Lecture 5: Convolutional Neural Networks

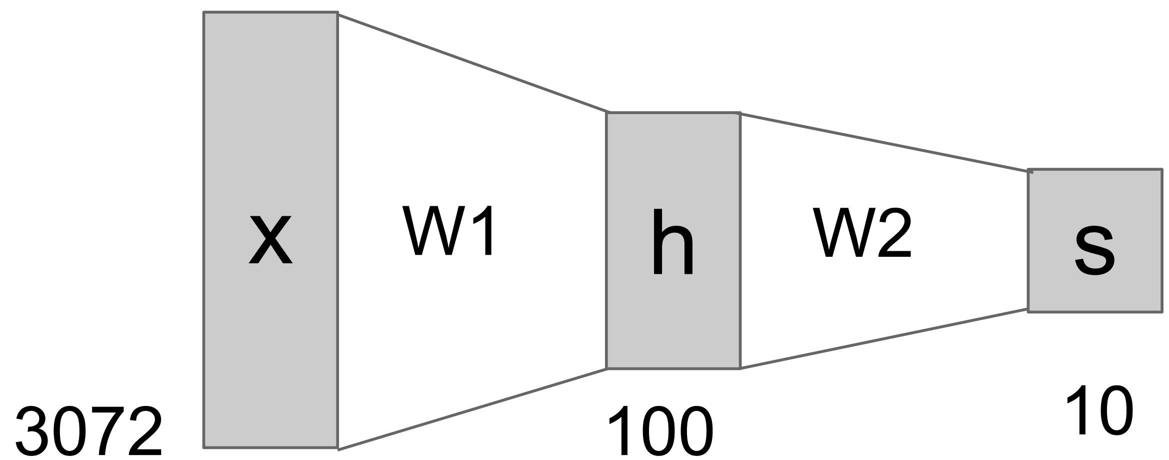
Slides adapted from Stanford course on ConvNets

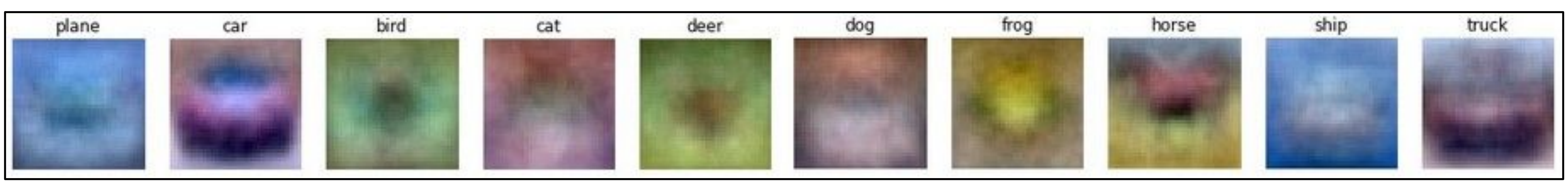
Gerard Pons-Moll

Last time: Neural Networks

Linear score function: $f = Wx$

2-layer Neural Network $f = W_2 \max(0, W_1 x)$





Next: Convolutional Neural Networks

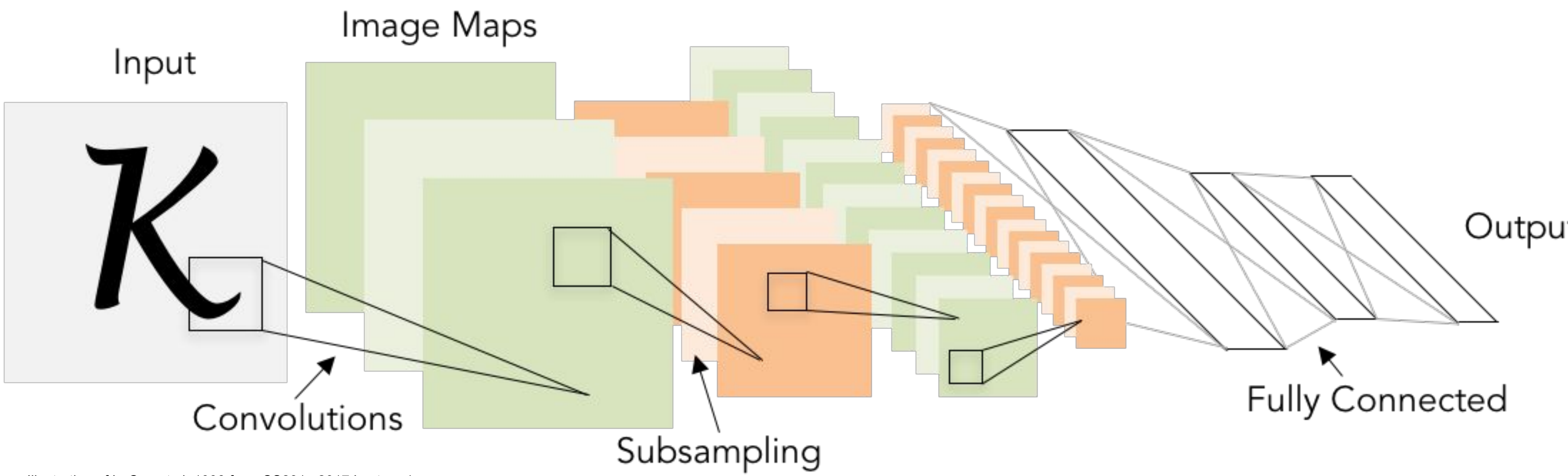


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

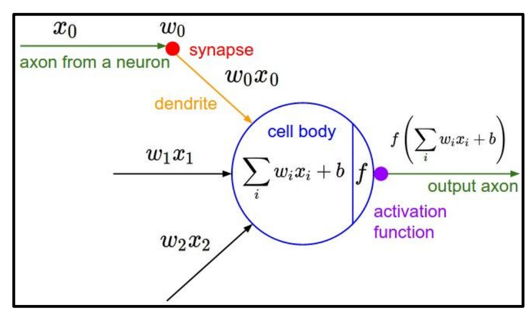
A bit of history...

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

recognized
letters of the alphabet

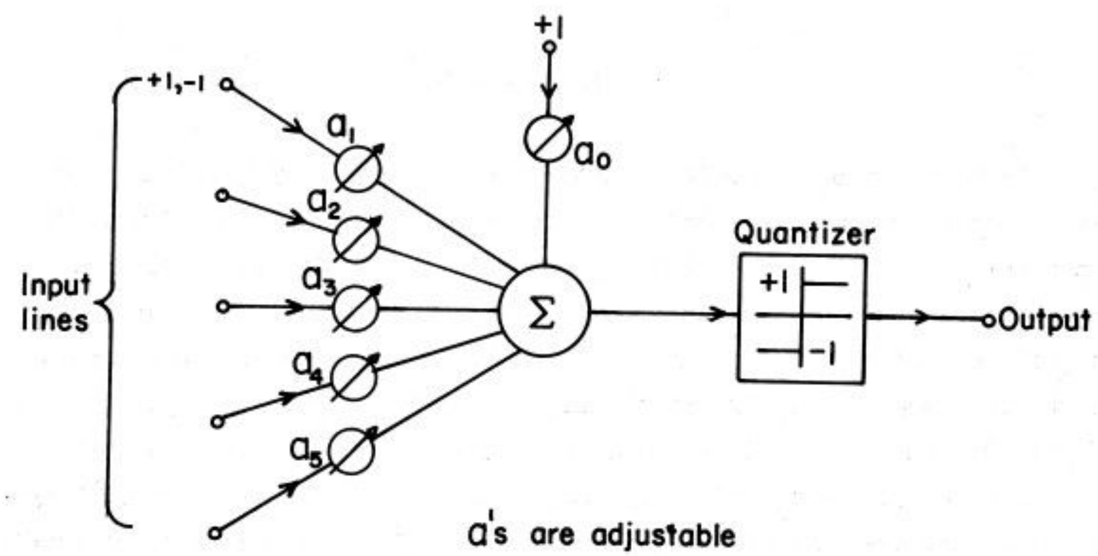


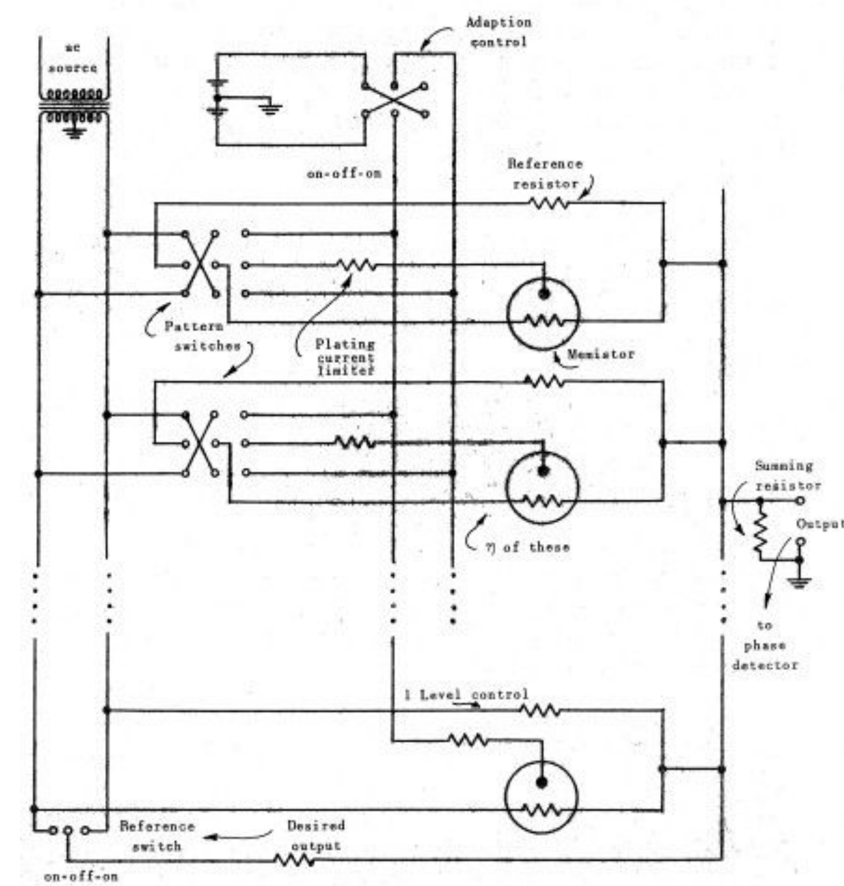
update rule:

$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i},$$

Frank Rosenblatt, ~1957: Perceptron

A bit of history...





Widrow and Hoff, ~1960: Adaline/Madaline

These figures are reproduced from Widrow 1960, Stanford Electronics Laboratories Technical Report with permission from Stanford University Special Collections.

A bit of history...

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$

recognizable math

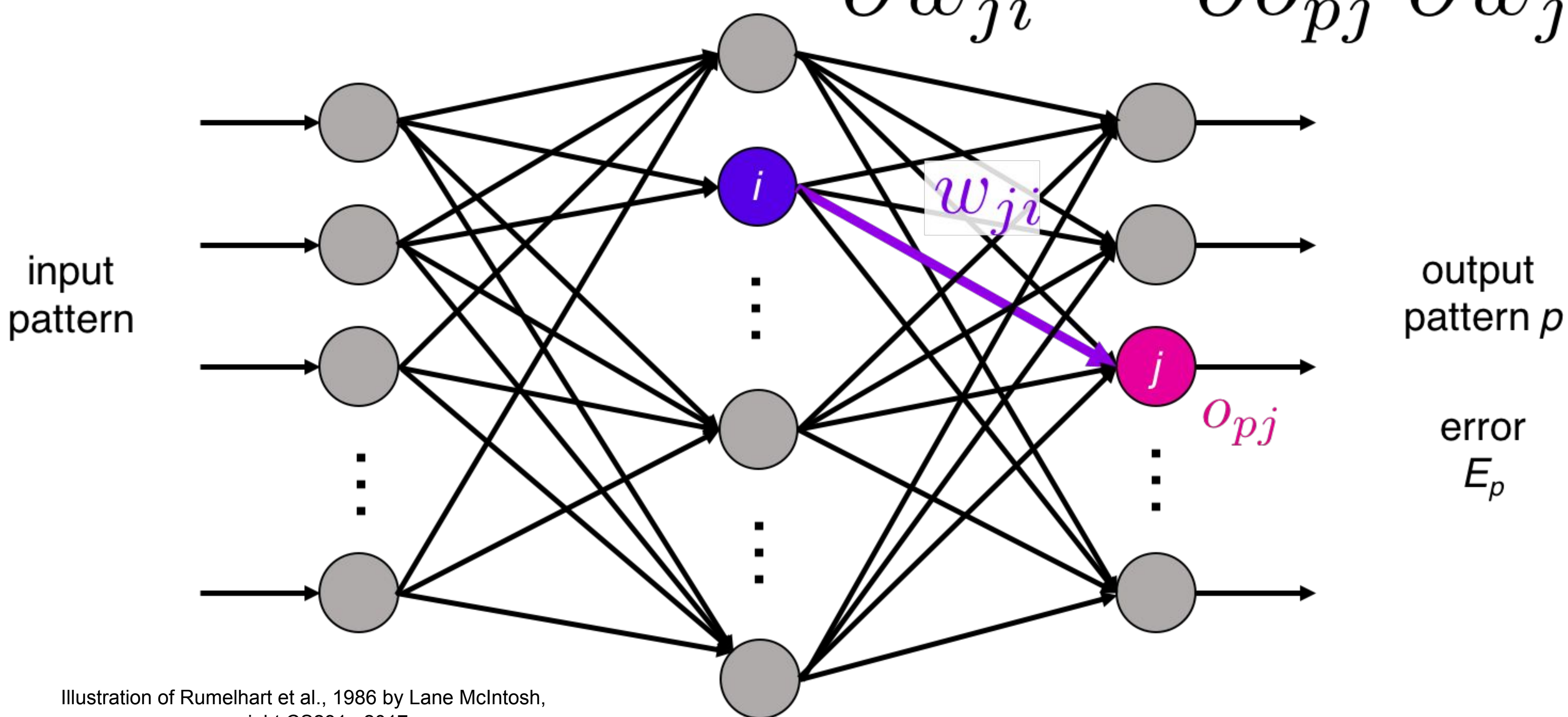


Illustration of Rumelhart et al., 1986 by Lane McIntosh, copyright CS231n 2017

Rumelhart et al., 1986: First time back-propagation became popular

A bit of history...

