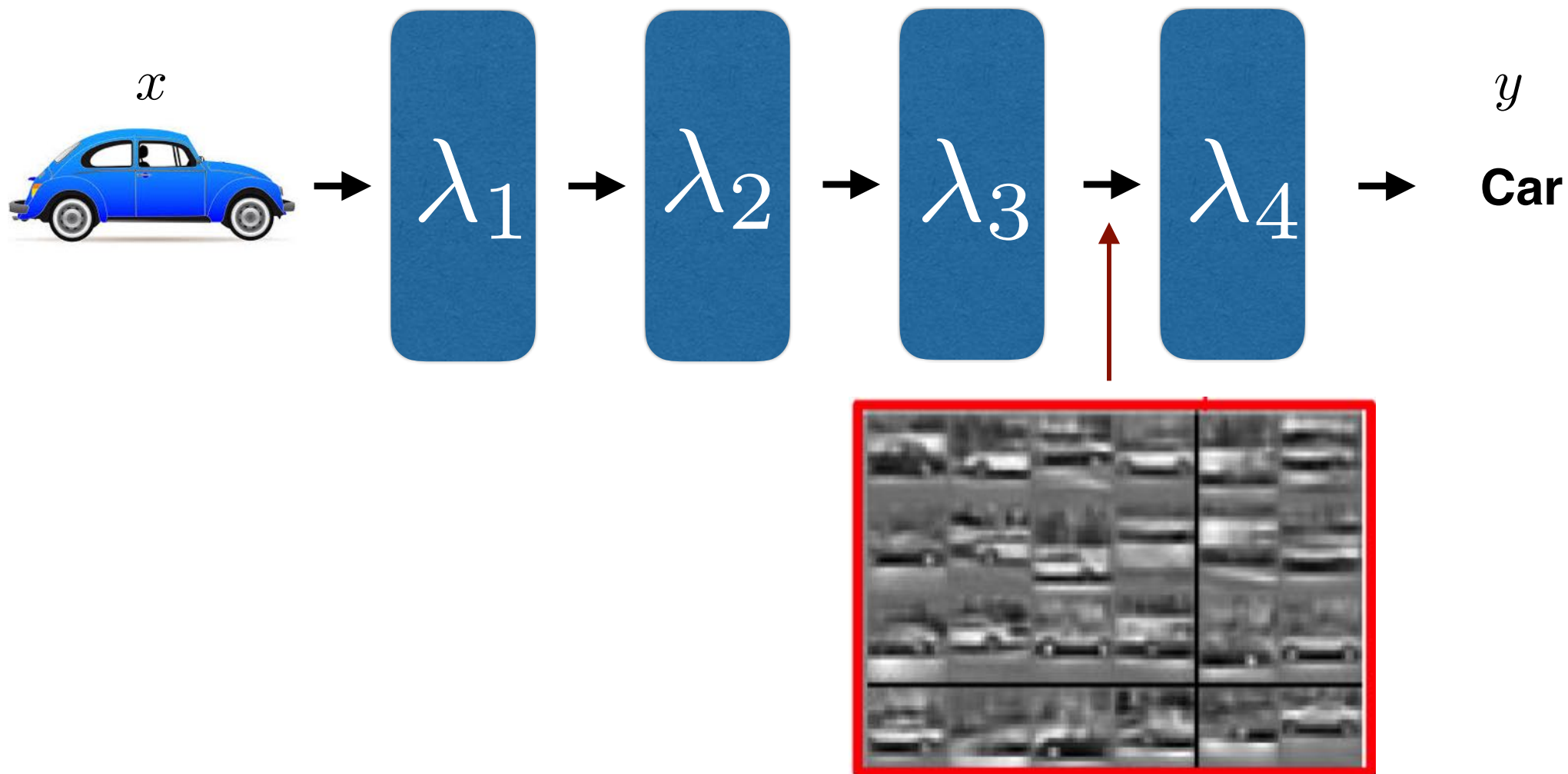
Lee et al. "Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations"

Deep Learning: Complex Functions by Composition



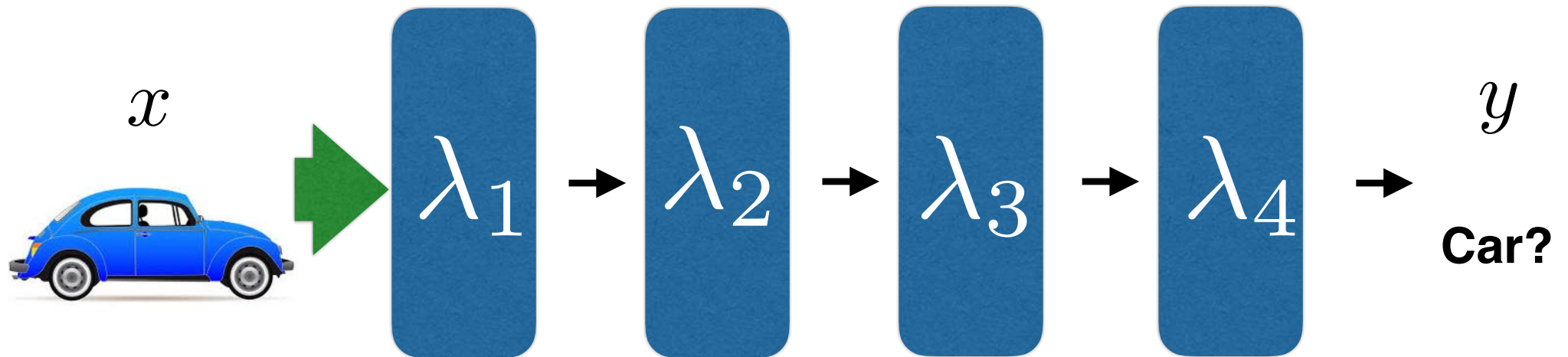
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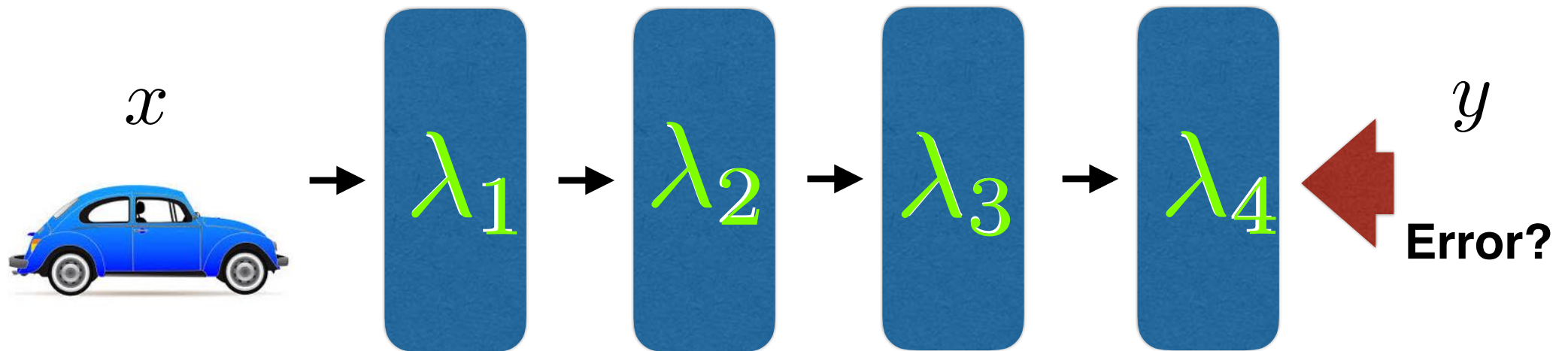
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Training: Overview



- Setting
 - ▶ generate output y for input x (forward pass)

Training: Overview



- Setting

- ▶ generate output y for input x (forward pass)
- ▶ when there is an error, propagate error backwards to update weights (error back propagation)

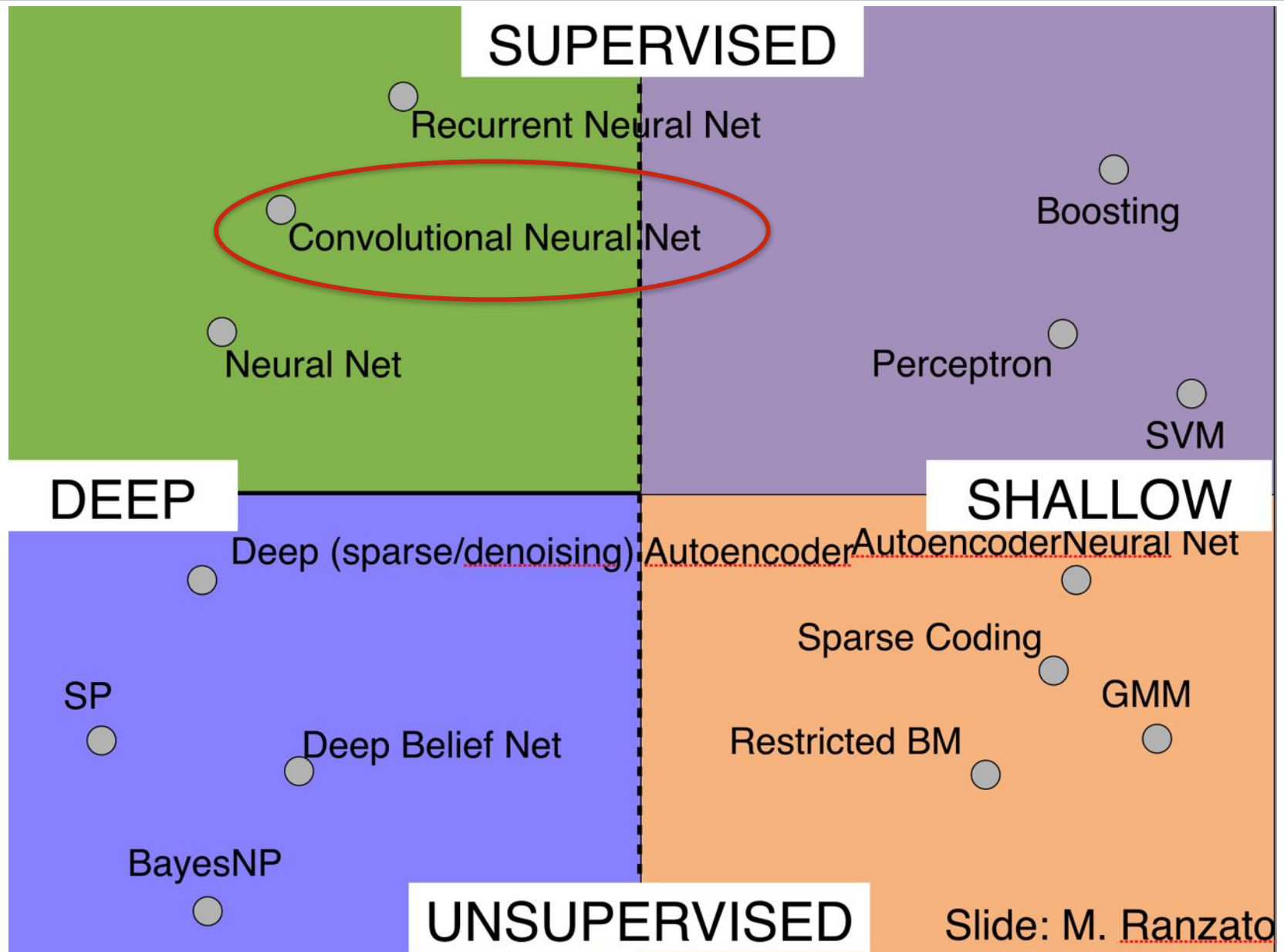
Summary of Main Ideas in Deep Learning

1. **Learning of feature extraction** (across many layers)
2. **Efficient** and **trainable** systems by **differentiable** building blocks
3. Composition of deep architectures via **non-linear modules**
4. “**End-to-End**” training: no differentiation between feature extraction and classification