[Hinton and Salakhutdinov 2006]

Reinvigorated research in Deep Learning

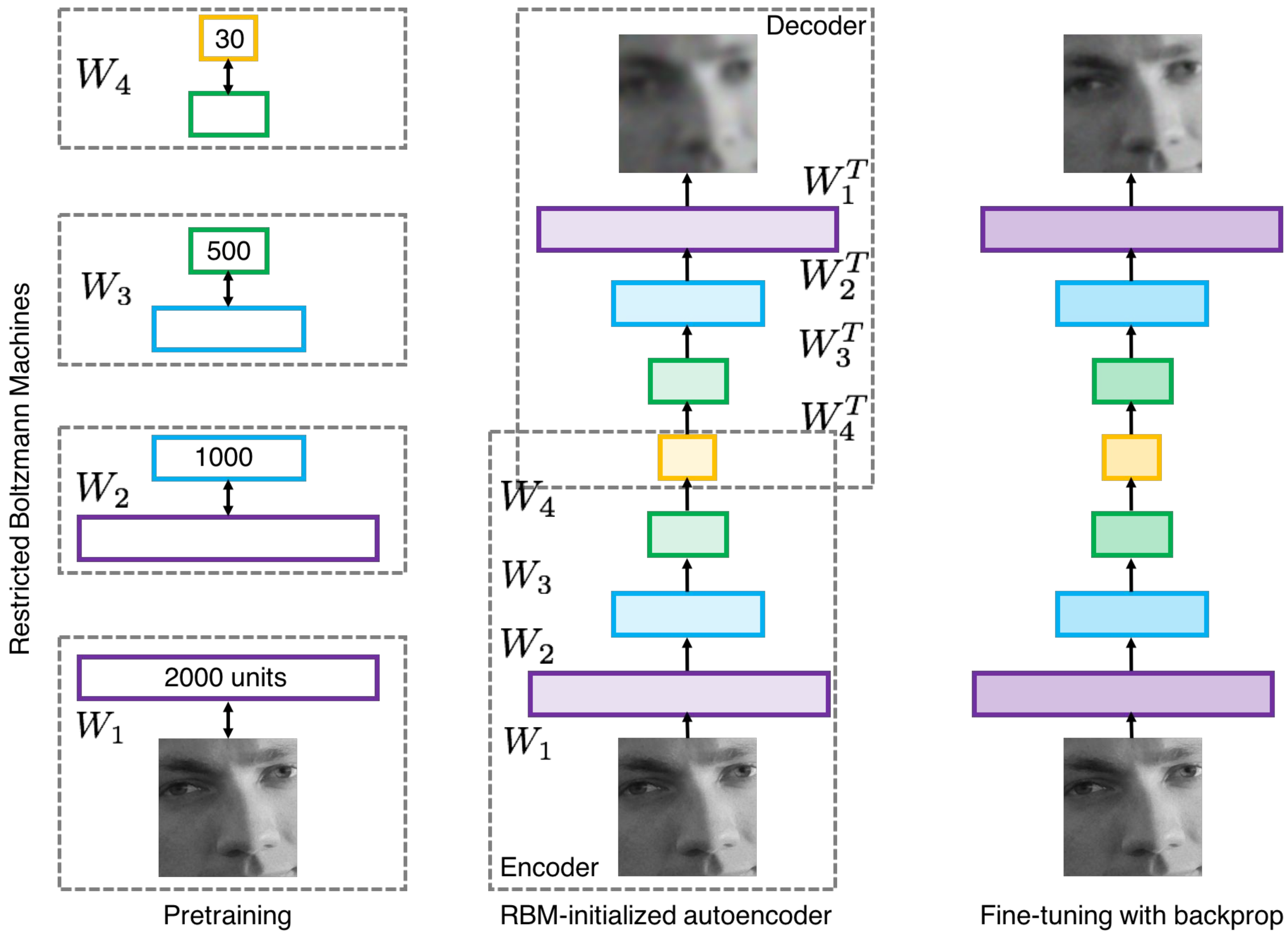


Illustration of Hinton and Salakhutdinov 2006 by Lane McIntosh, copyright CS231n 2017

First strong results

Acoustic Modeling using Deep Belief Networks
Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010
Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition
George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

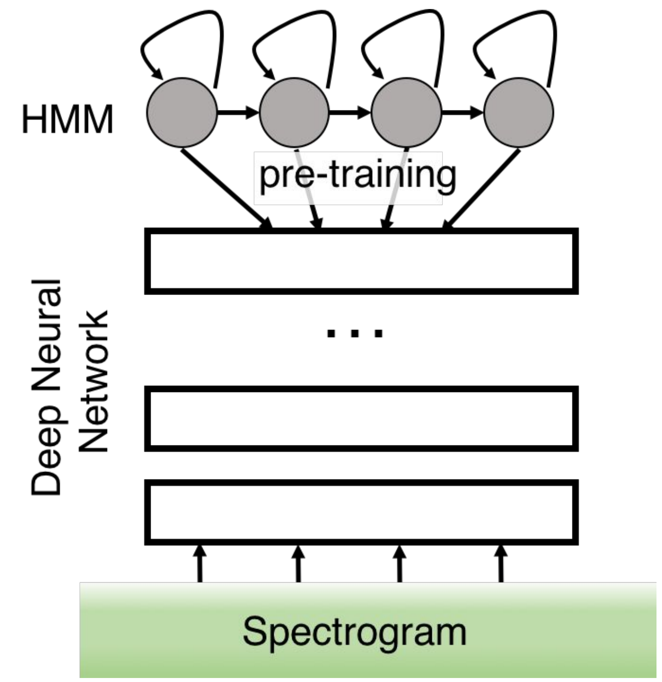
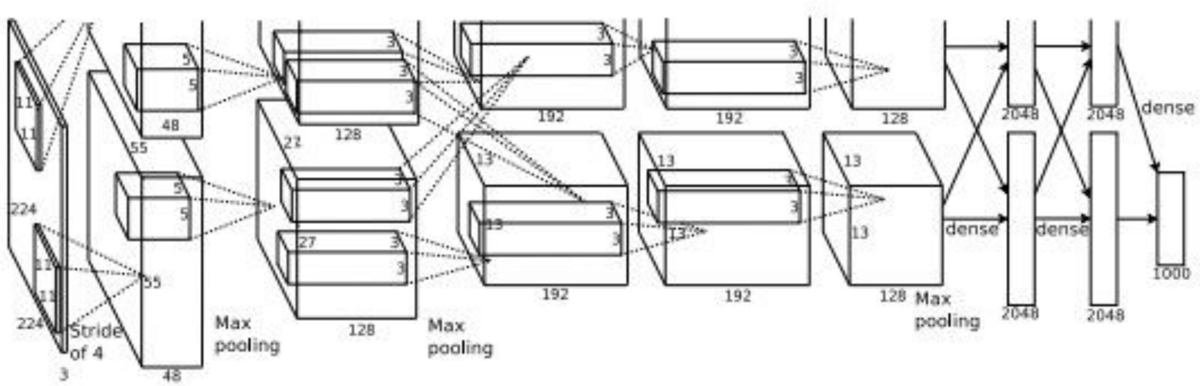


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

Imagenet classification with deep convolutional neural networks
Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012



Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

A bit of history:

Hubel & Wiesel,

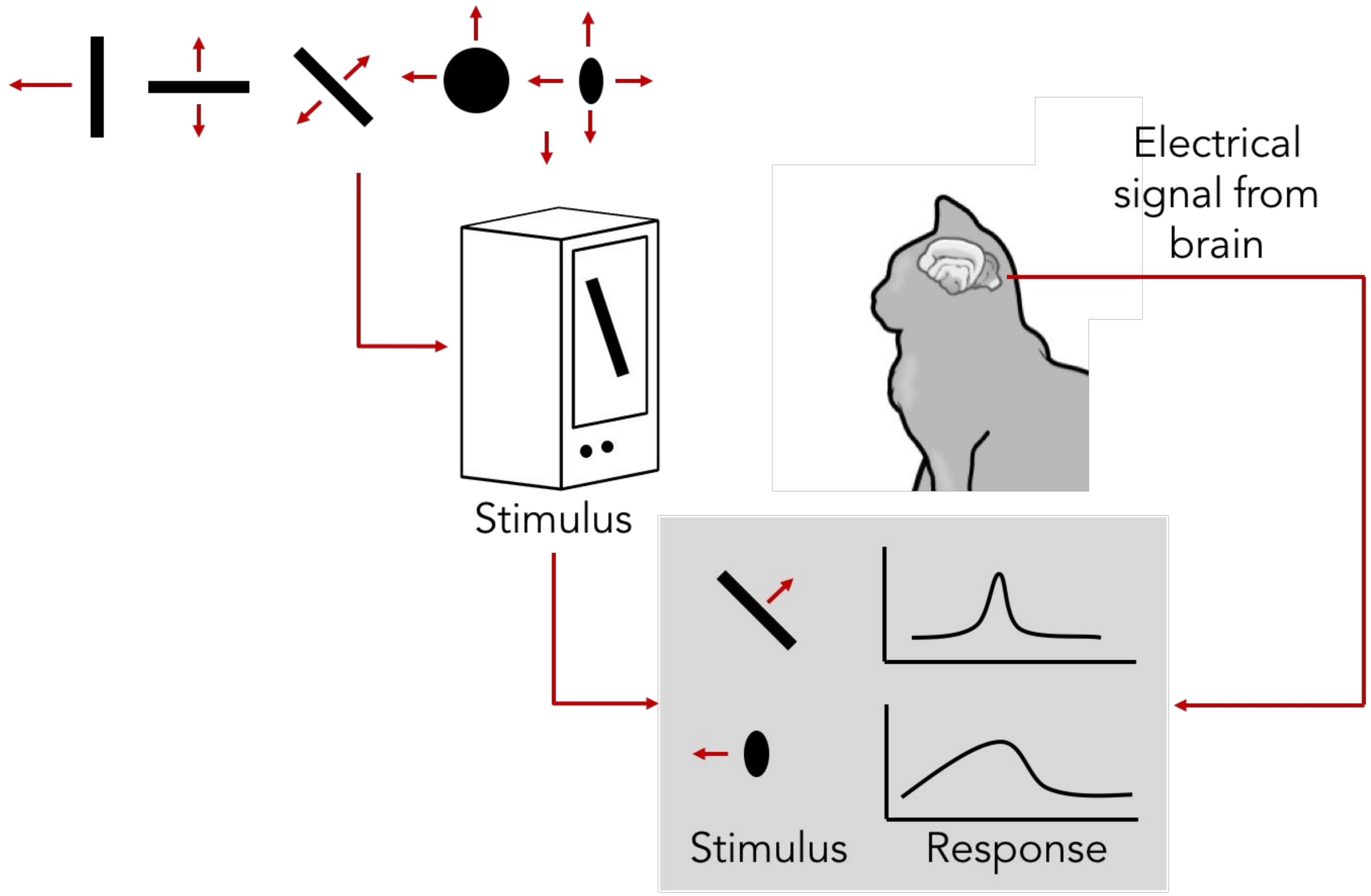
1959

RECEPTIVE FIELDS OF SINGLE NEURONES IN
THE CAT'S STRIATE CORTEX

1962

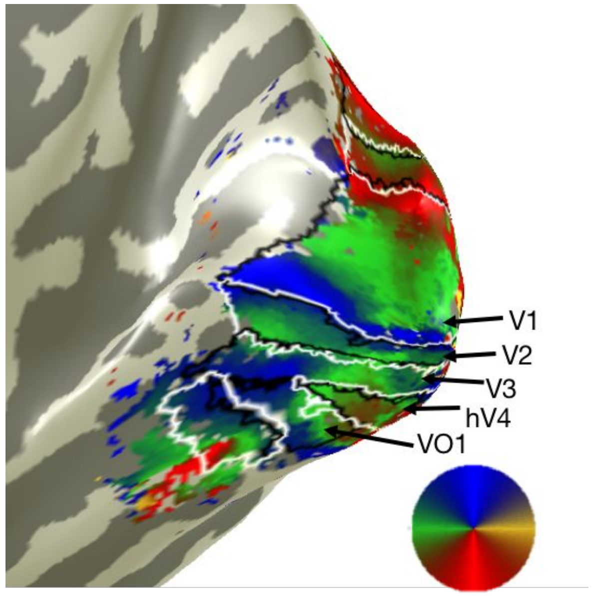
RECEPTIVE FIELDS, BINOCULAR INTERACTION
AND FUNCTIONAL ARCHITECTURE IN THE CAT'S VISUAL CORTEX

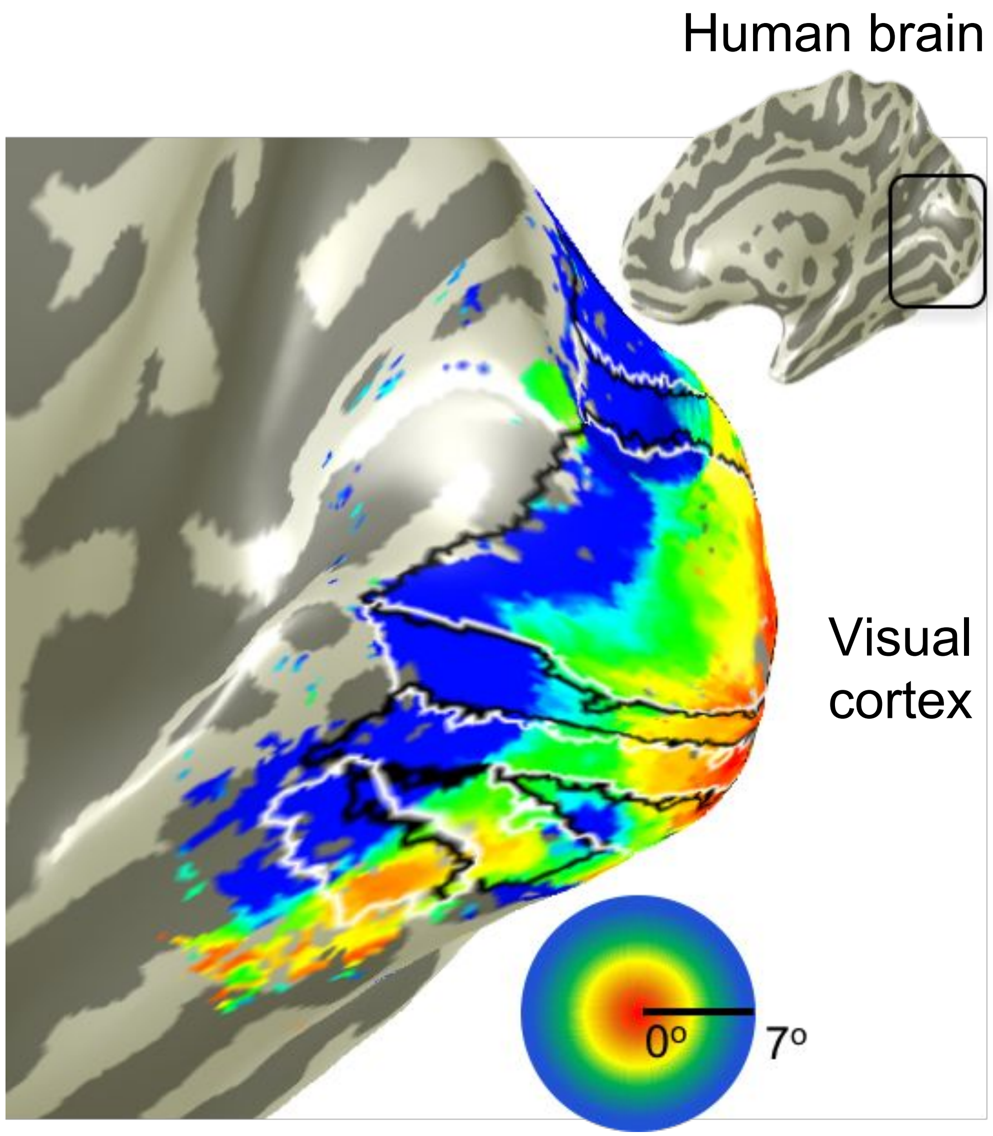
1968...



A bit of history

Topographical mapping in the cortex:
nearby cells in cortex represent
nearby regions in the visual field





Retinotopy images courtesy of Jesse Gomez in the
Stanford Vision & Perception Neuroscience Lab.

Hierarchical organization

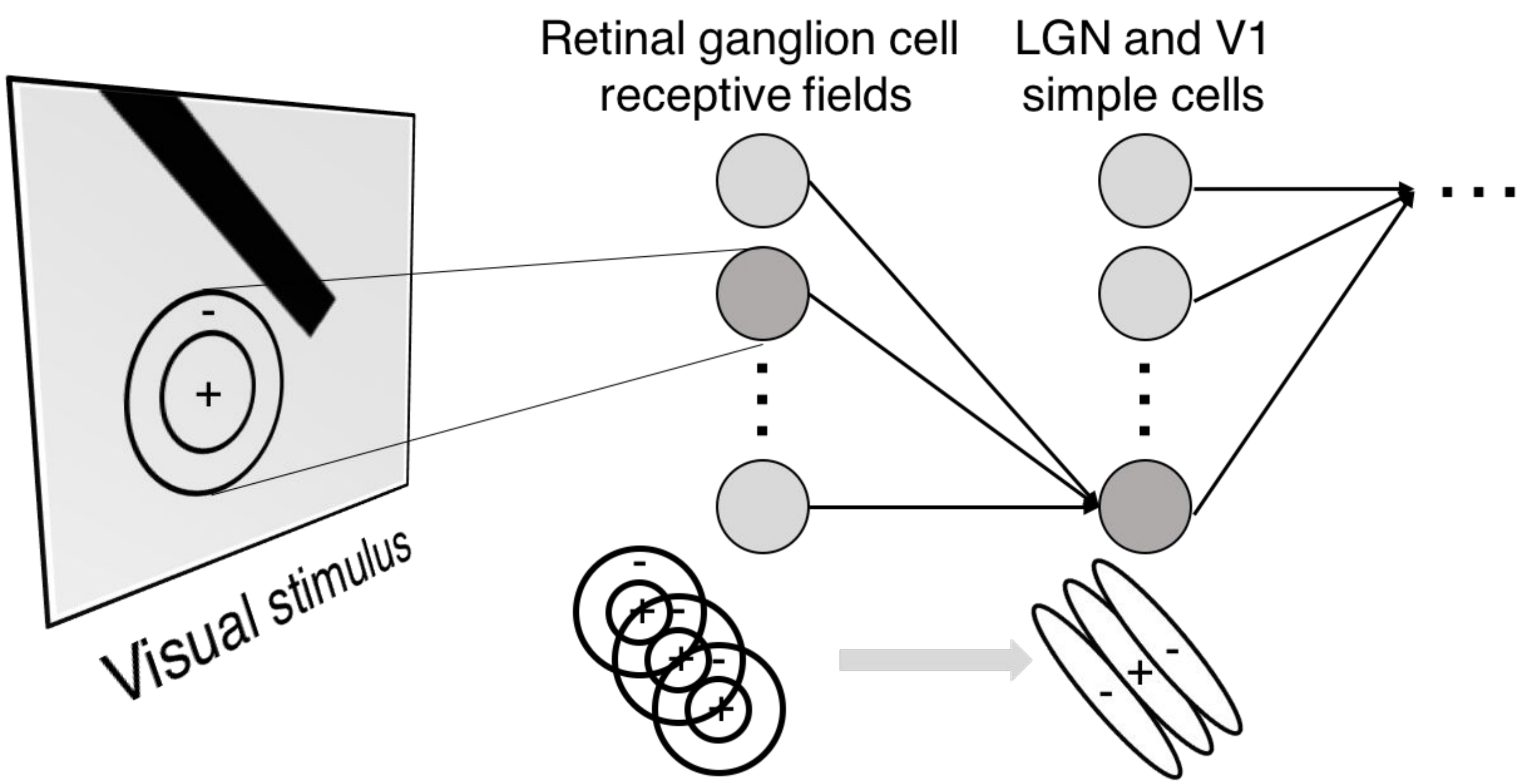
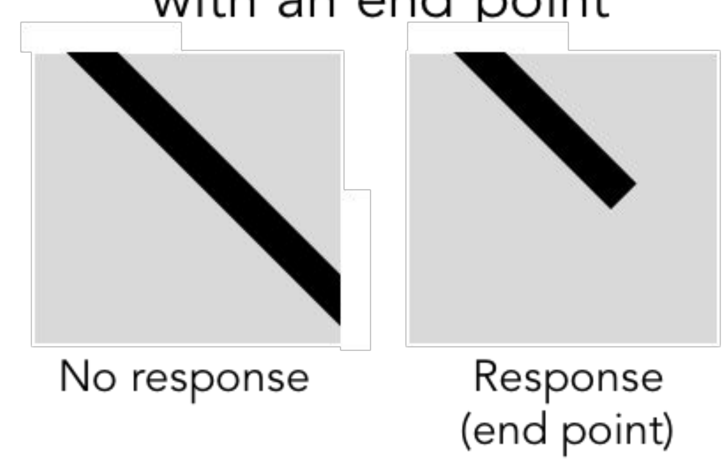


Illustration of hierarchical organization in early visual pathways by Lane McIntosh, copyright CS231n 2017

Simple cells:
Response to light orientation

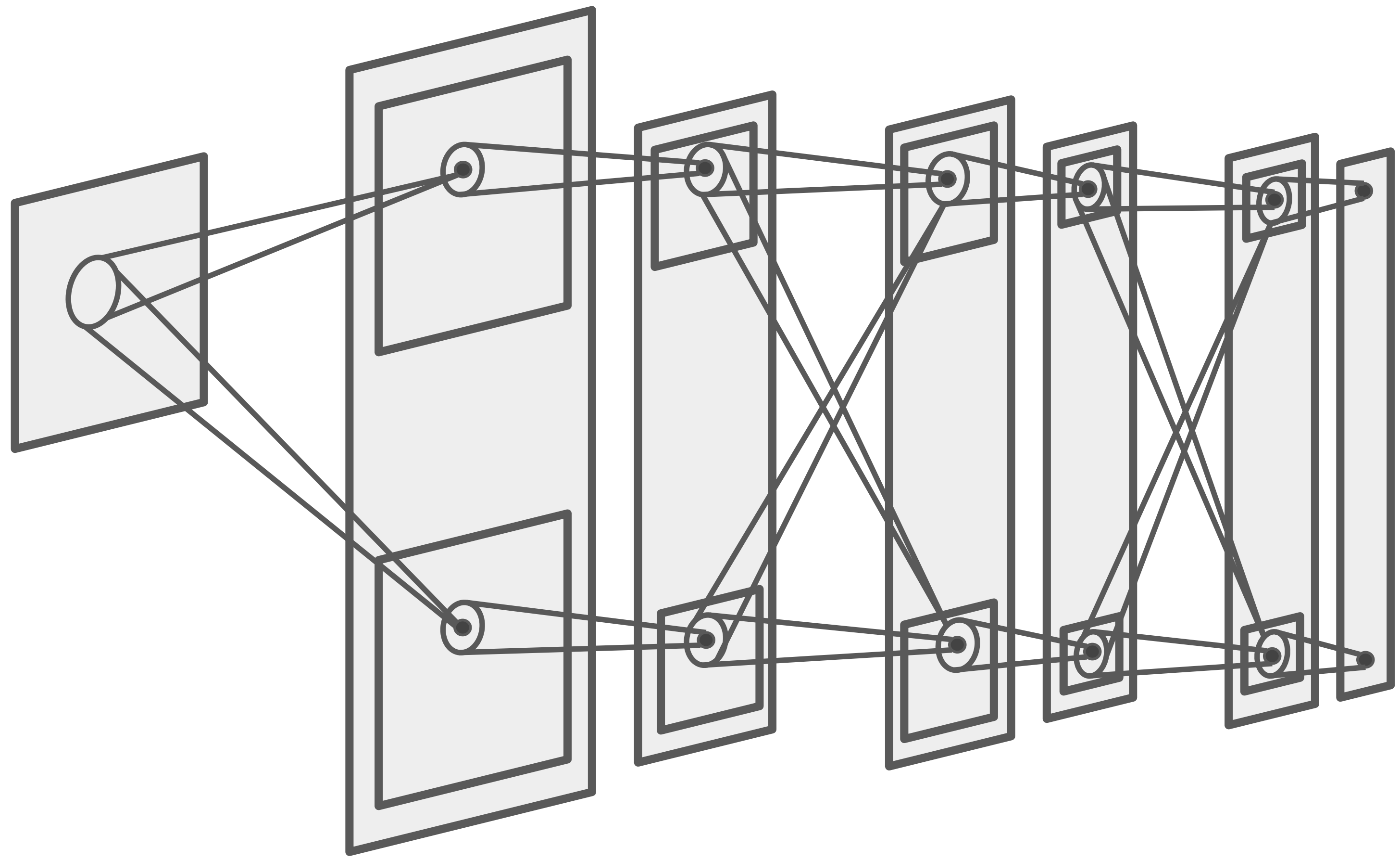
Complex cells:
Response to light orientation and movement

Hypercomplex cells:
response to movement with an end point



A bit of history:

Neocognitron
[Fukushima 1980]

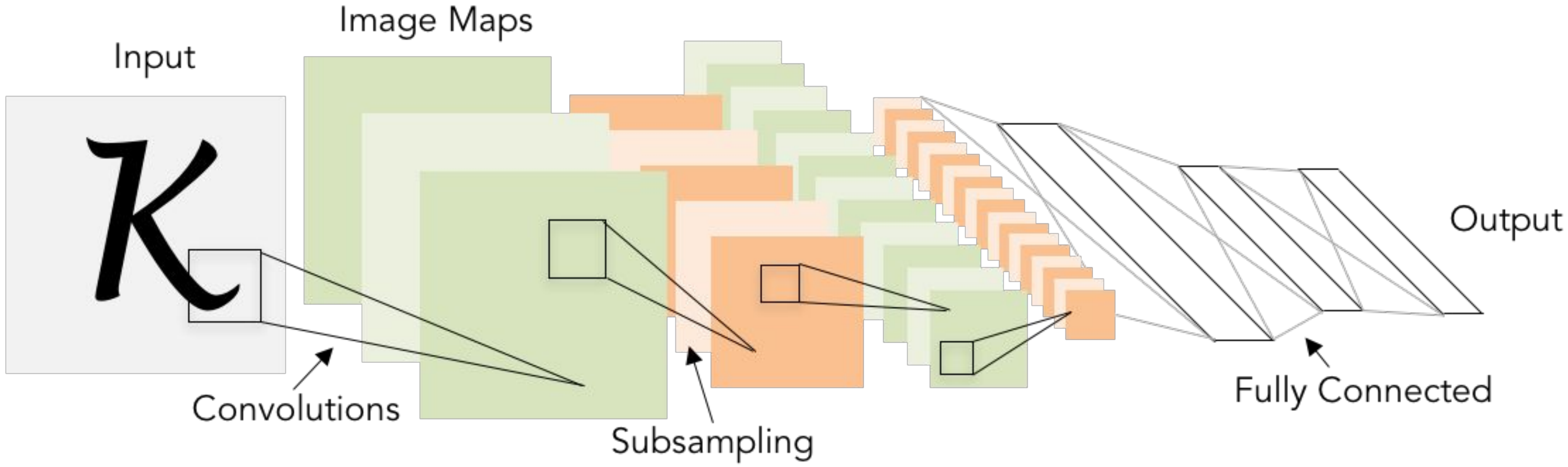


“sandwich” architecture (SCSCSC…)
simple cells: modifiable parameters
complex cells: perform pooling

A bit of history:

Gradient-based learning applied to document recognition

[LeCun, Bottou, Bengio, Haffner 1998]

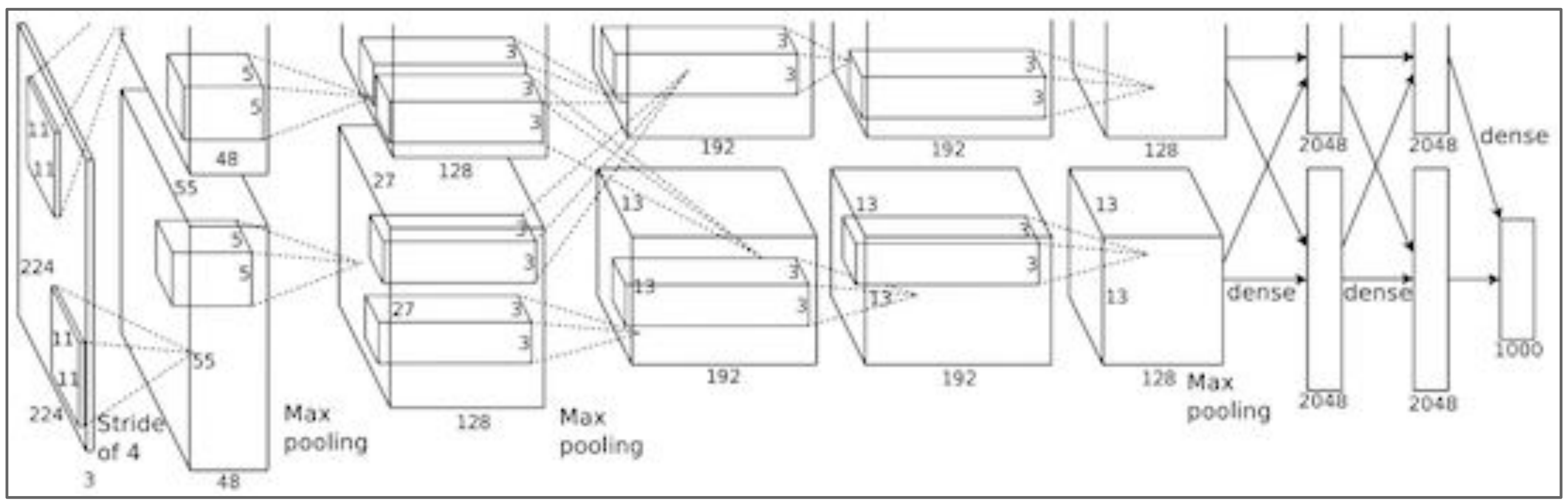


LeNet-5

A bit of history:

ImageNet Classification with Deep Convolutional Neural Networks

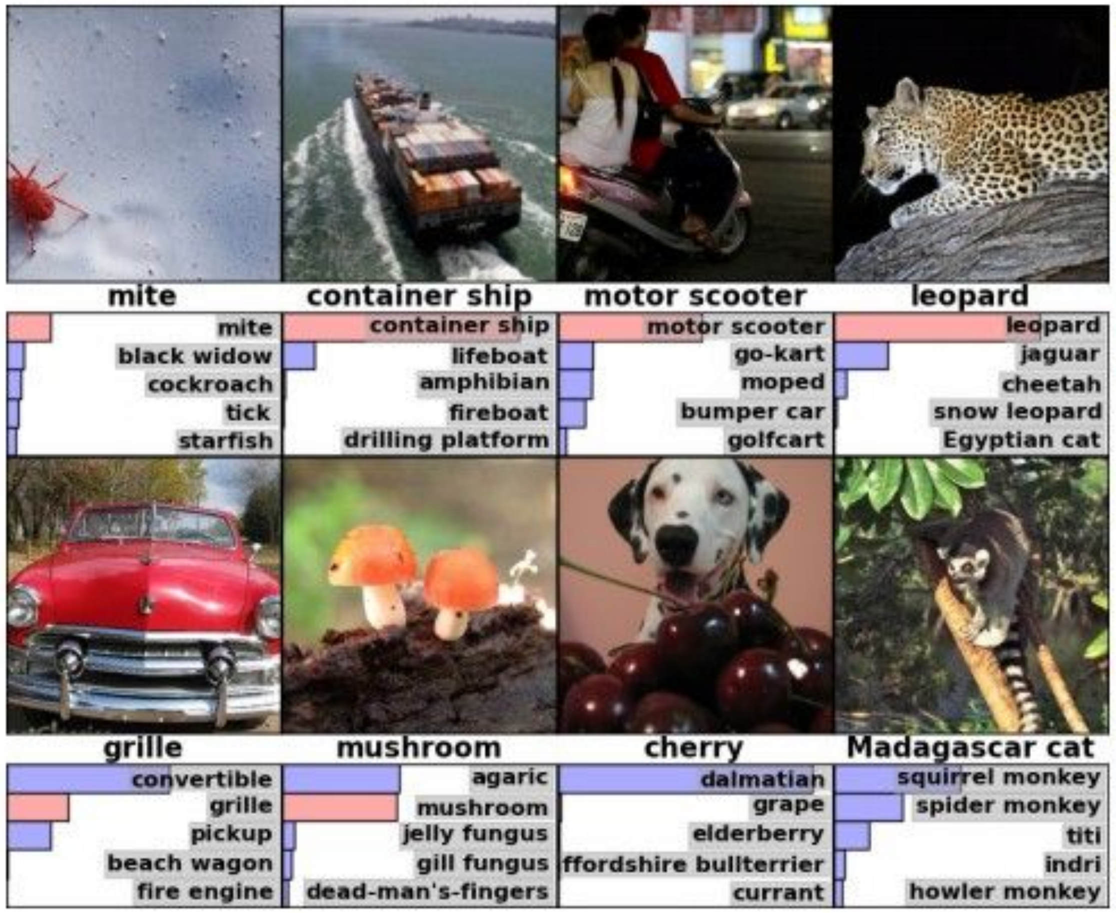
[Krizhevsky, Sutskever, Hinton, 2012]