Overview Today's Lecture

- Backpropagation - Gradient Descent
 - ▶ illustrated using computational graphs
 - ▶ chain rule - upstream and local gradients
 - ▶ modularization simple
- What is Deep Learning
 - ▶ intuition why deep learning can help
 - ▶ integrated learning of features and classifier
- **Convolutional Neural Networks (CNNs)**
 - ▶ one of the (few) highly successful NNs

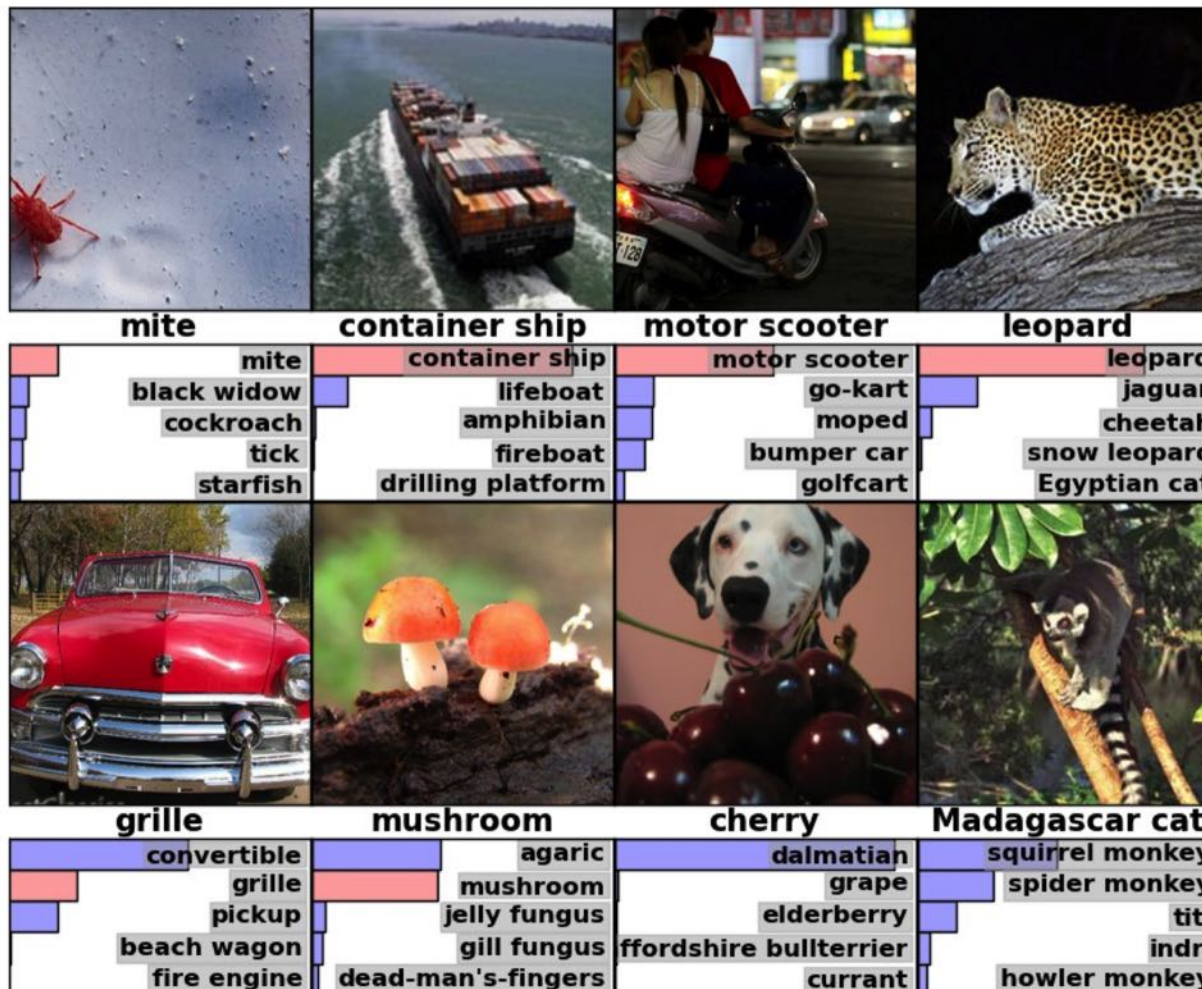
Overview of Deep Learning



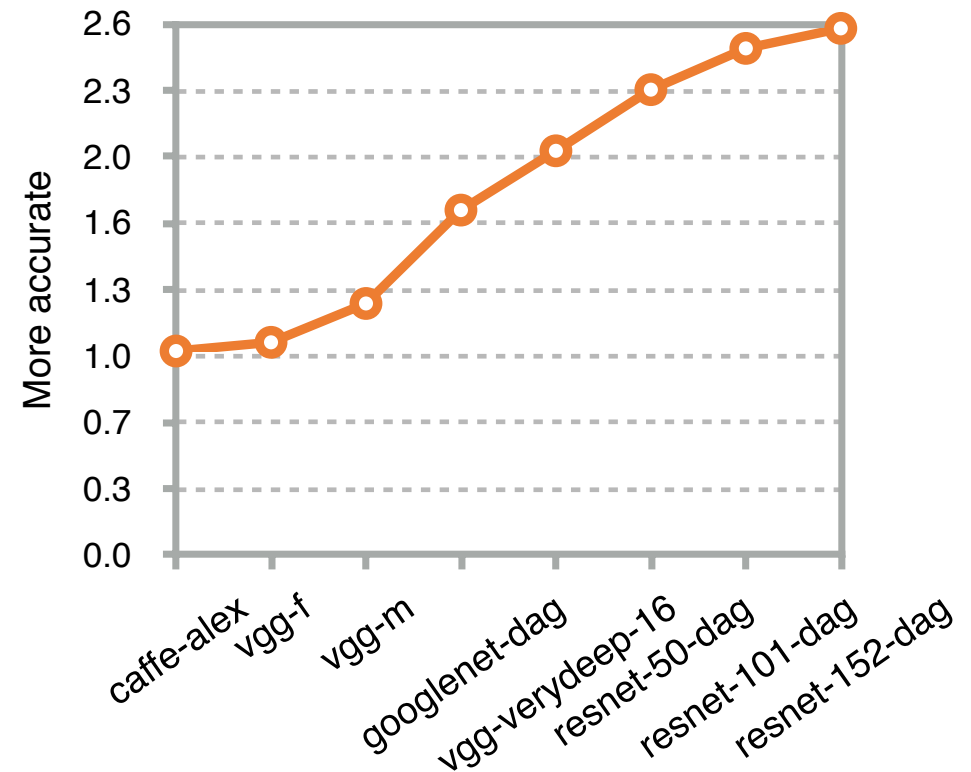
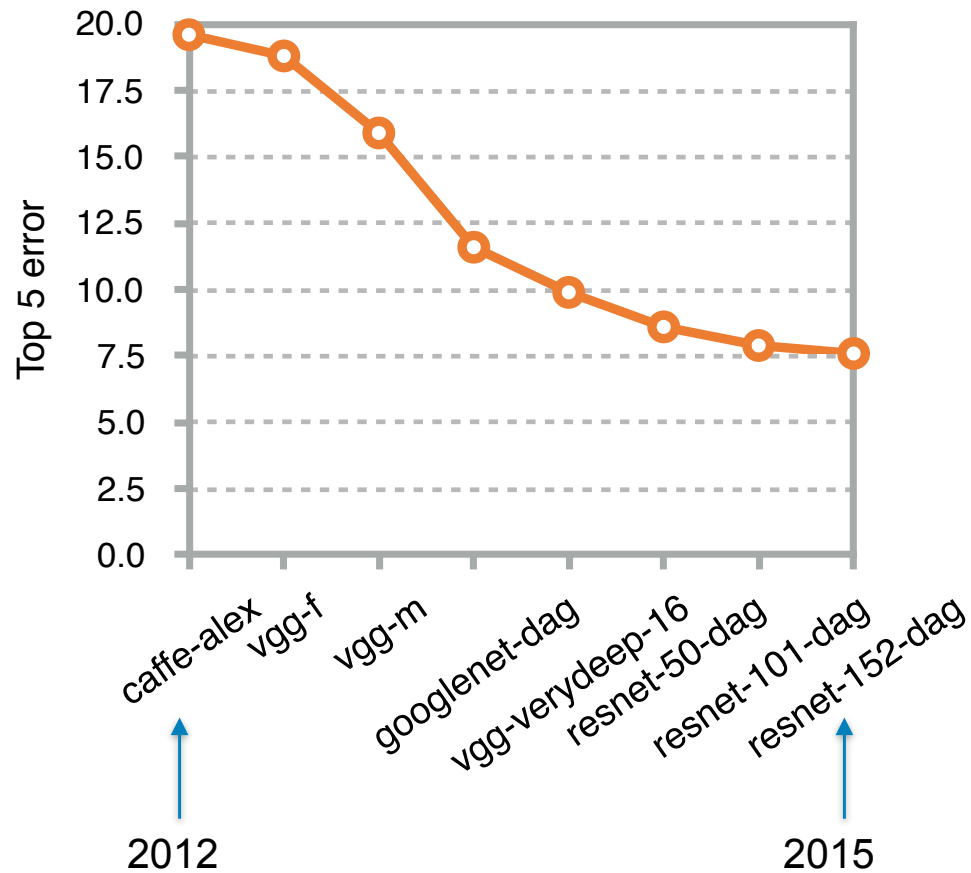
Computer Vision: Image Classification