“AlexNet”

Fast-forward to today: ConvNets are everywhere

Classification

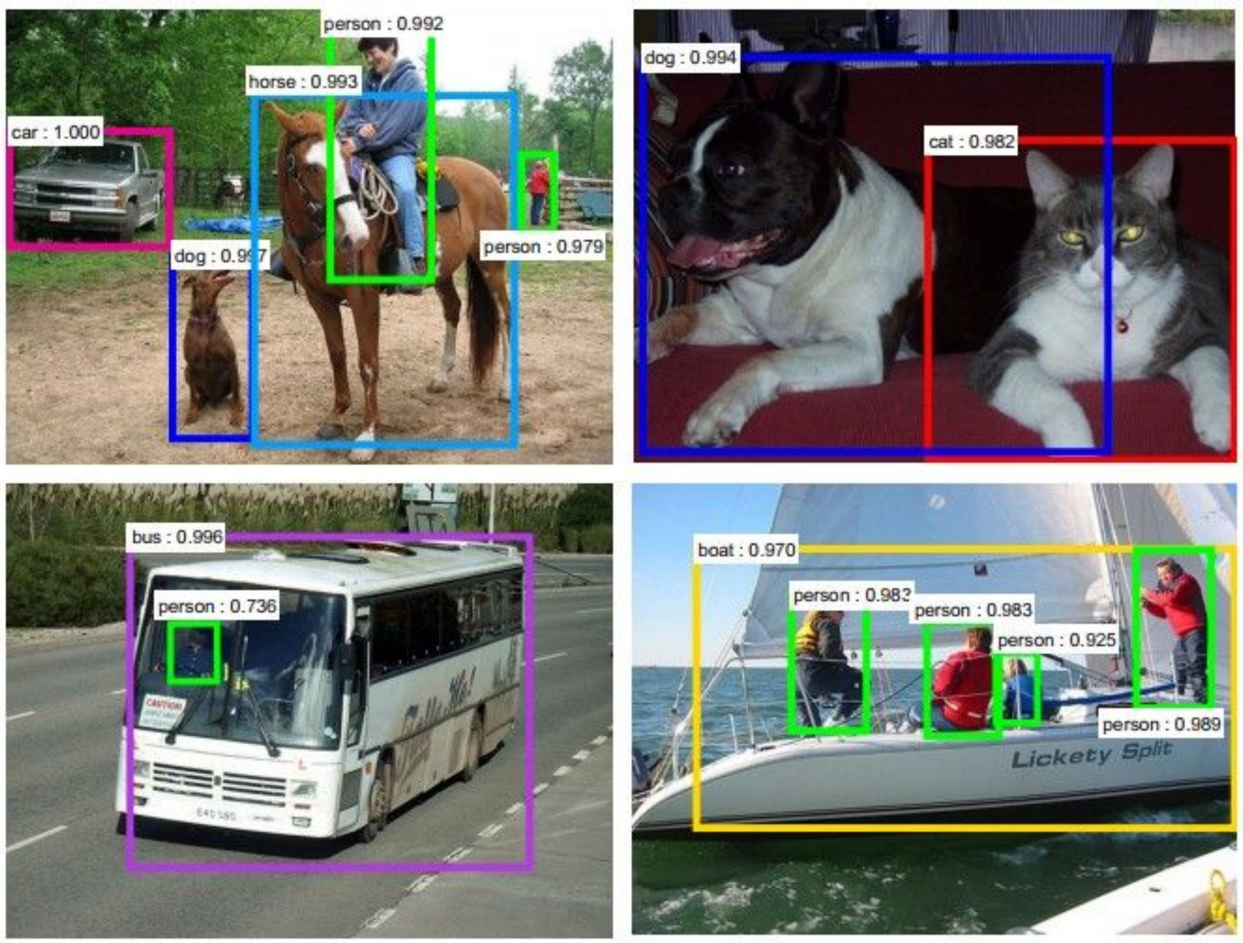


Retrieval

Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

Fast-forward to today: ConvNets are everywhere

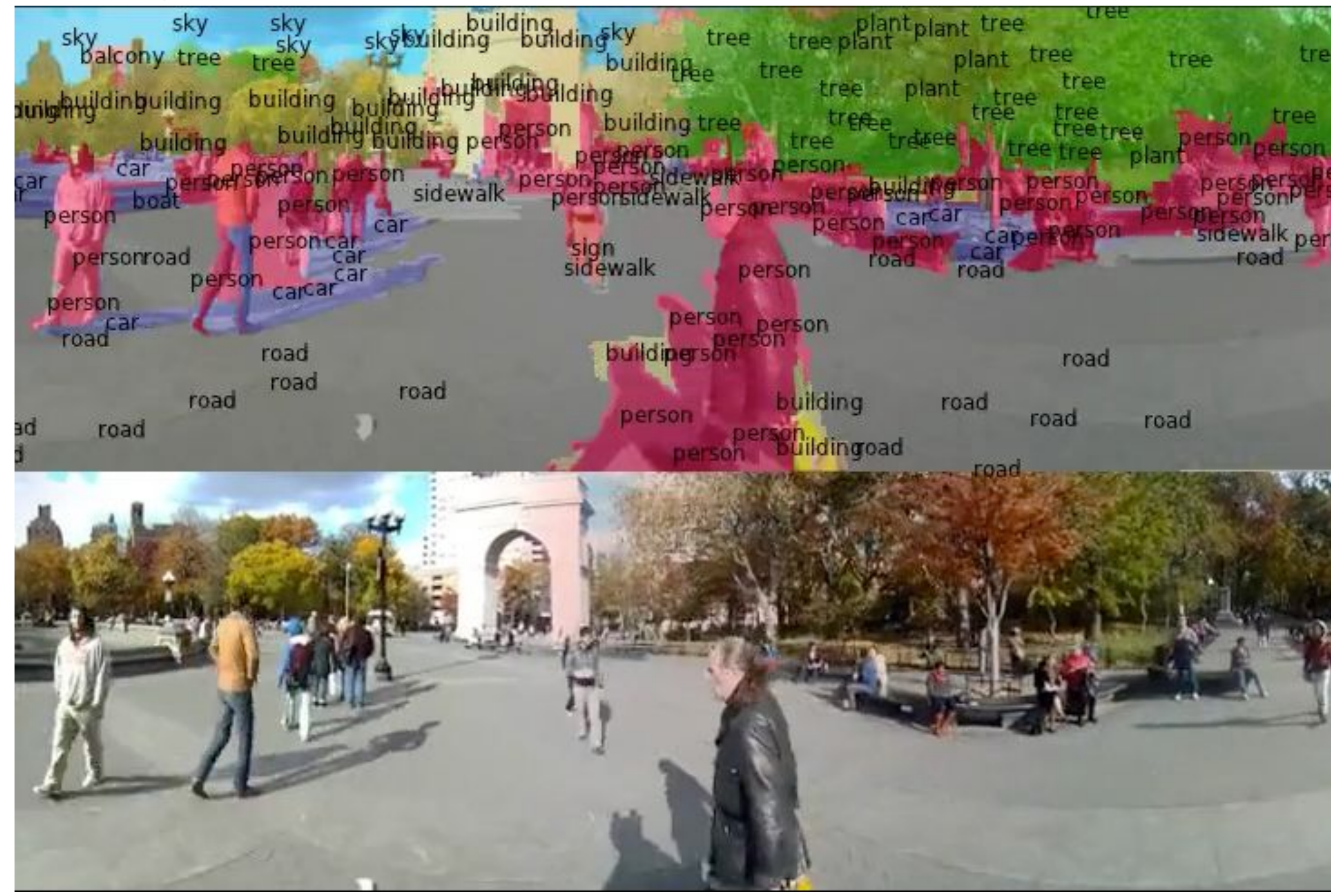
Detection



Figures copyright Shaoqing Ren, Kaiming He, Ross Girschick, Jian Sun, 2015. Reproduced with permission.

[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Segmentation



Figures copyright Clement Farabet, 2012. Reproduced with permission.

[Farabet et al., 2012]

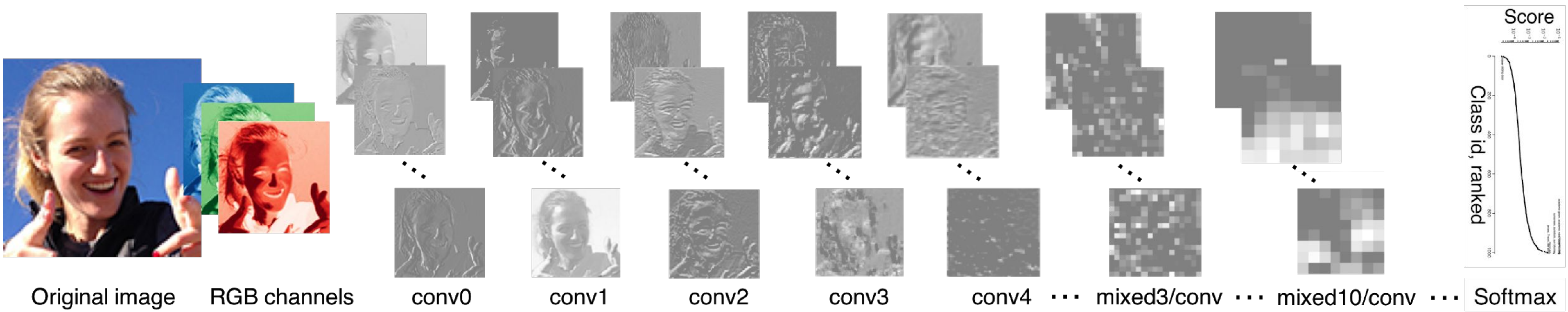
Fast-forward to today: ConvNets are everywhere

self-driving cars

NVIDIA Tesla line
(these are the GPUs on rye01.stanford.edu)

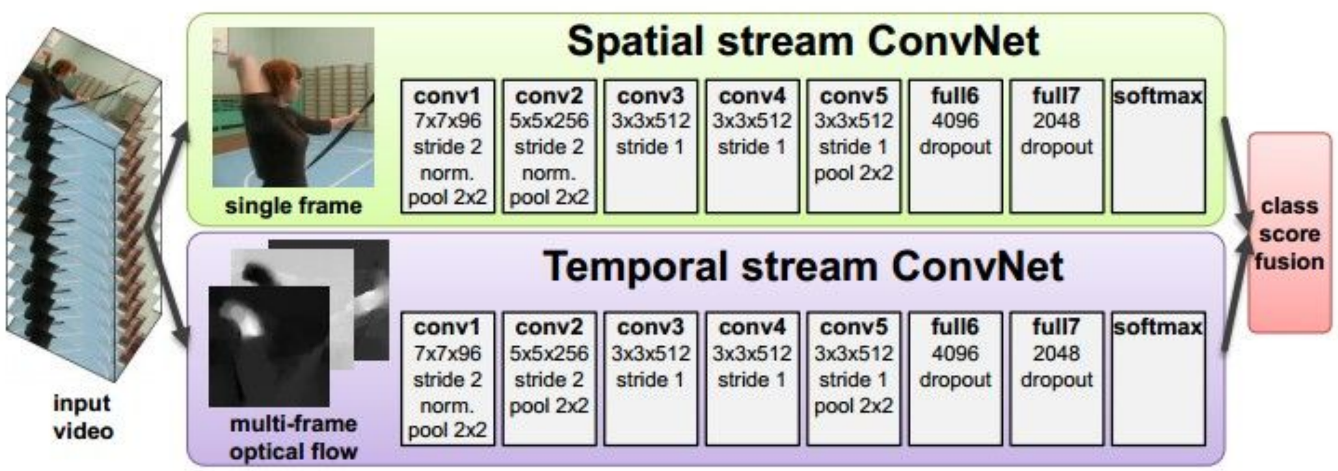
Note that for embedded systems a typical setup would involve NVIDIA Tegras, with integrated GPU and ARM-based CPU cores.

Fast-forward to today: ConvNets are everywhere



[Taigman et al. 2014]

Activations of inception-v3 architecture [Szegedy et al. 2015] to image of Emma McIntosh, used with permission. Figure and architecture not from Taigman et al. 2014.



[Simonyan et al. 2014]

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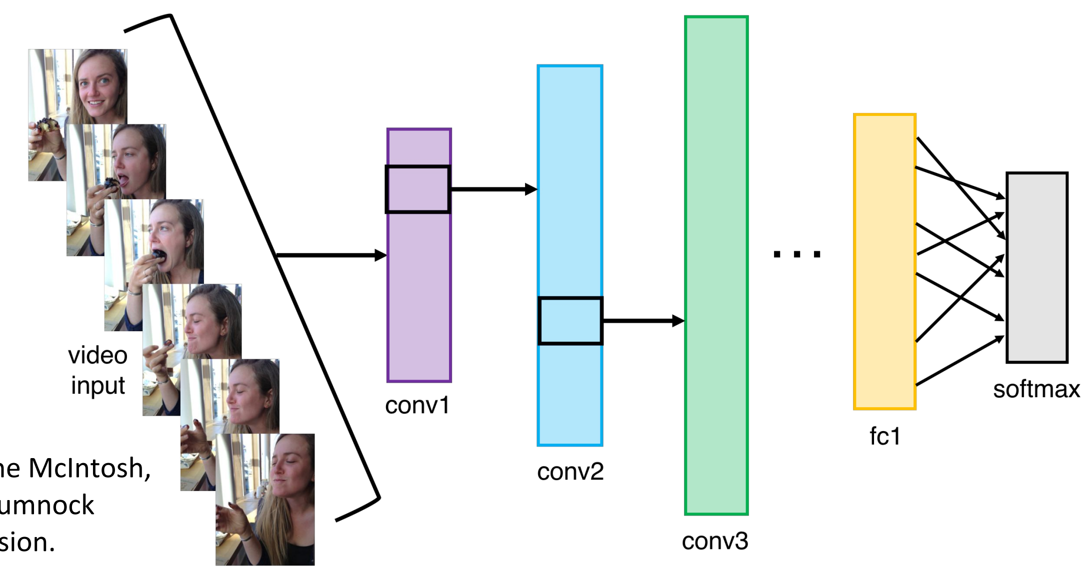
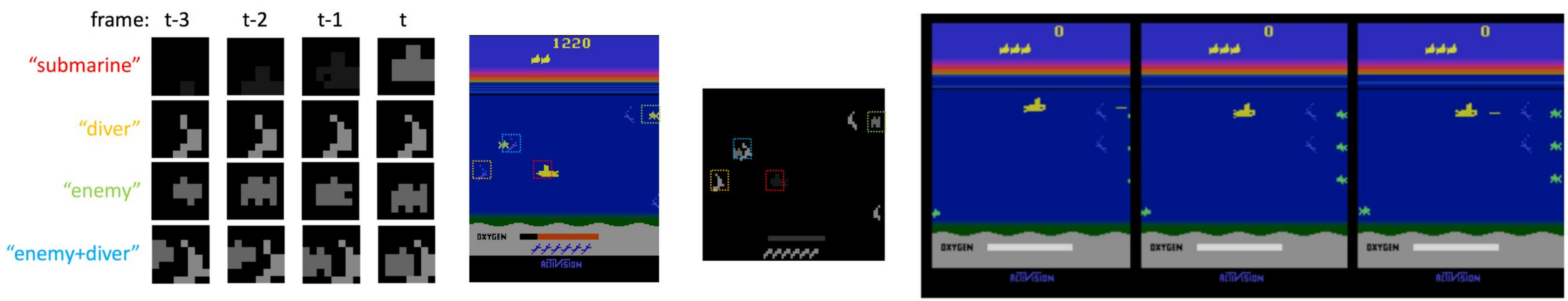


Illustration by Lane McIntosh, photos of Katie Cumnock used with permission.