1000 classes; 1ms for an image; 92.5% recognition rate

<http://demo.caffe.berkeleyvision.org>

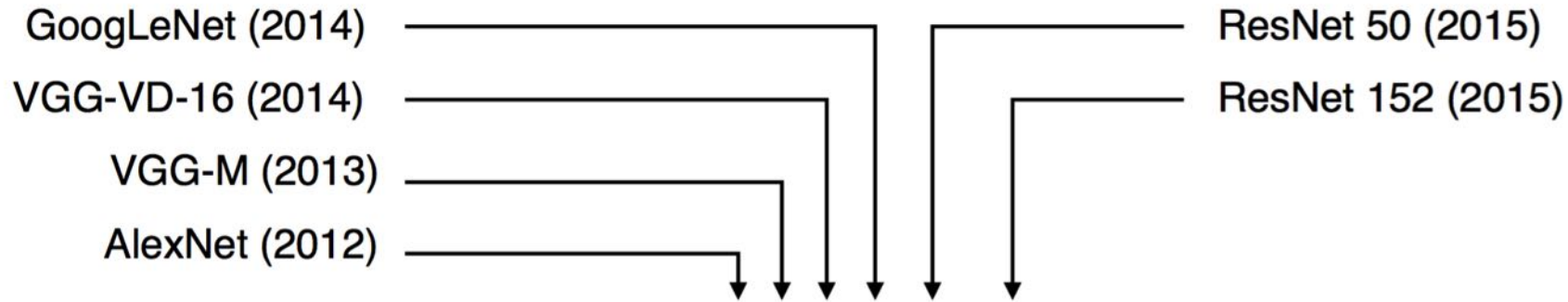


Architectures get deeper



~ 2.6x improvement in 3 years

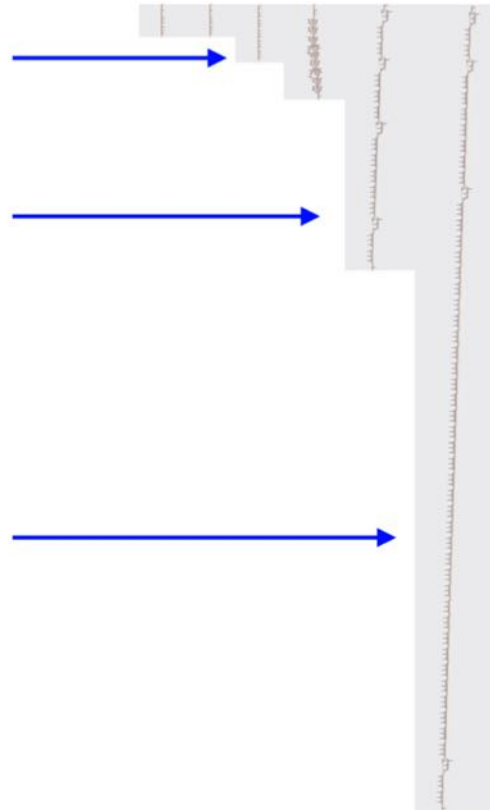
Architectures get deeper



16 convolutional layers

50 convolutional layers

152 convolutional layers



Krizhevsky, I. Sutskever, and G. E. Hinton. *ImageNet classification with deep convolutional neural networks*. In Proc. NIPS, 2012.

C. Szegedy, W. Liu, Y. Jia, P. Sermanet, S. Reed, D. Anguelov, D. Erhan, V. Vanhoucke, and A. Rabinovich. *Going deeper with convolutions*. In Proc. CVPR, 2015.

K. Simonyan and A. Zisserman. *Very deep convolutional networks for large-scale image recognition*. In Proc. ICLR, 2015.

K. He, X. Zhang, S. Ren, and J. Sun. *Deep residual learning for image recognition*. In Proc. CVPR, 2016.

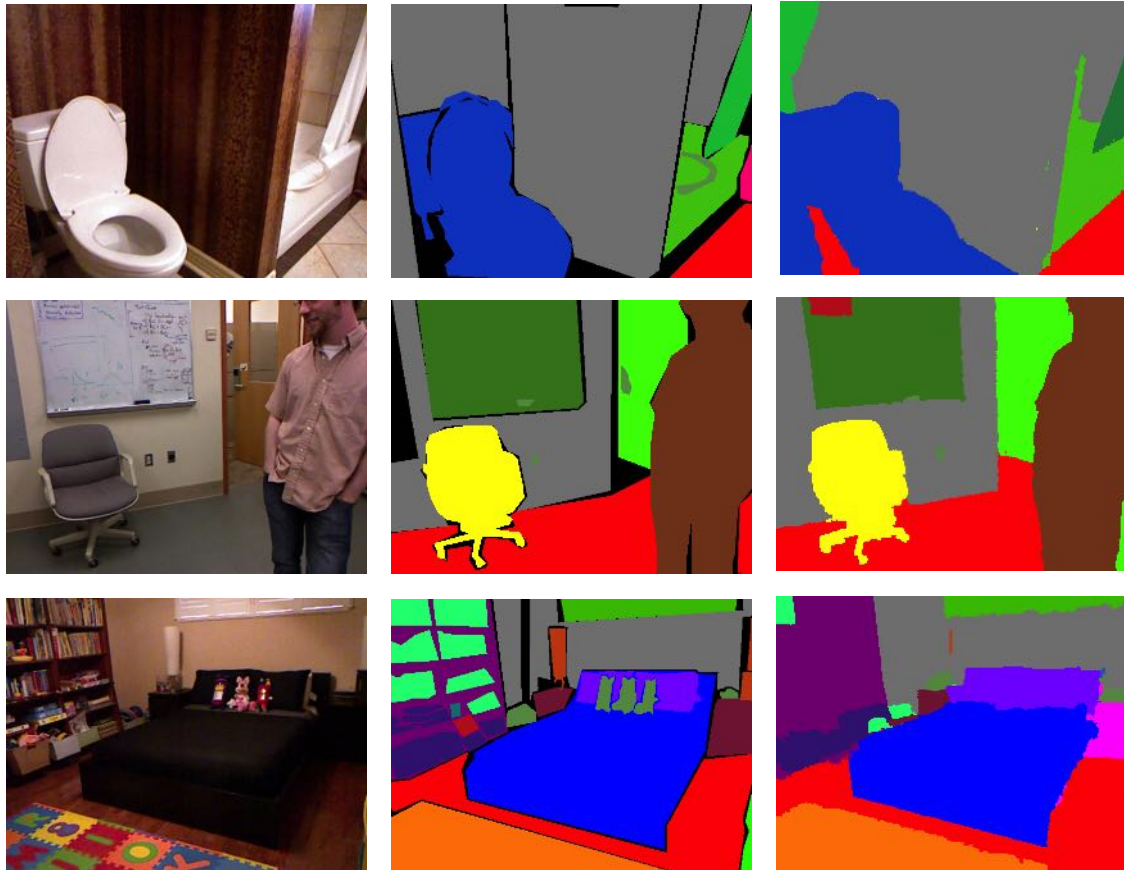
Computer Vision: Semantic Segmentation



Badrinarayanan et al. 2015

SegNet: A Deep Convolutional Encoder-Decoder Architecture for Robust Semantic Pixel-Wise Labelling

Computer Vision: Semantic Segmentation



Input

Groundtruth

Output

Yang He; Wei-Chen Chiu; Margret Keuper; Mario Fritz

STD2P: RGBD Semantic Segmentation Using Spatio-Temporal Data-Driven Pooling

IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2017



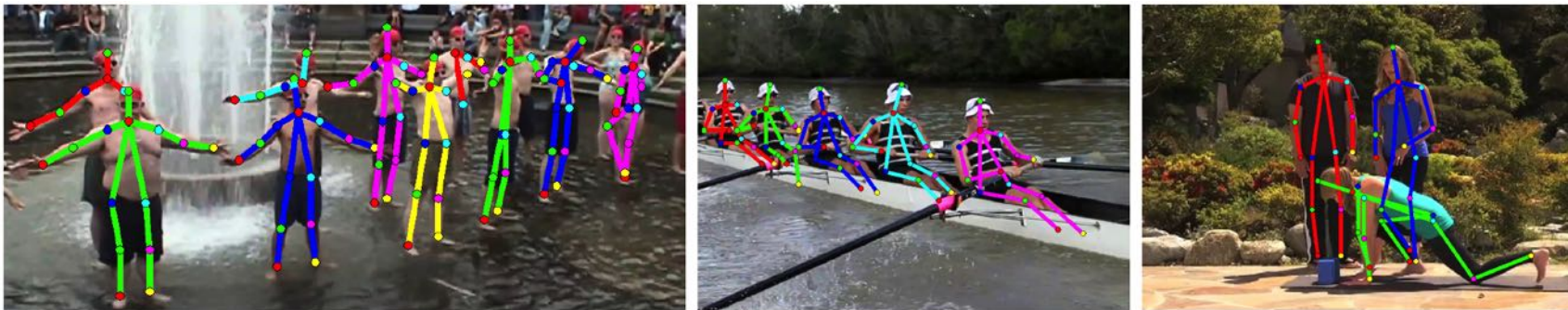
Deep(er)Cut: Joint Subset Partition & Labeling for Multi Person Pose Estimation

DeepCut: Joint Subset Partition and Labeling for Multi-Person Pose Estimation

L. Pishchulin, E. Insafutdinov, S. Tang, B. Andres, M. Andriluka, P. Gehler, and B. Schiele, CVPR'16

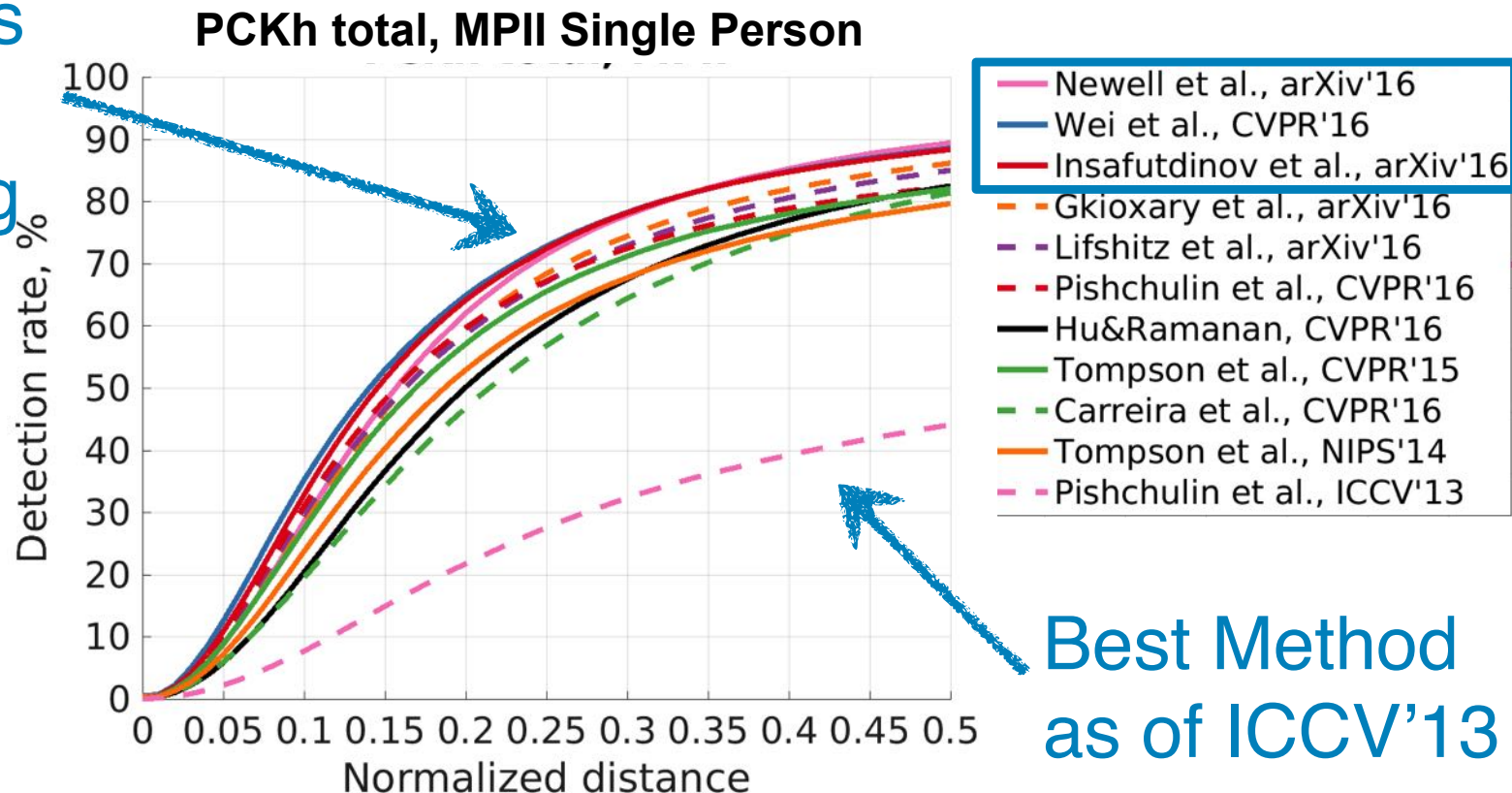
DeeperCut: A Deeper, Stronger, and Faster Multi-Person Pose Estimation Model

E. Insafutdinov, L. Pishchulin, B. Andres, M. Andriluka, and B. Schiele, ECCV'16



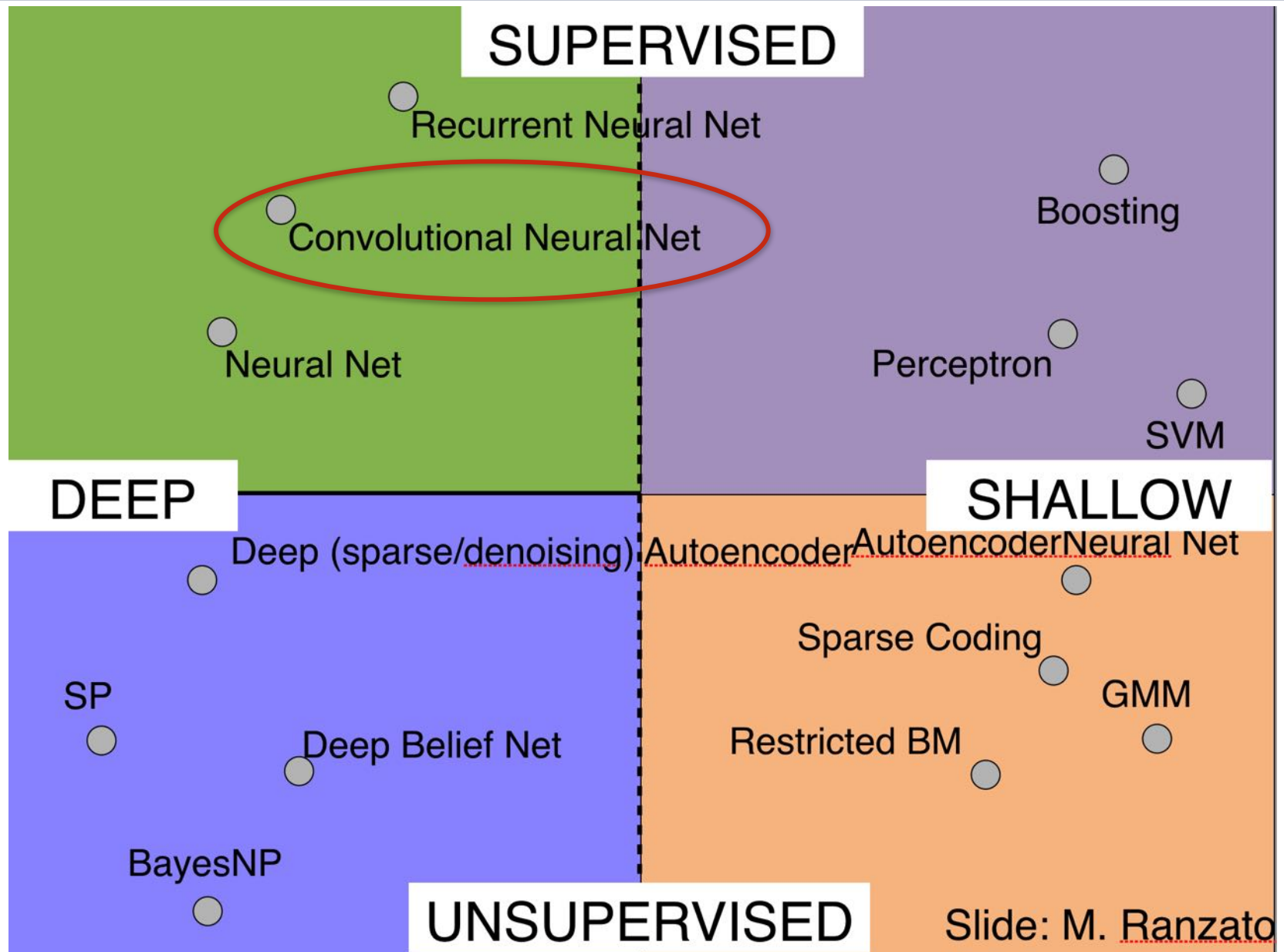
Analysis - overall performance

Best Methods
now:
deep learning
“takes” over



- ✓ since CVPR'14, dataset has become **de-facto standard benchmark**
- ✓ **large training set** facilitated development of **deep learning methods**

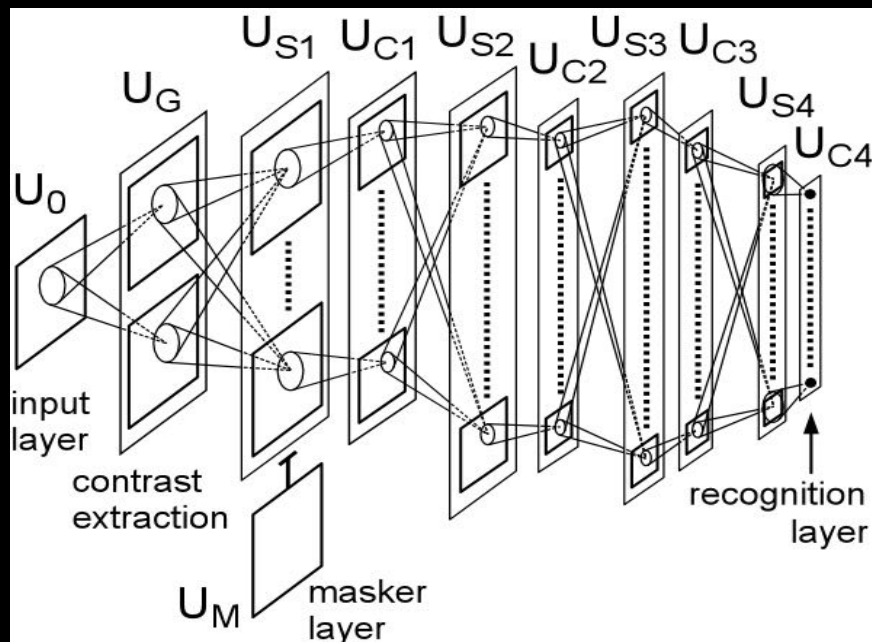
Overview of Deep Learning



Multistage Hubel&Wiesel Architecture

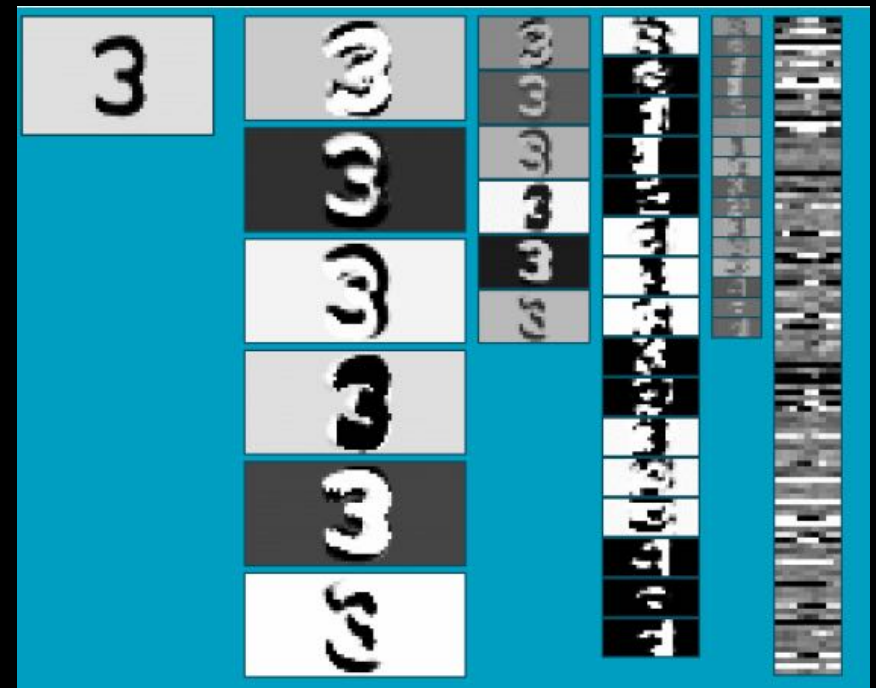
Slide: Y.LeCun

- [Hubel & Wiesel 1962]
 - simple cells detect local features
 - complex cells “pool” the outputs of simple cells within a retinotopic neighborhood.