Fast-forward to today: ConvNets are everywhere

[Toshev, Szegedy 2014]

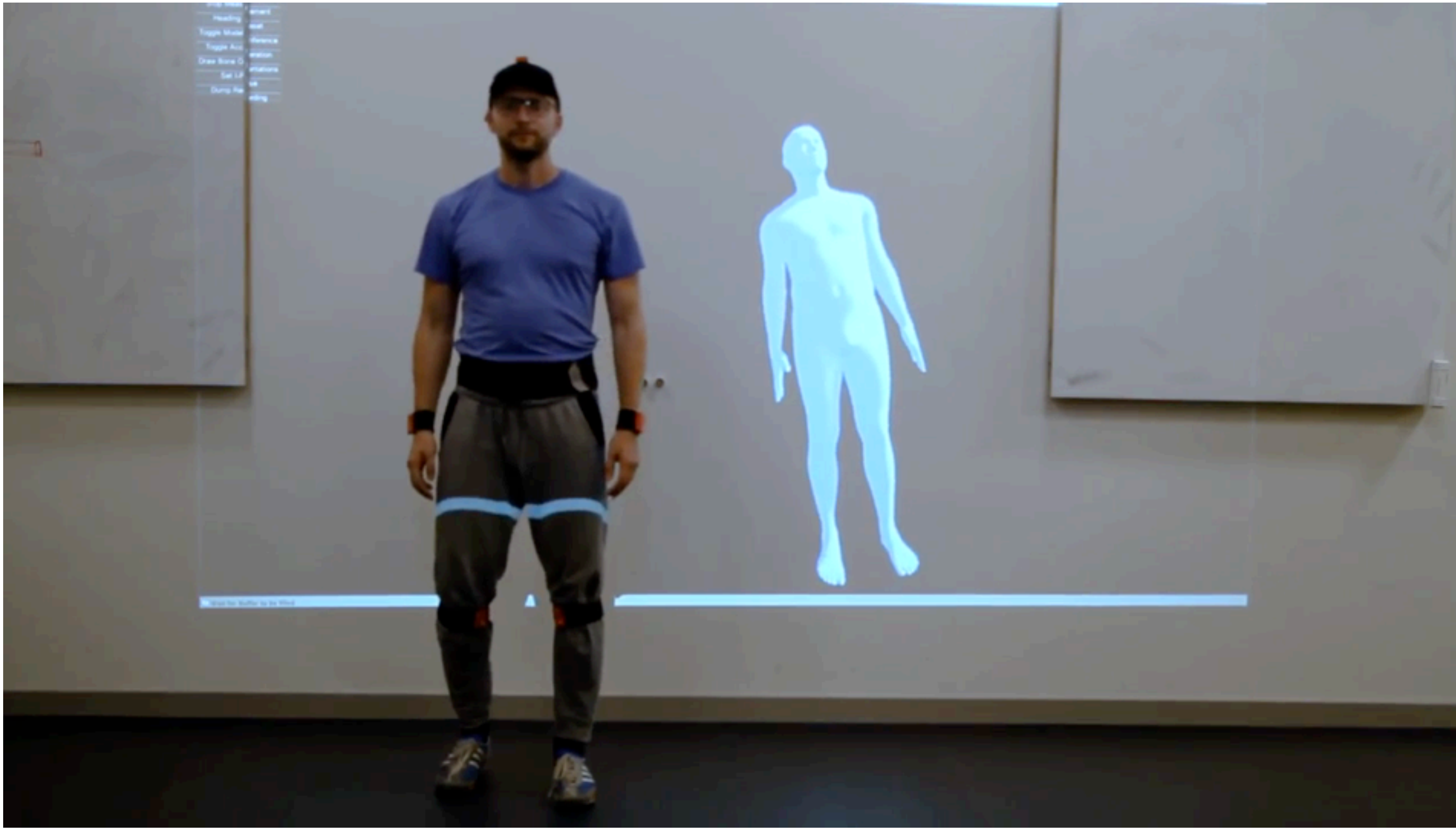


[Guo et al. 2014]

Fast forward to today: ConvNets are everywhere

Omran et al. 2018

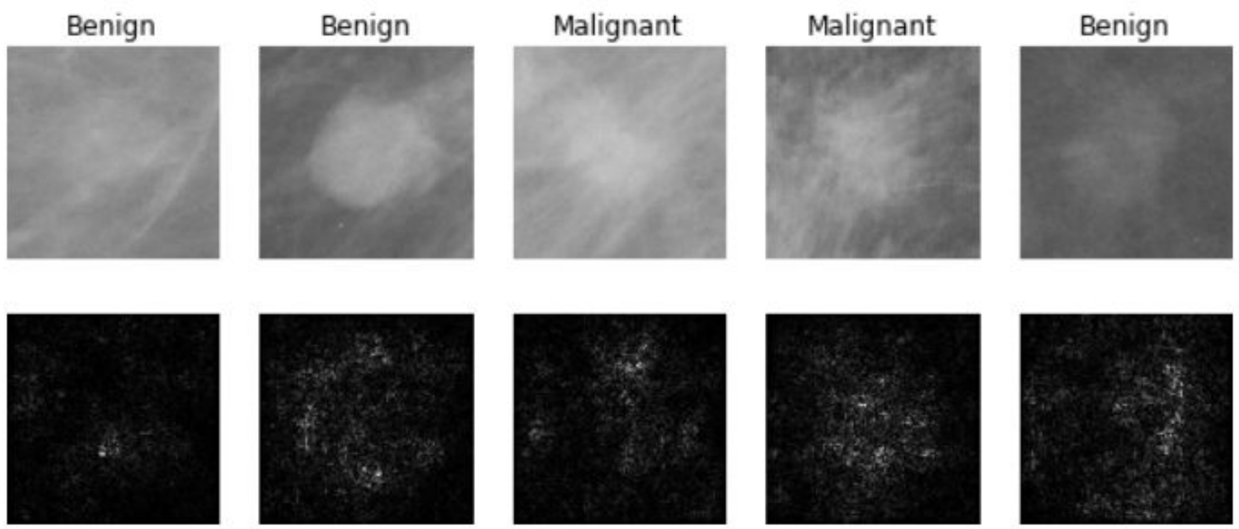
Fast forward to today: ConvNets are everywhere



Huang et al. 2018

Fast forward to today: ConvNets are everywhere

Fast-forward to today: ConvNets are everywhere

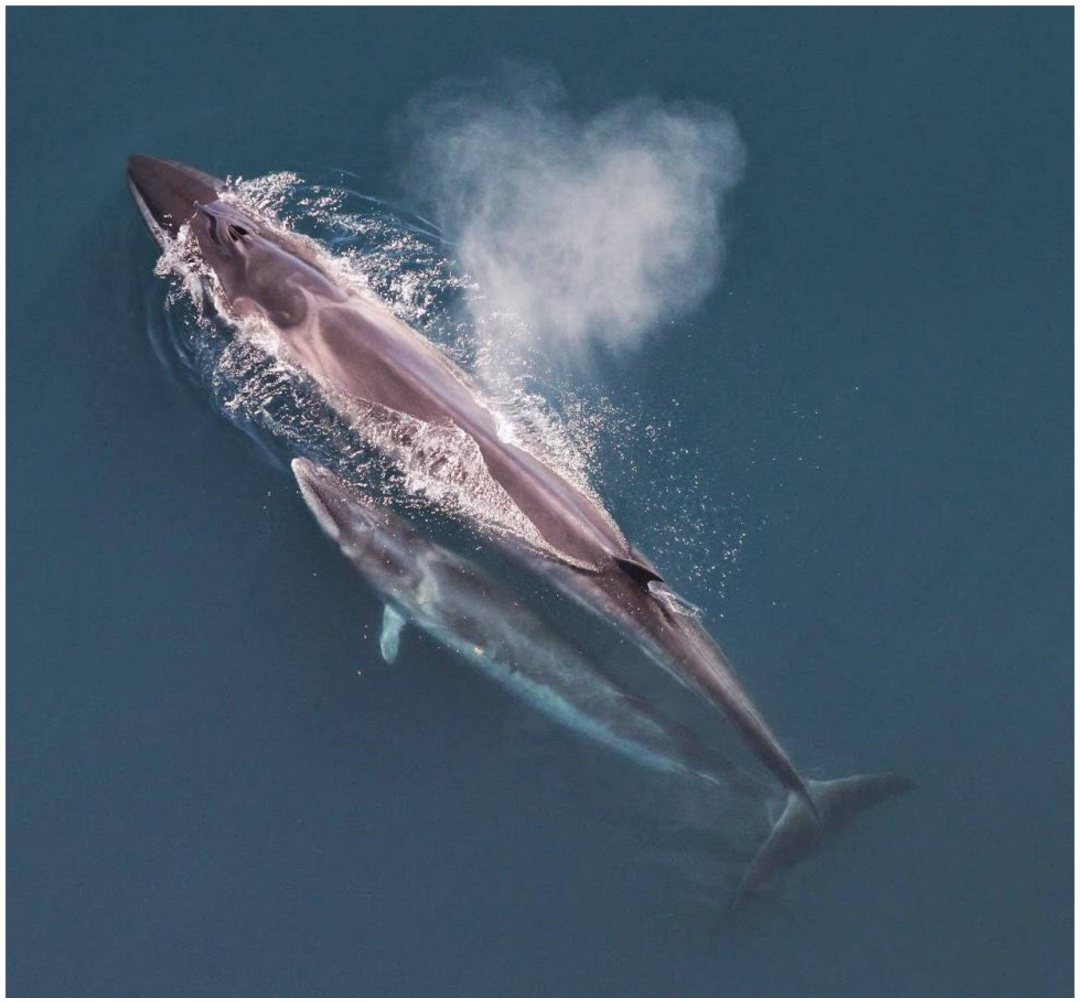


[Levy et al. 2016]

[Dieleman et al. 2014]

[Sermanet et al. 2011]
[Ciresan et al.]

This image by Christin Khan is in the public domain and originally came from the U.S. NOAA.



Whale recognition, Kaggle Challenge

Photo and figure by Lane McIntosh; not actual example from Mnih and Hinton, 2010 paper.

Mnih and Hinton, 2010

Image Captioning

[Vinyals et al., 2015]
[Karpathy and Fei-Fei, 2015]

No errors

A white teddy bear sitting in the grass

A man riding a wave on top of a surfboard

Minor errors

A man in a baseball uniform throwing a ball

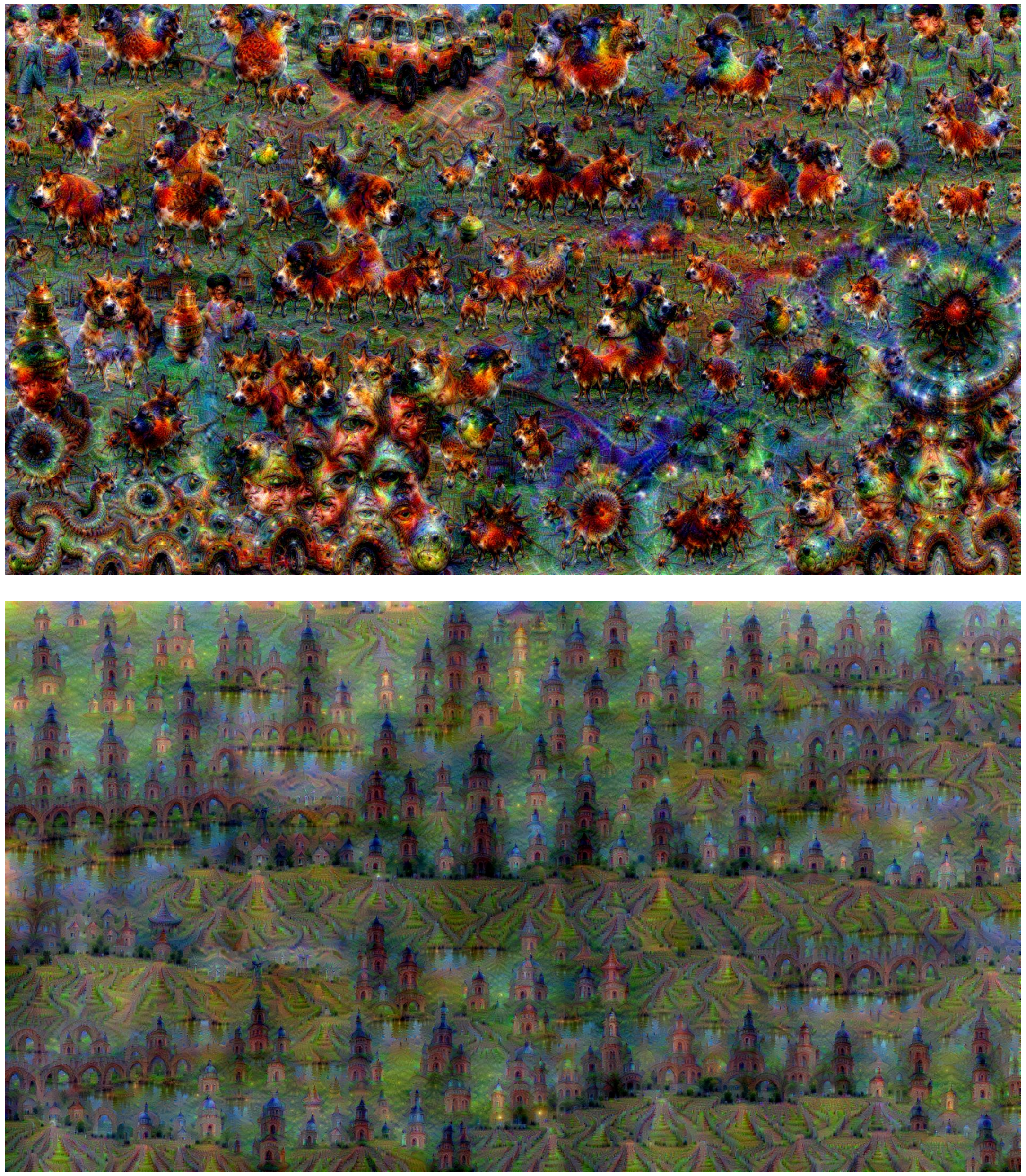
A cat sitting on a suitcase on the floor