Cognitron / Neocognitron
[Fukushima 1971-1982]

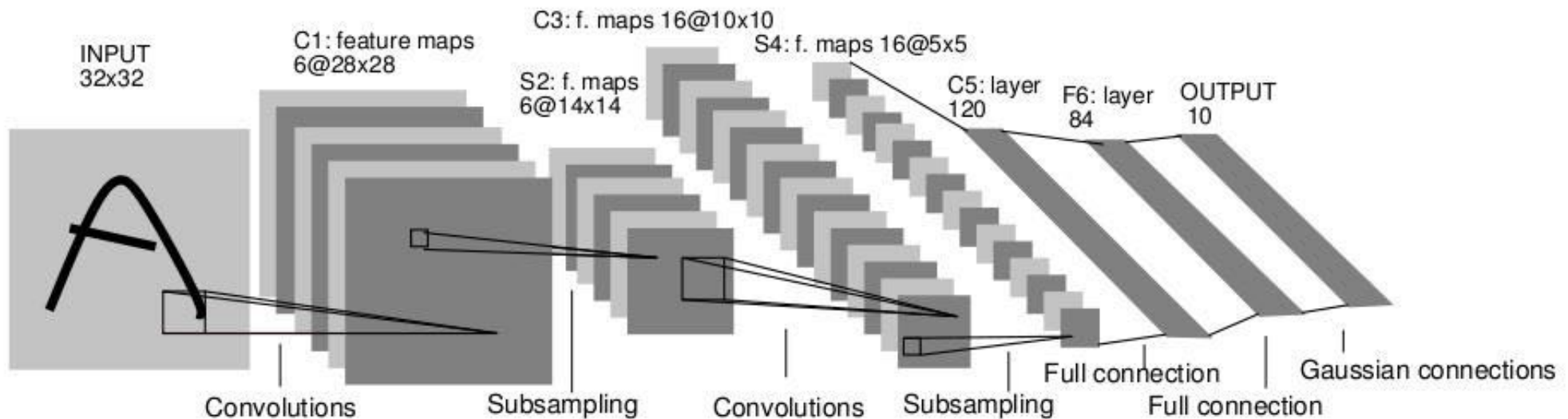
- Also HMAX [Poggio 2002-2006]



Convolutional Networks
[LeCun 1988-present]

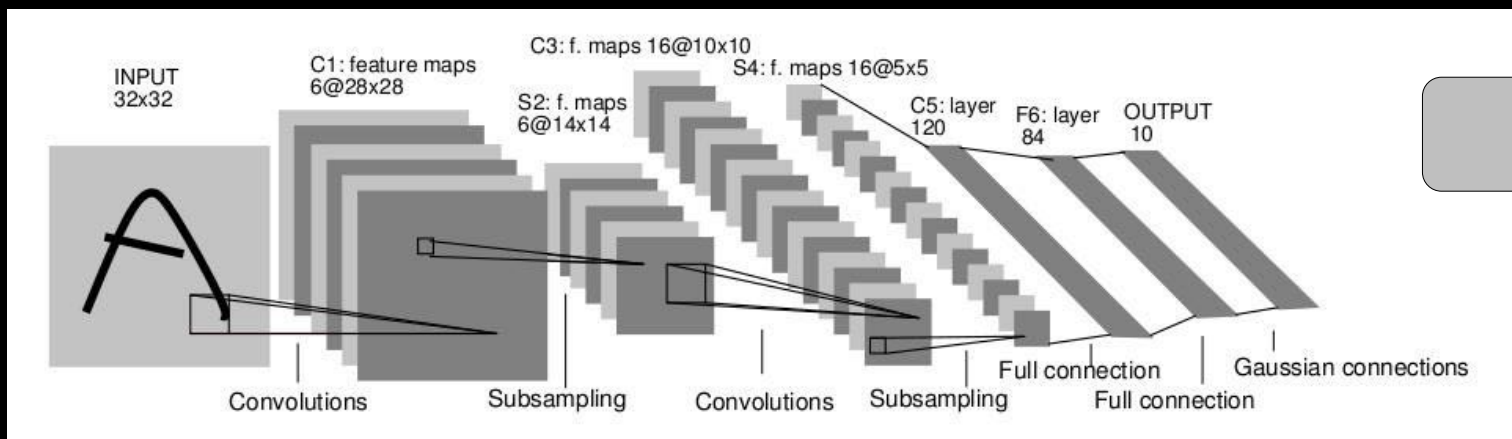
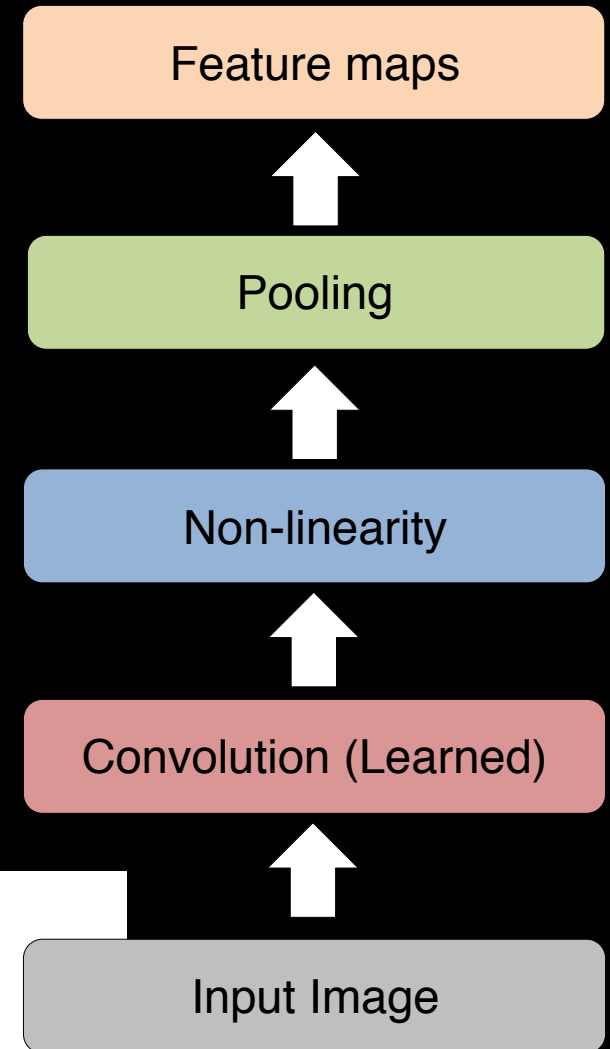
Convolutional Neural Networks

- LeCun et al. 1989
- Neural network with specialized connectivity structure



Characteristics of Convnets

- Feed-forward:
 - Convolve input
 - Non-linearity (rectified linear)
 - Pooling (local max) / (=subsampling)
- Supervised
- Train convolutional filters by back-propagating classification error

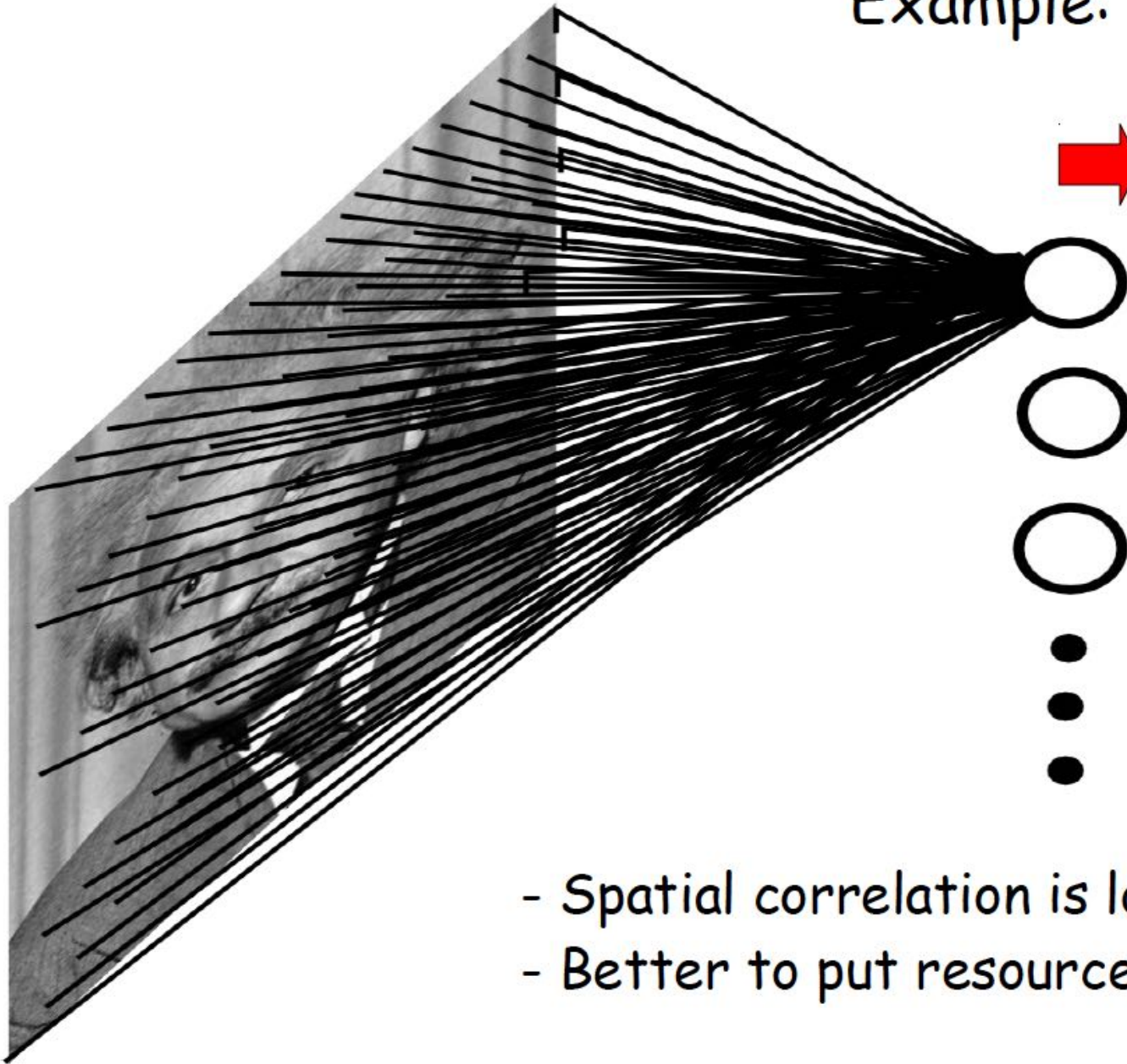


[LeCun et al. 1989]

FULLY CONNECTED NEURAL NET

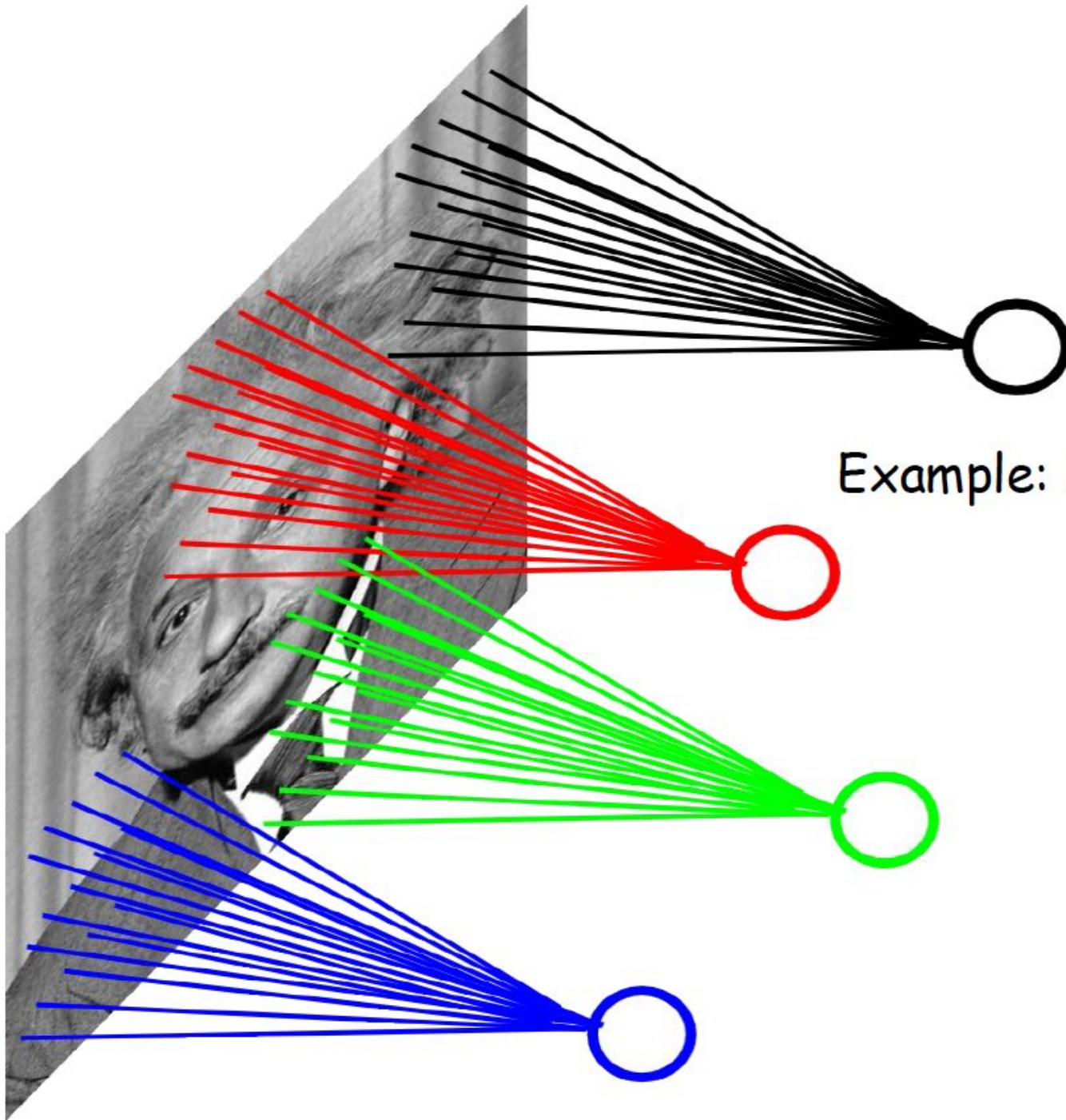
Example: 1000x1000 image
1M hidden units

➔ **10^{12} parameters!!!**



- Spatial correlation is local
- Better to put resources elsewhere!