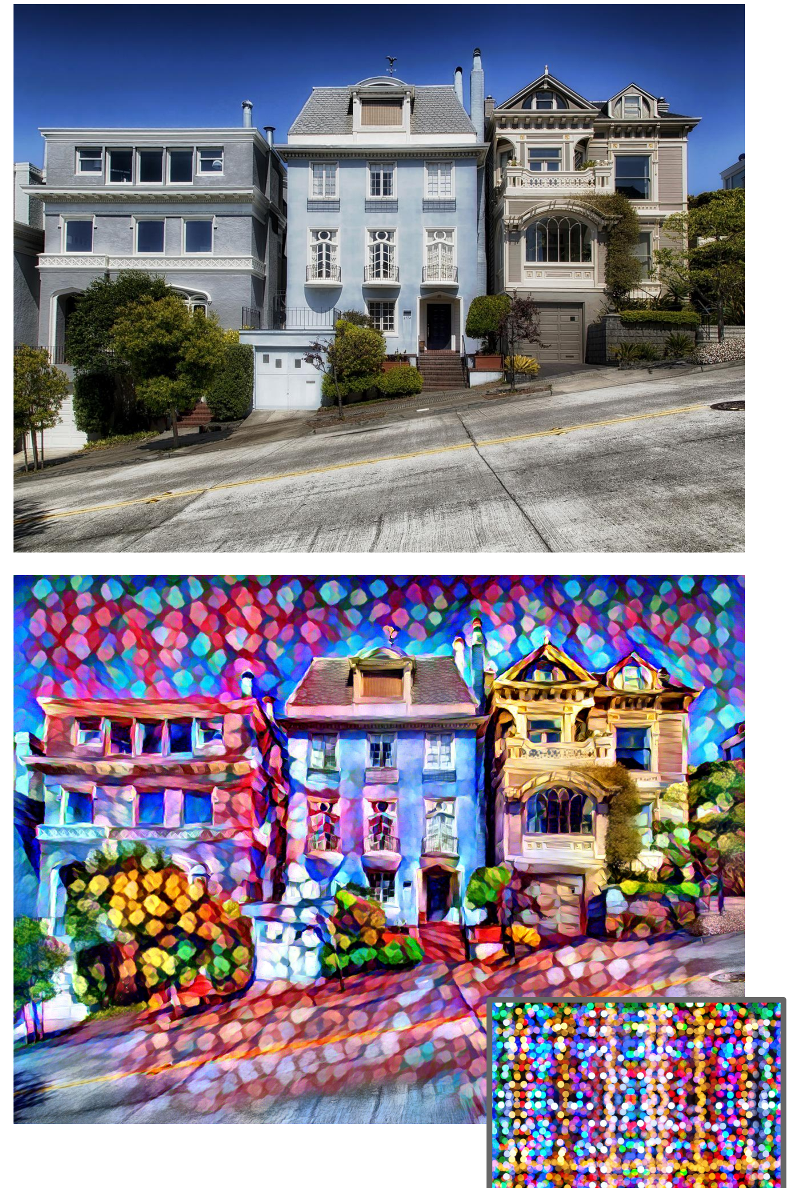
Somewhat related

A woman is holding a cat in her hand

A woman standing on a beach holding a surfboard

Captions generated by Justin Johnson using Neuraltalk2





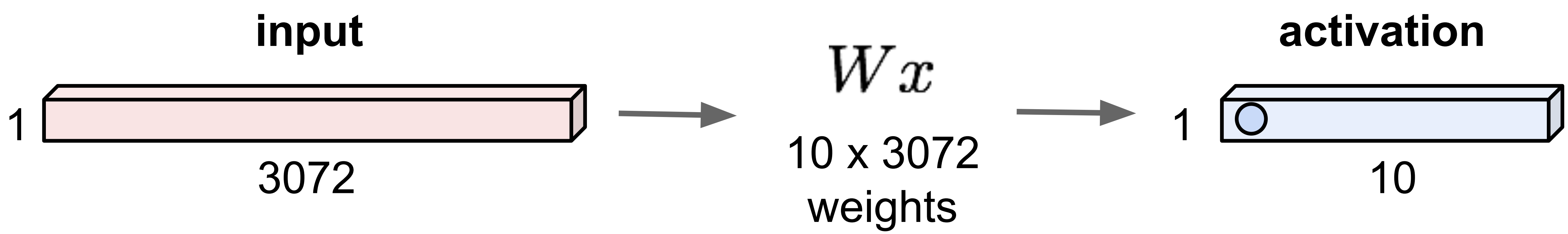
Gatys et al, "Image Style Transfer using Convolutional Neural Networks", CVPR 2016
Gatys et al, "Controlling Perceptual Factors in Neural Style Transfer", CVPR 2017

Convolutional Neural Networks

(First without the brain stuff)

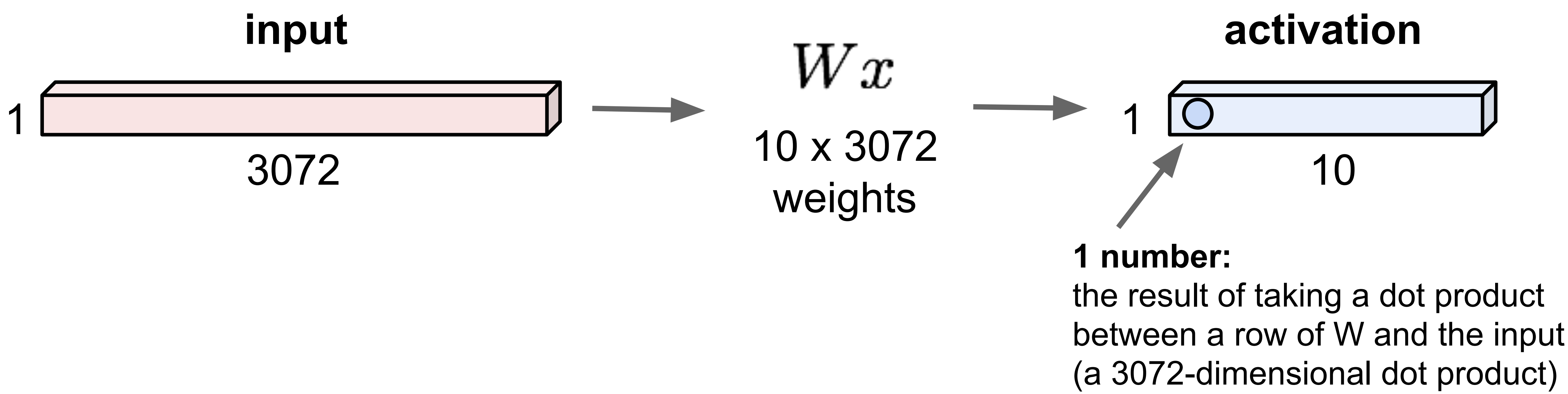
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



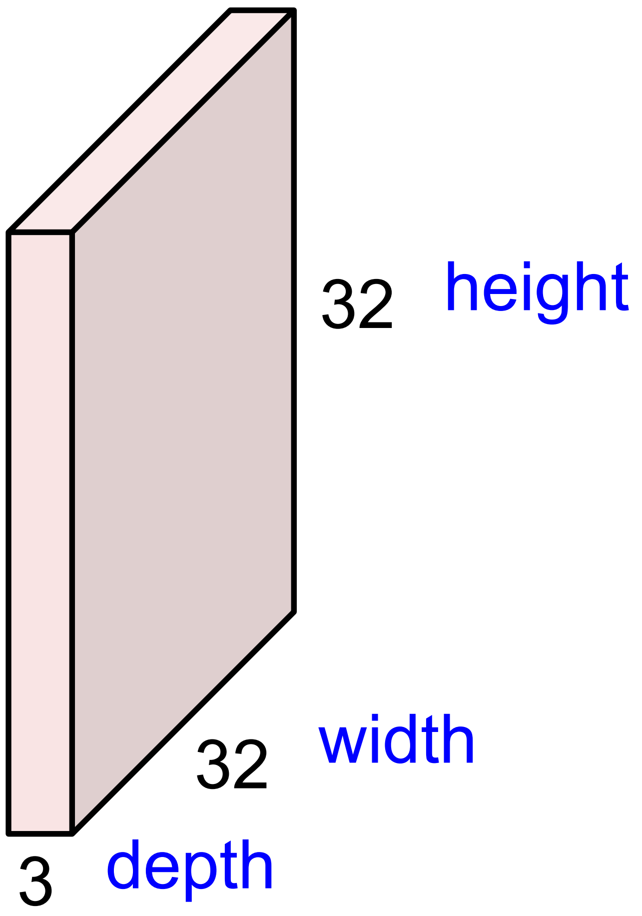
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



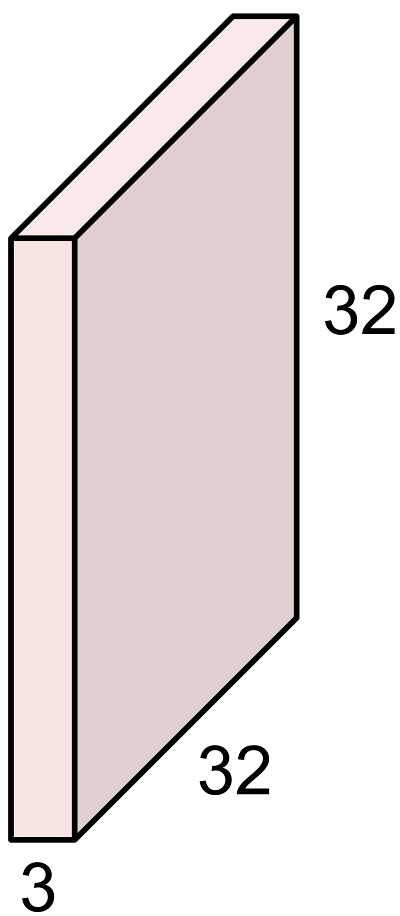
Convolution Layer

32x32x3 image -> preserve spatial structure

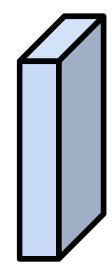


Convolution Layer

32x32x3 image



5x5x3 filter

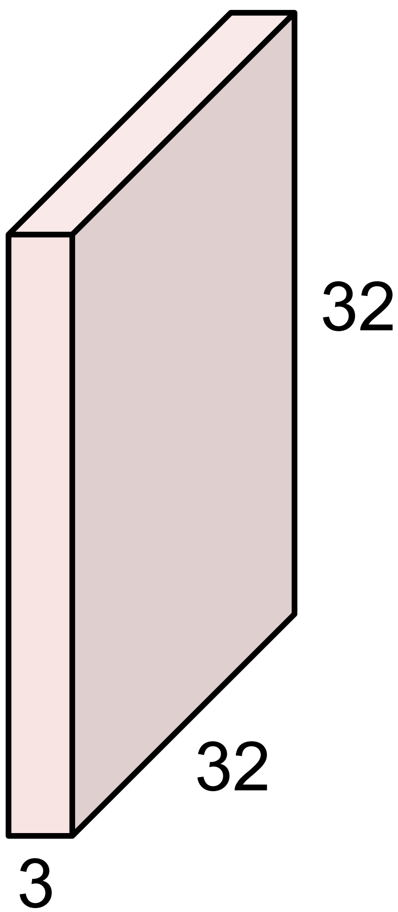


Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

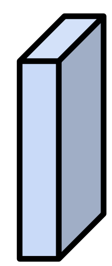
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image



5x5x3 filter



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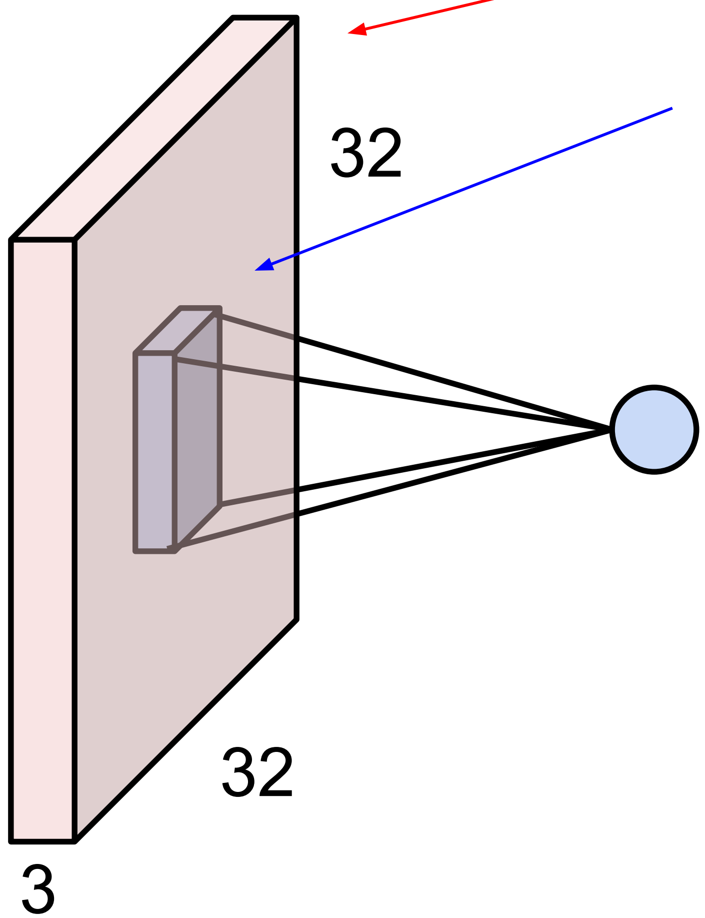
Convolution Layer