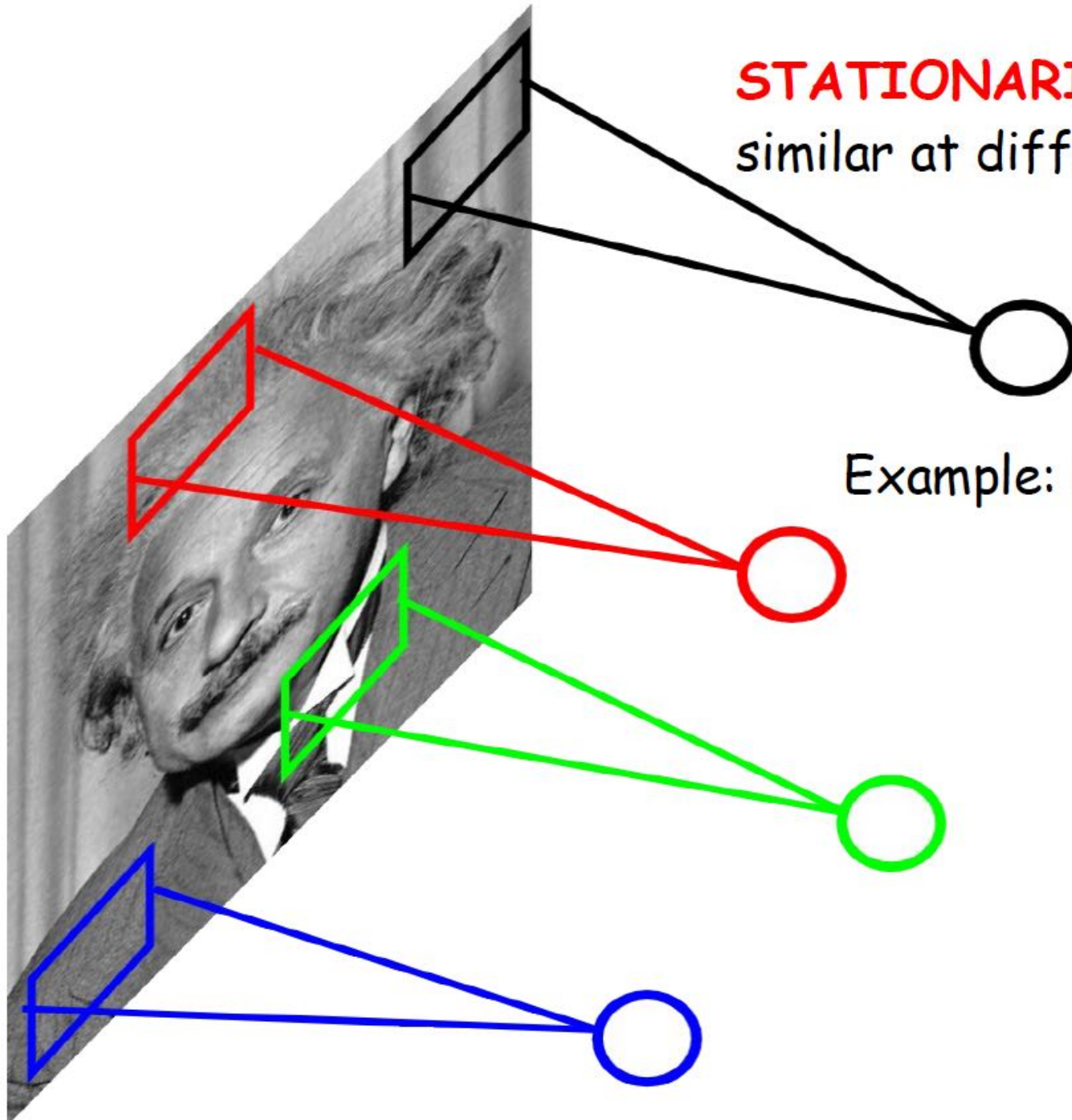
LOCALLY CONNECTED NEURAL NET



Example: 1000x1000 image
1M hidden units
Filter size: 10x10
100M parameters

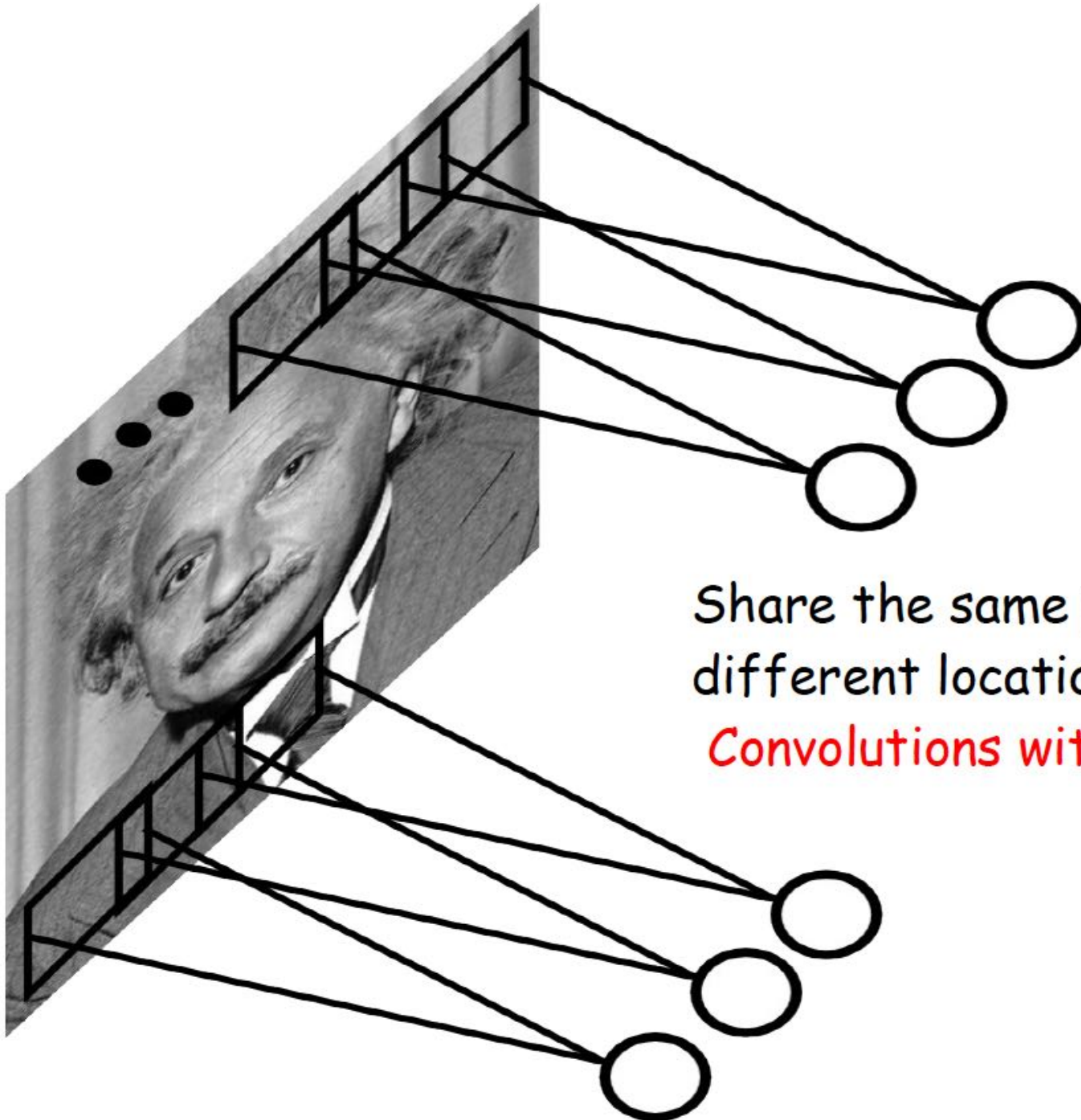
LOCALLY CONNECTED NEURAL NET

STATIONARITY? Statistics is similar at different locations



Example: 1000x1000 image
1M hidden units
Filter size: 10x10
100M parameters

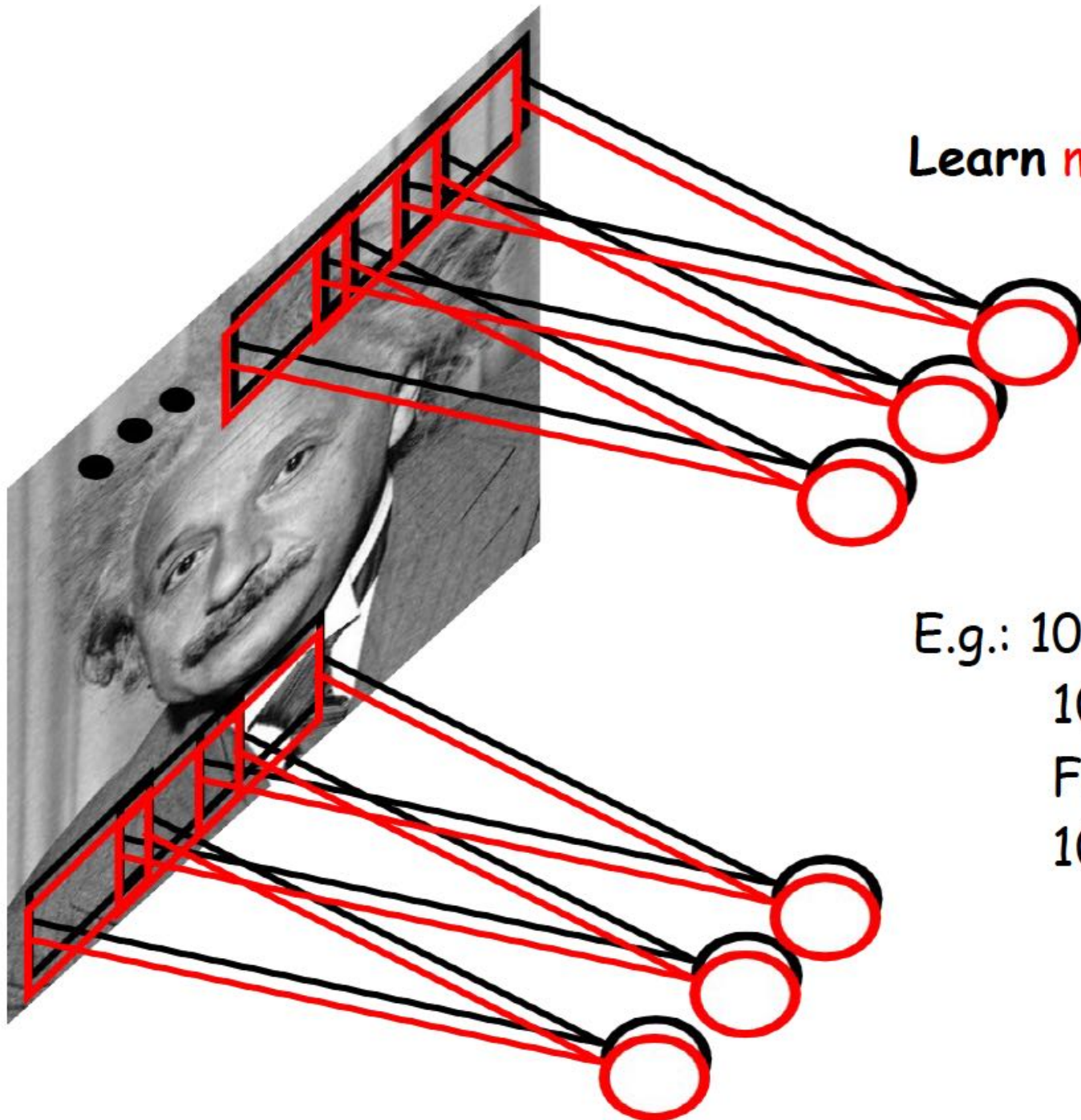
CONVOLUTIONAL NET



Share the same parameters across different locations:

Convolutions with learned kernels

CONVOLUTIONAL NET



Learn **multiple filters**.

E.g.: 1000x1000 image
100 Filters
Filter size: 10x10
10K parameters

NEURAL NETS FOR VISION

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

Solution:

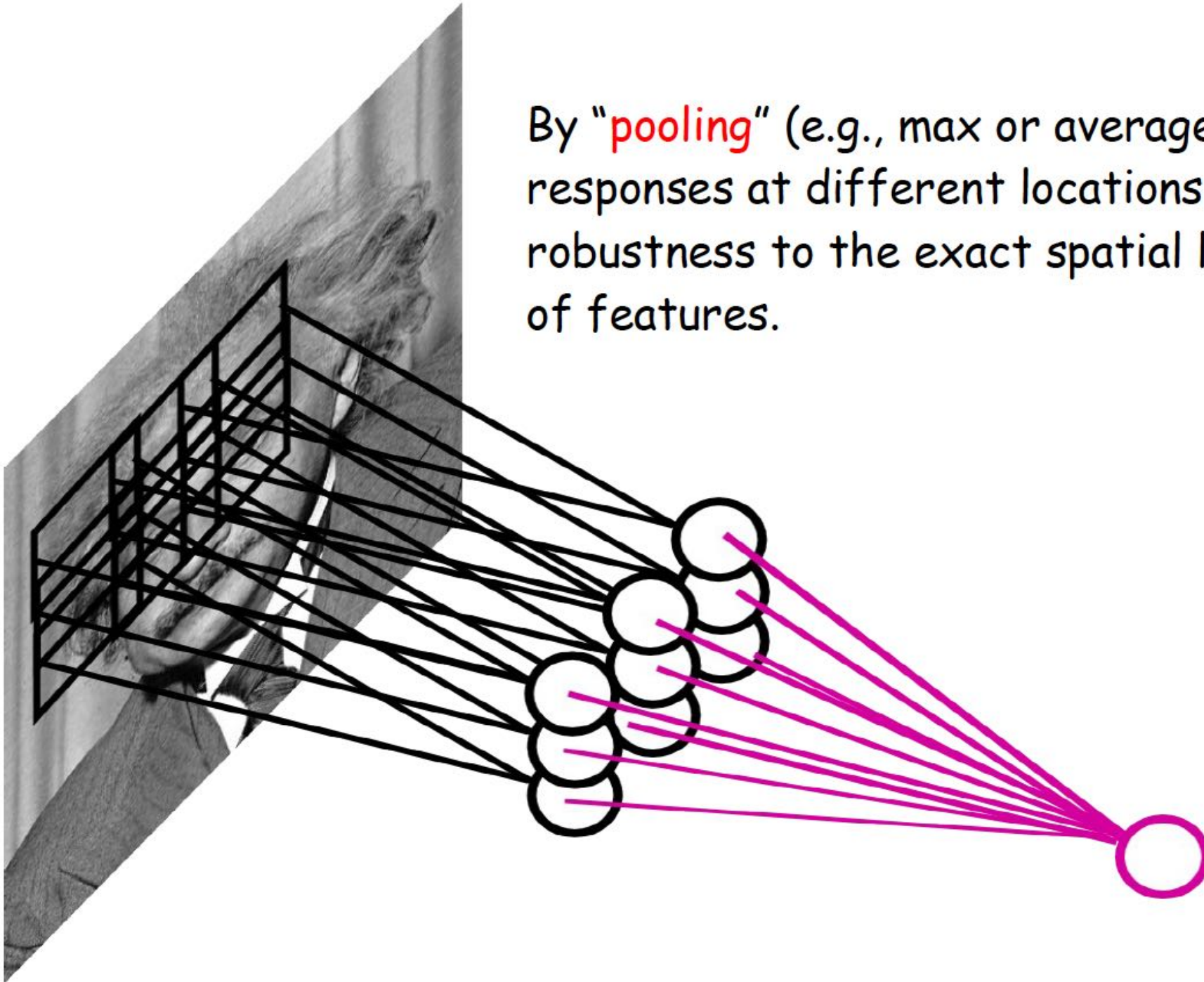
- connect each hidden unit to a small patch of the input
- share the weight across hidden units