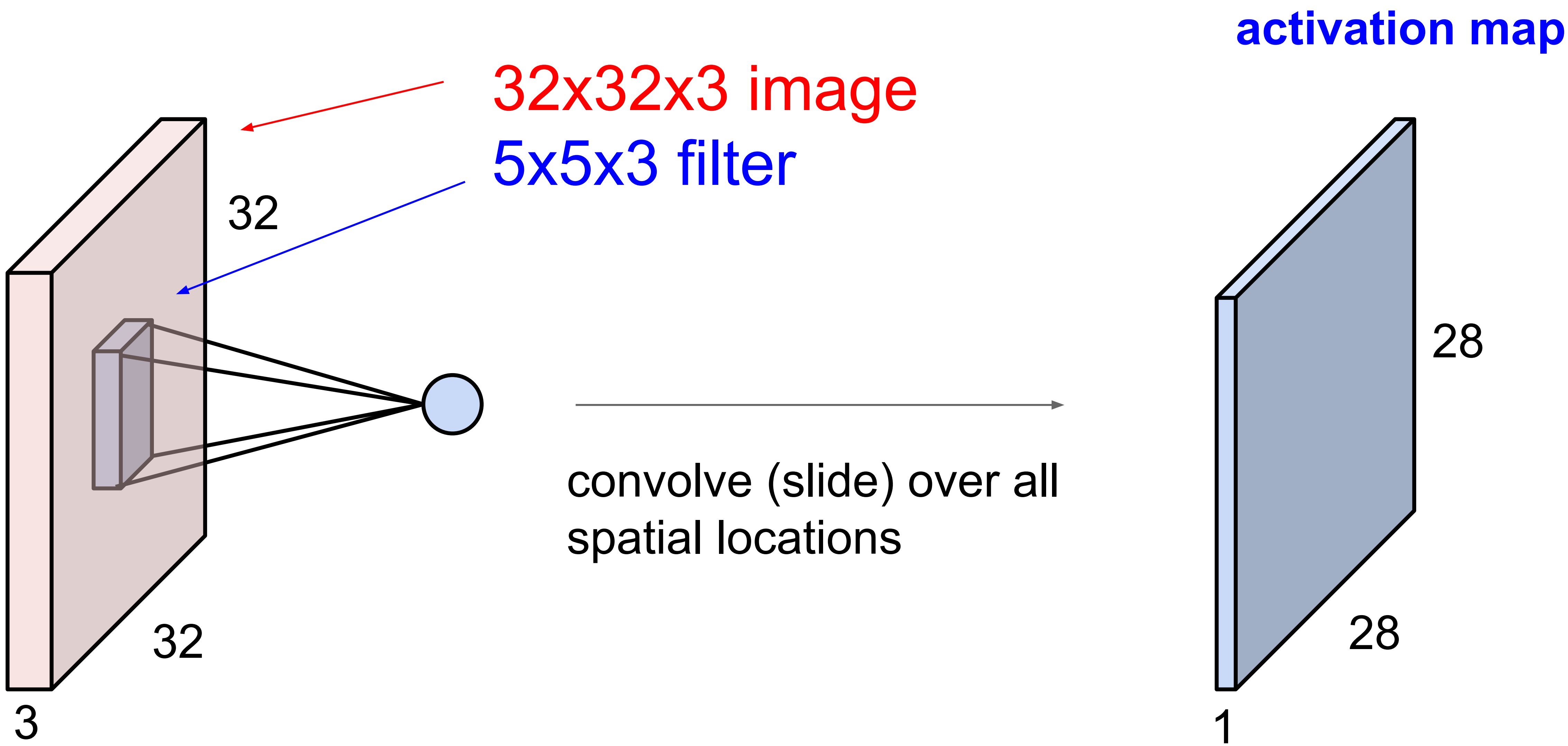
32x32x3 image

5x5x3 filter w

1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

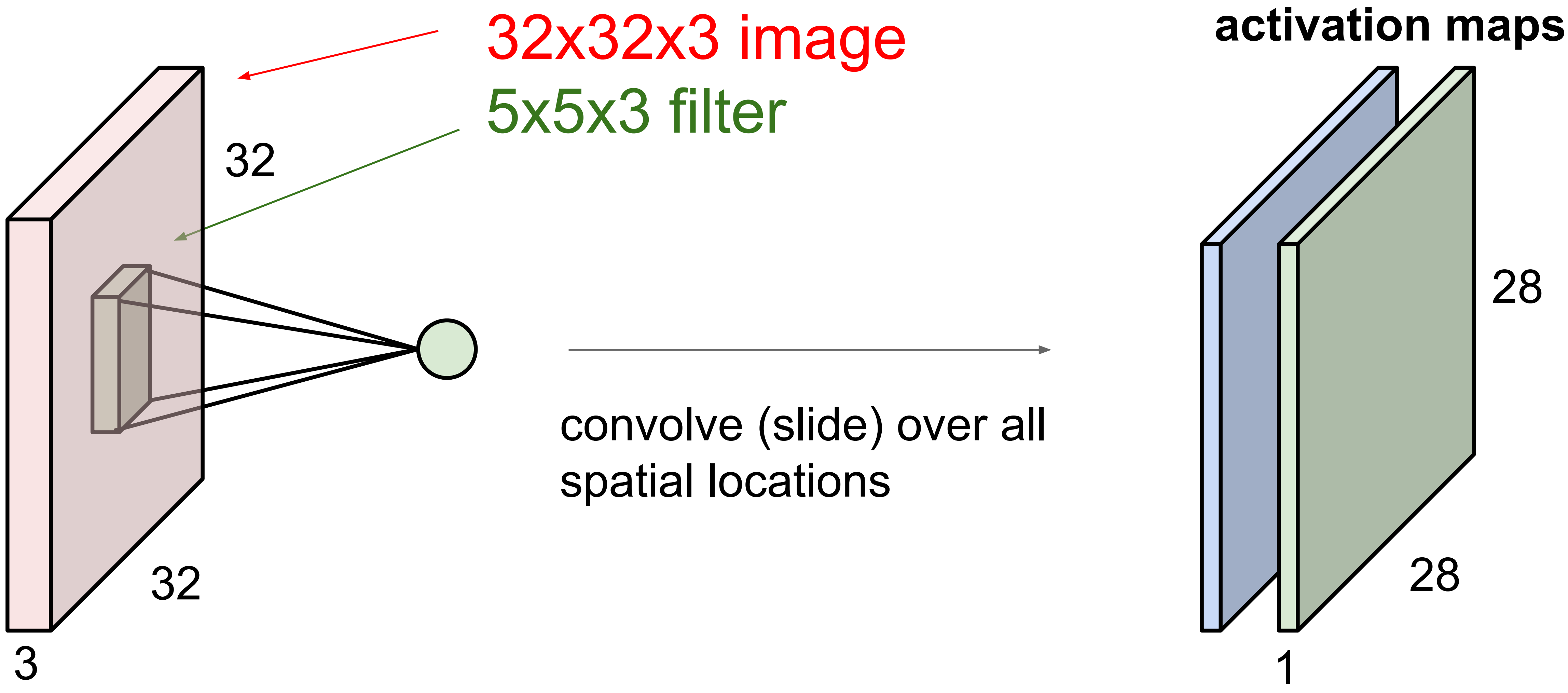
$$w^T x + b$$

Convolution Layer

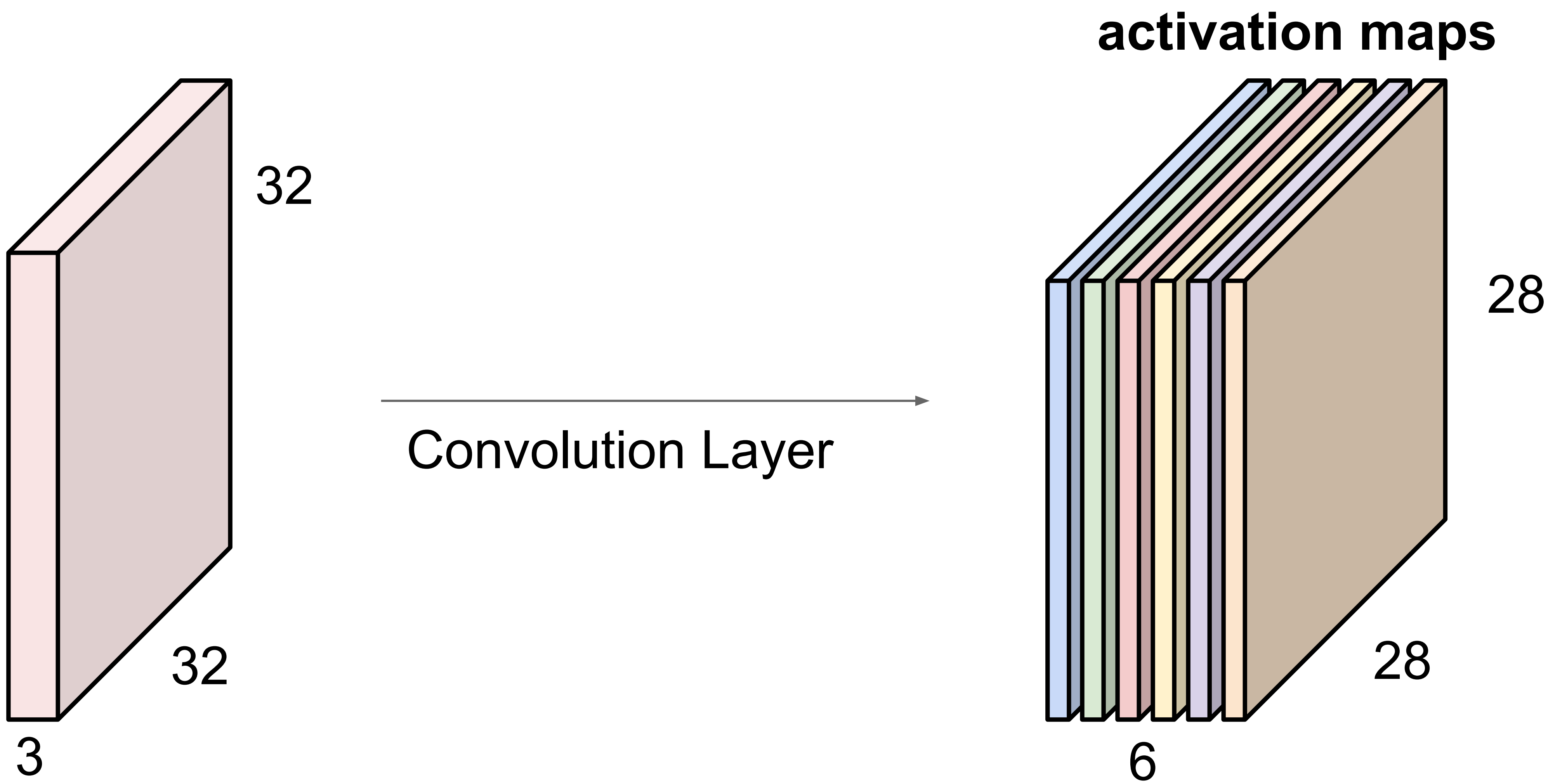


Convolution Layer

consider a second, green filter

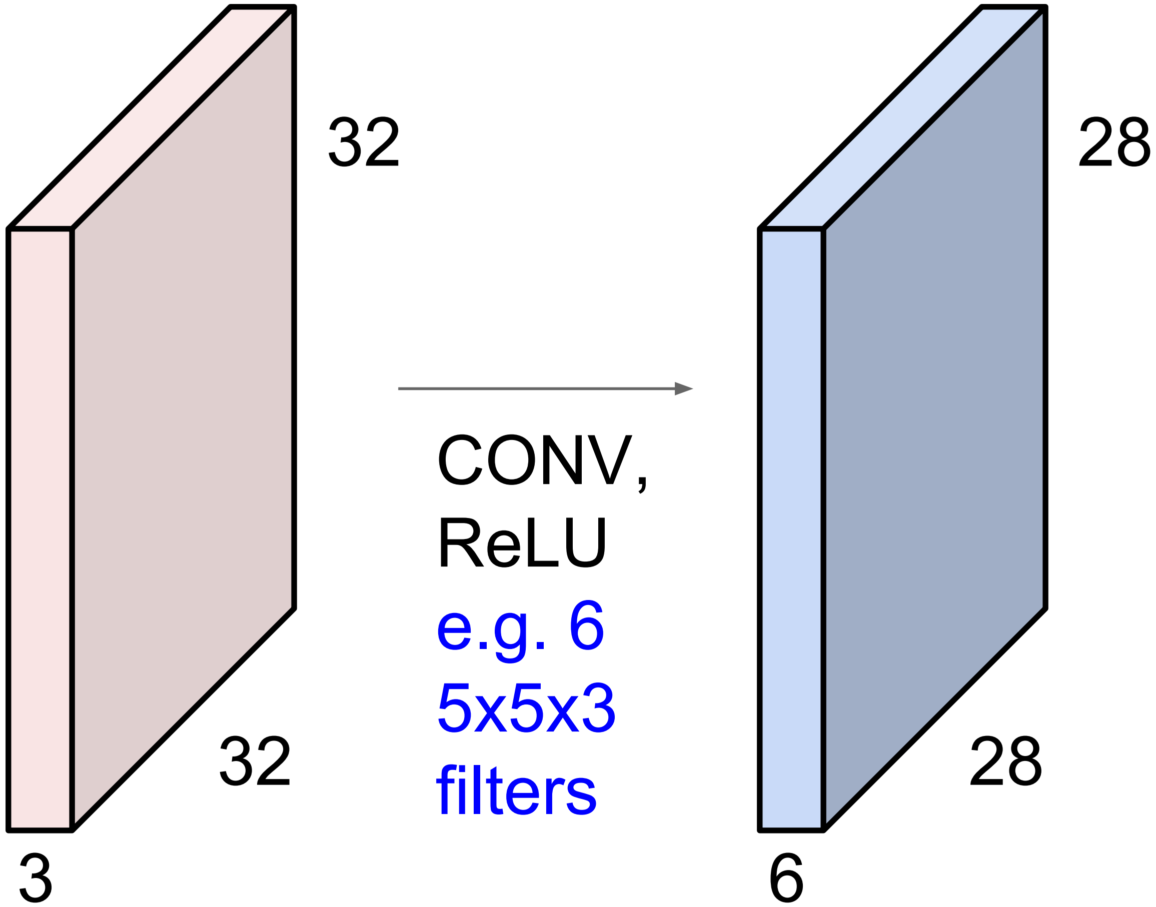


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

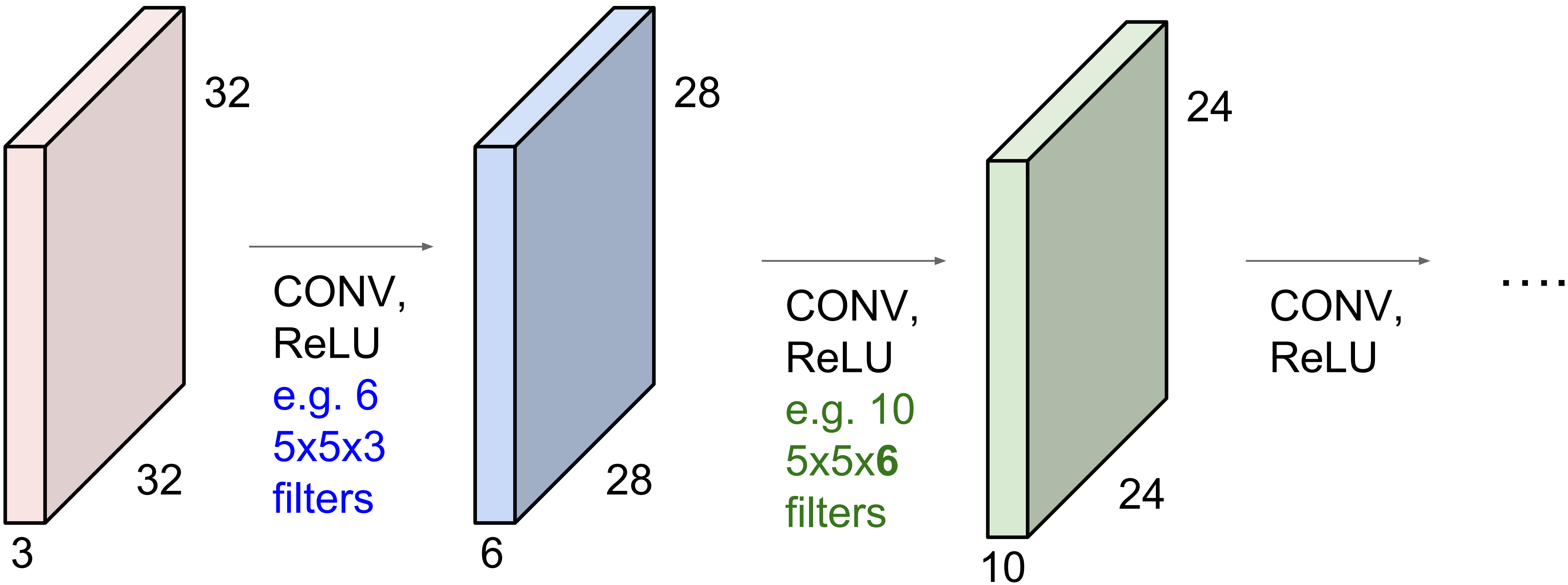


We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



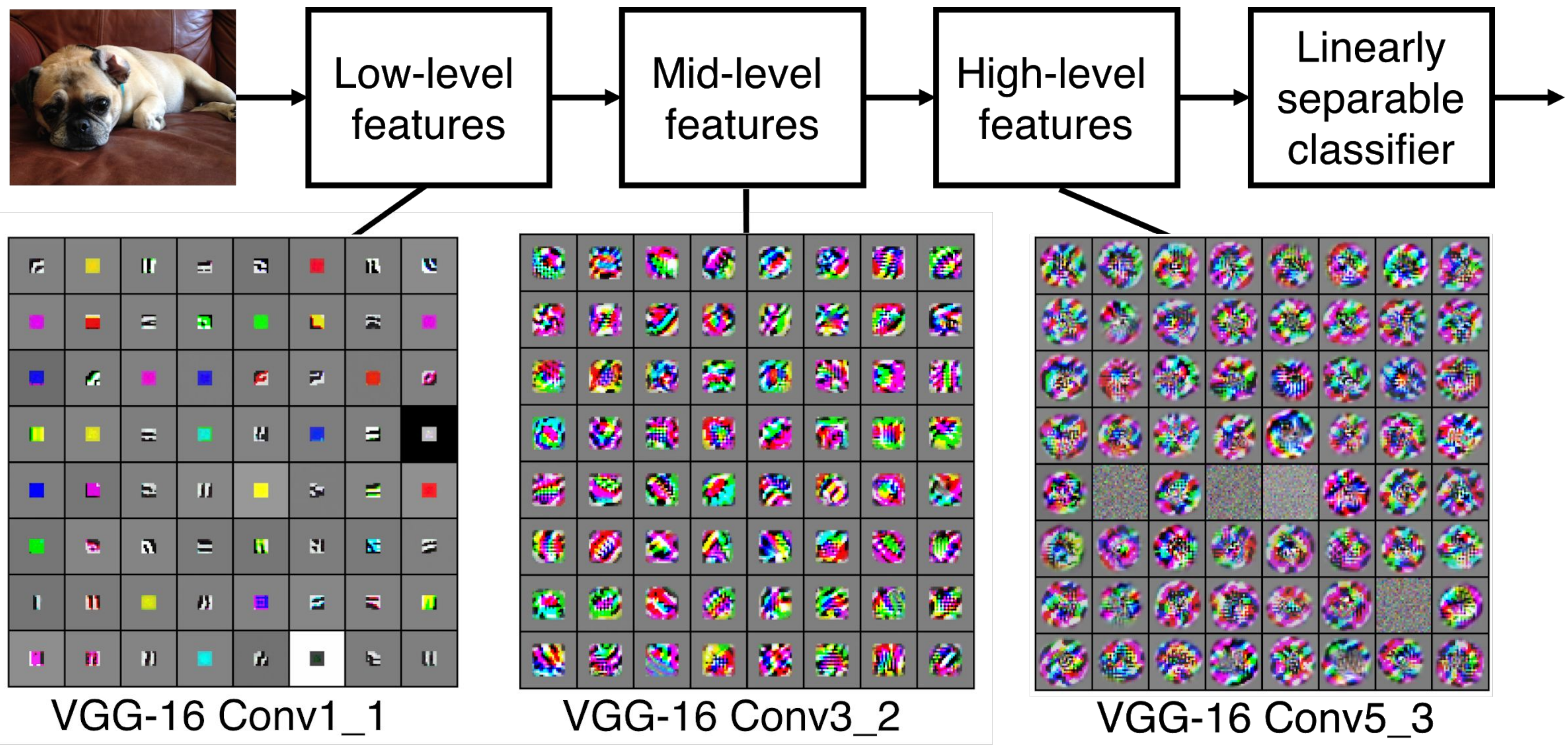
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