This is called: **convolutional network.**

LeCun et al. "Gradient-based learning applied to document recognition" IEEE 1998

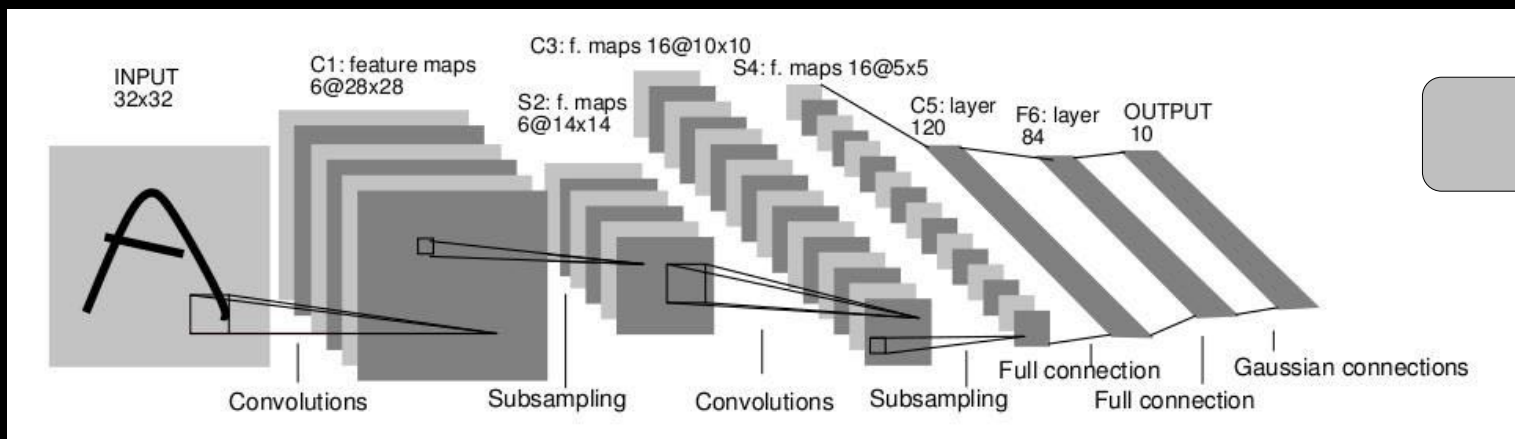
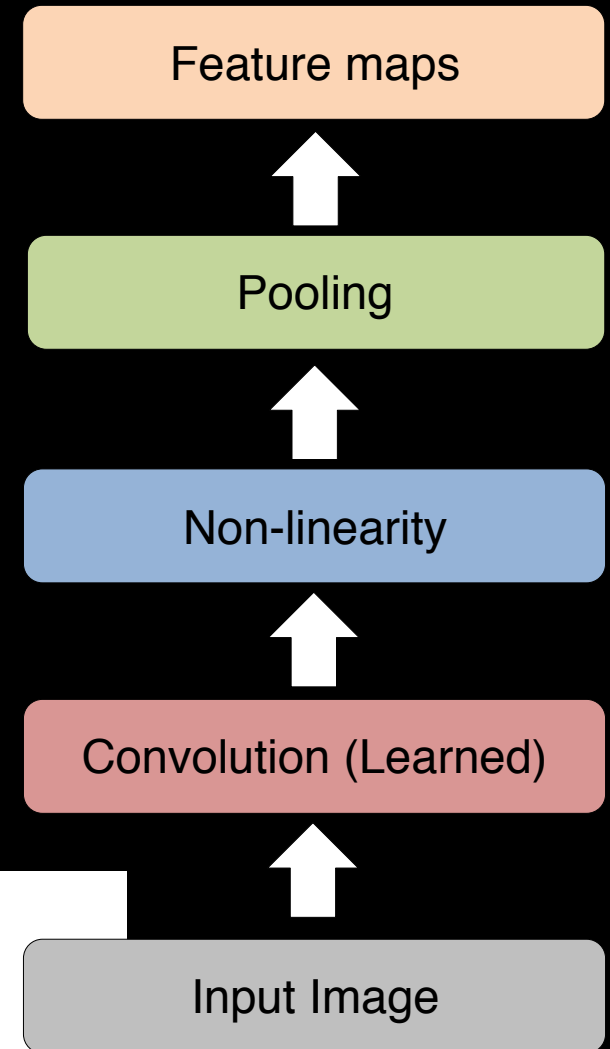
CONVOLUTIONAL NET

By “pooling” (e.g., max or average) filter responses at different locations we gain robustness to the exact spatial location of features.



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 - Pooling (local max) / (=subsampling)
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[LeCun et al. 1989]

Krizhevsky et al. [NIPS2012]

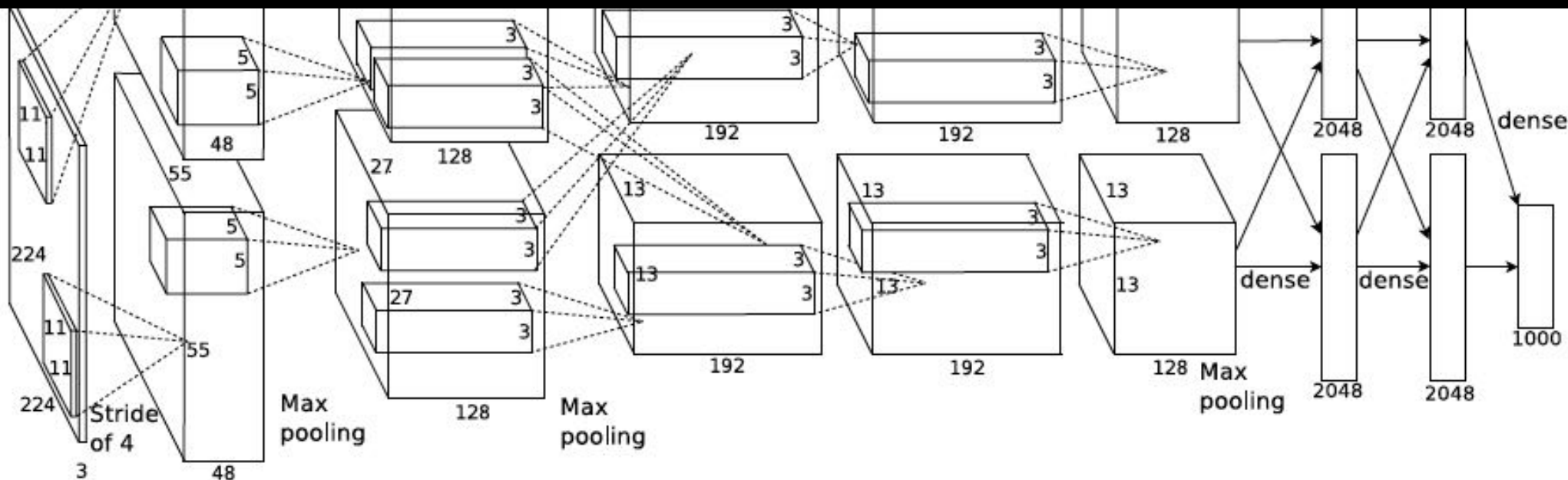
- Same model as LeCun'98 but:
 - Bigger model (8 layers)
 - More data (10^6 vs 10^3 images)
 - GPU implementation (50x speedup over CPU)
 - Better regularization (DropOut)

ImageNet Classification with Deep Convolutional Neural Networks

Alex Krizhevsky
University of Toronto
kriz@cs.utoronto.ca

Ilya Sutskever
University of Toronto
ilya@cs.utoronto.ca

Geoffrey E. Hinton
University of Toronto
hinton@cs.utoronto.ca



- 7 hidden layers, 650,000 neurons, 60,000,000 parameters
- Trained on 2 GPUs for a week

IMAGENET Large Scale Visual Recognition Challenge

The Image Classification Challenge:
1,000 object classes
1,431,167 images



Output:
Scale
T-shirt
Steel drum
Drumstick
Mud turtle



Output:
Scale
T-shirt
Giant panda
Drumstick
Mud turtle

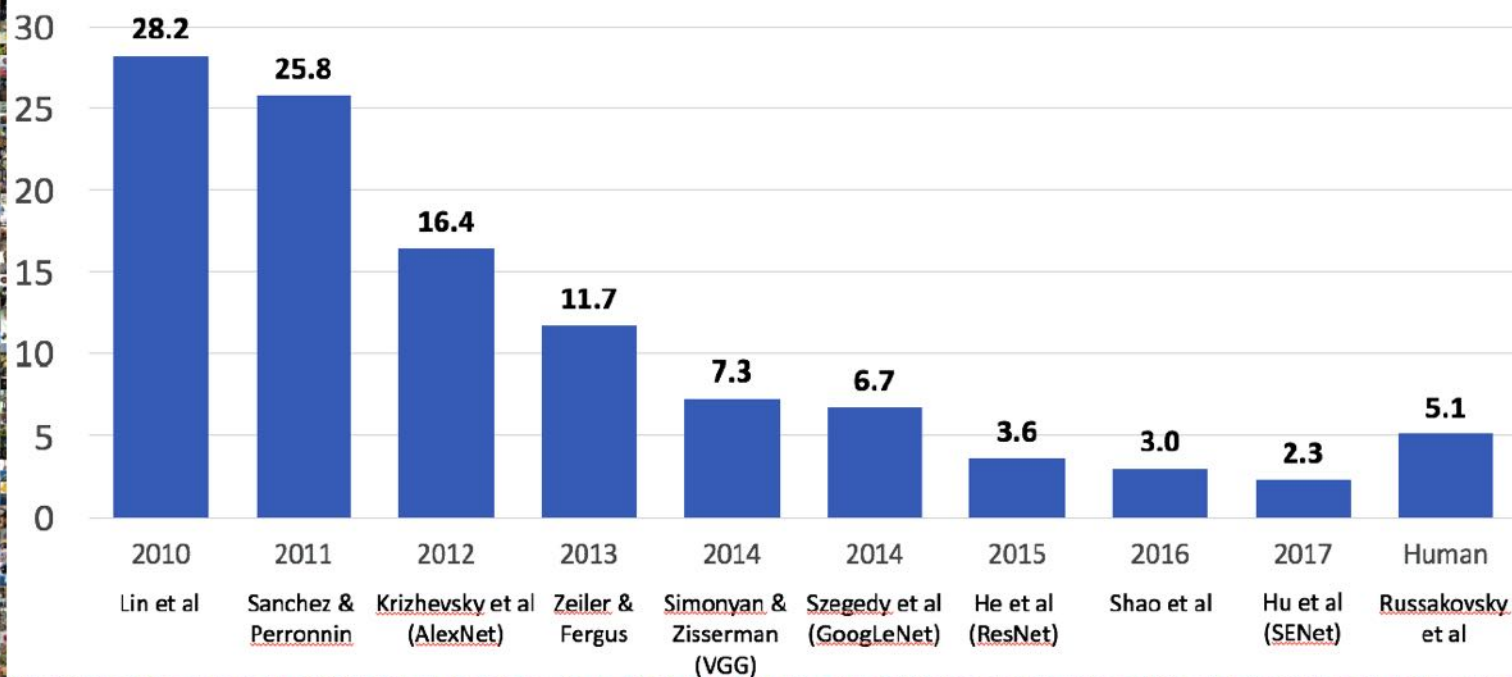


Russakovsky et al. IJCV 2015

slide credit: Fei-Fei, Justin Johnson, Serena Yeung

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