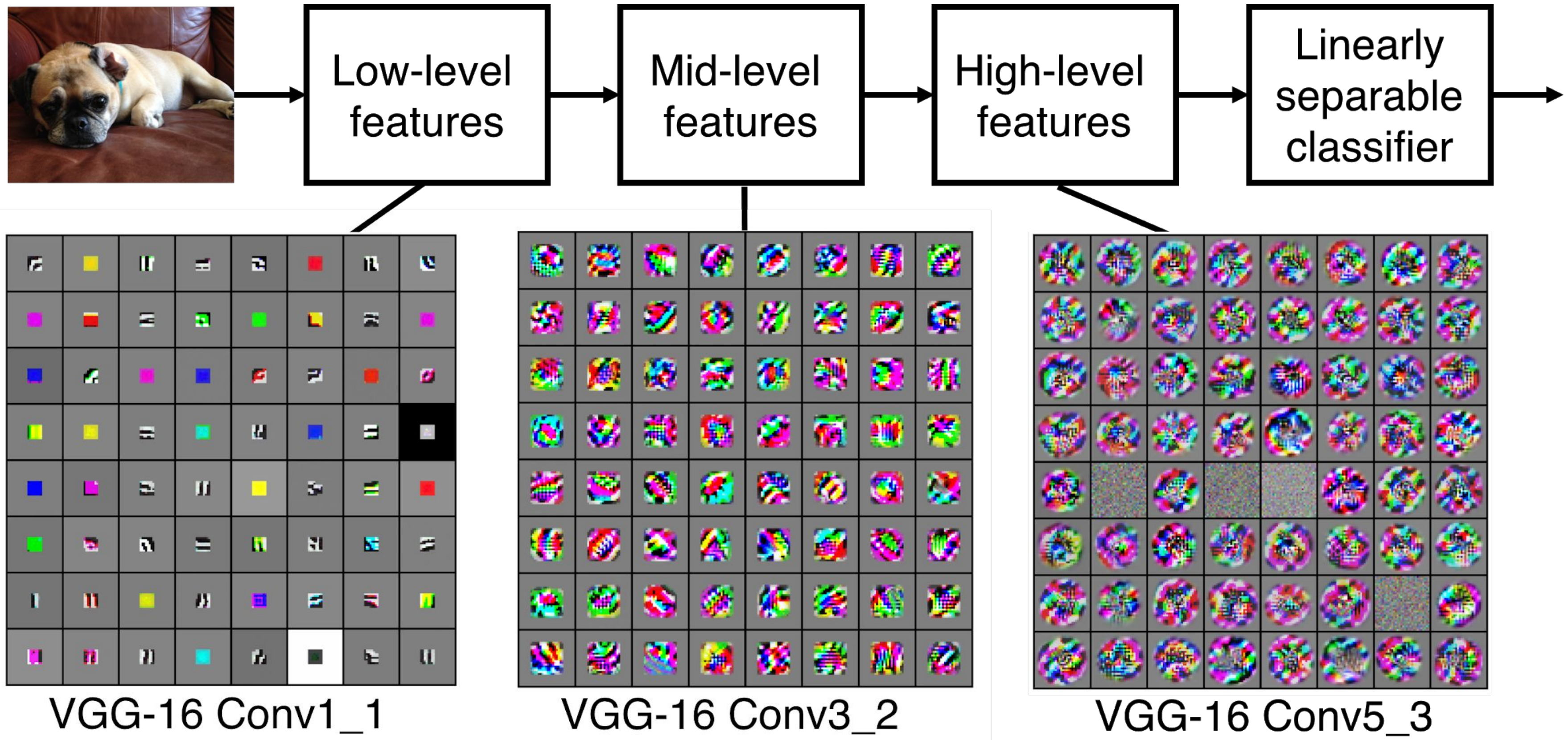
Preview

[Zeiler and Fergus 2013]

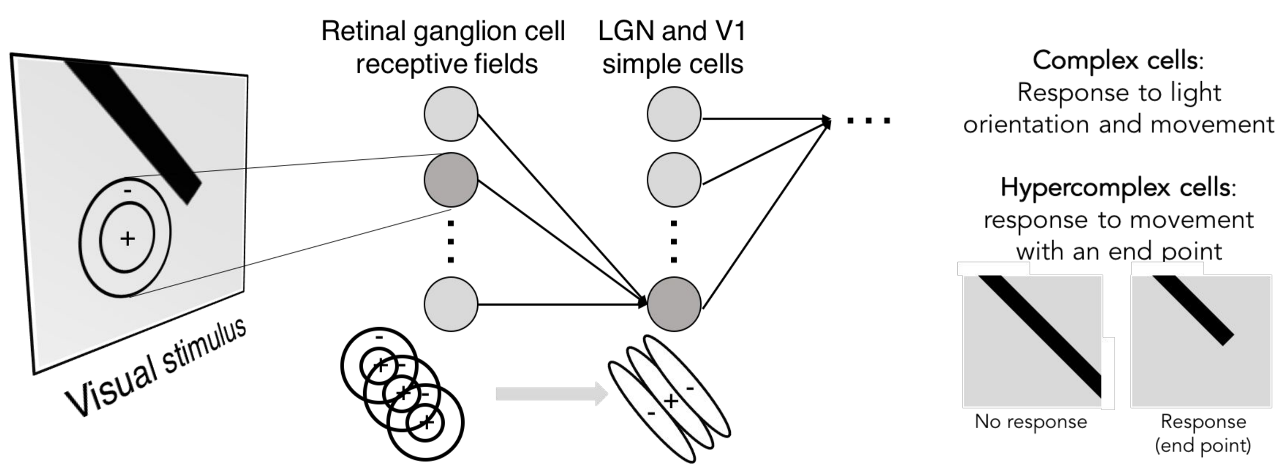
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



Preview



Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3



Retinal ganglion cell receptive fields
LGN and V1 simple cells
Visual stimulus
Complex cells: Response to light orientation and movement
Hypercomplex cells: response to movement with an end point
No response
Response (end point)

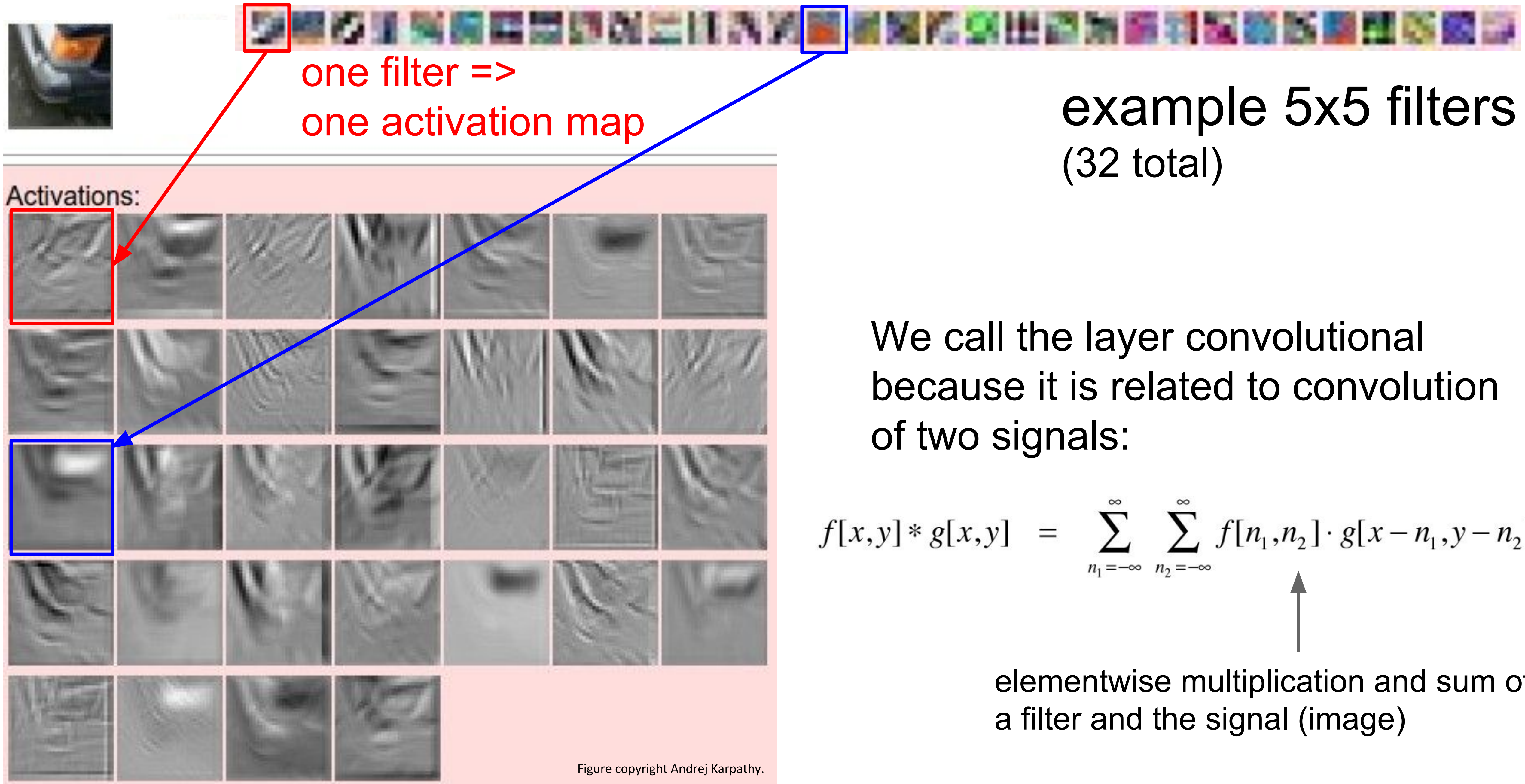


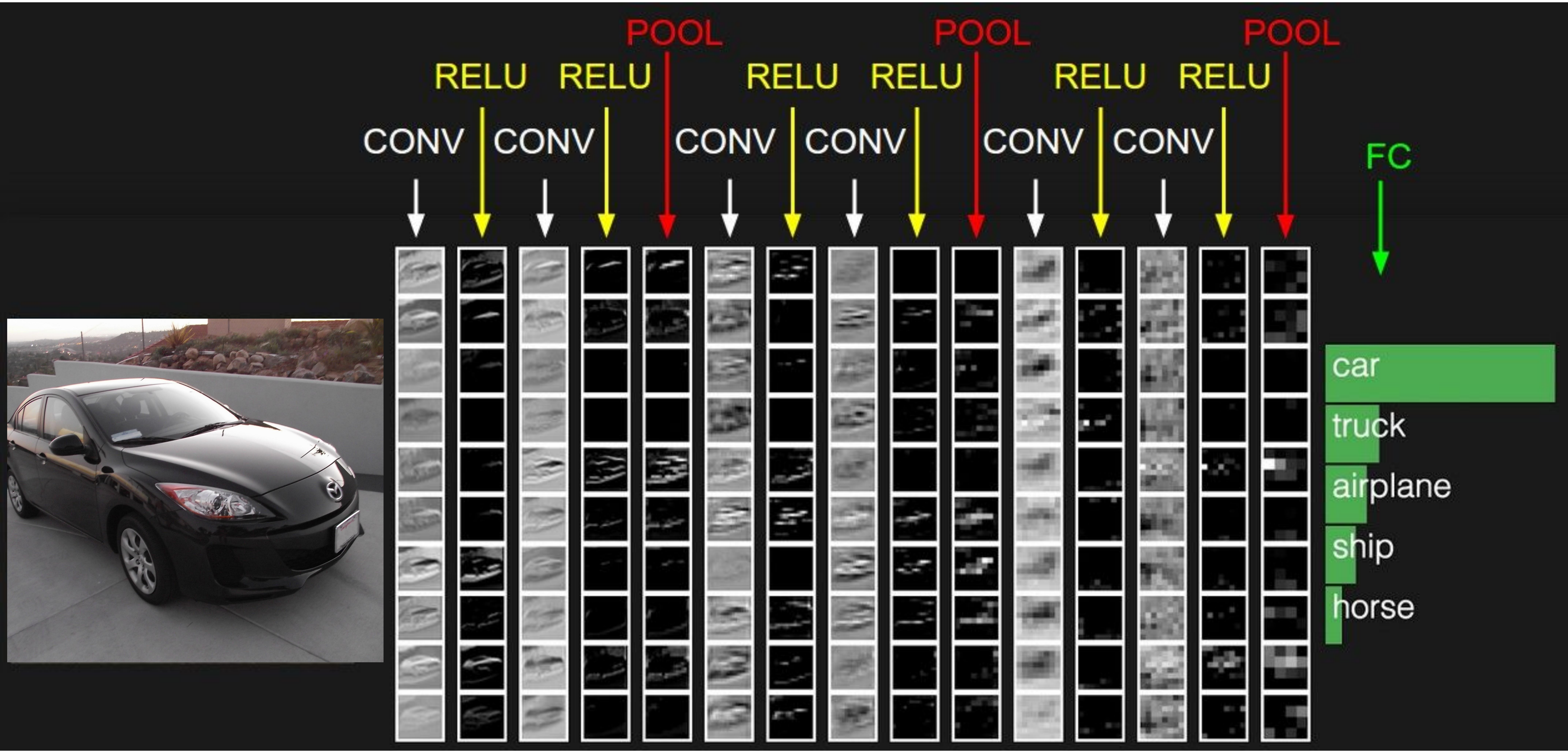
Figure copyright Andrej Karpathy.

example 5x5 filters
(32 total)

We call the layer convolutional because it is related to convolution of two signals:

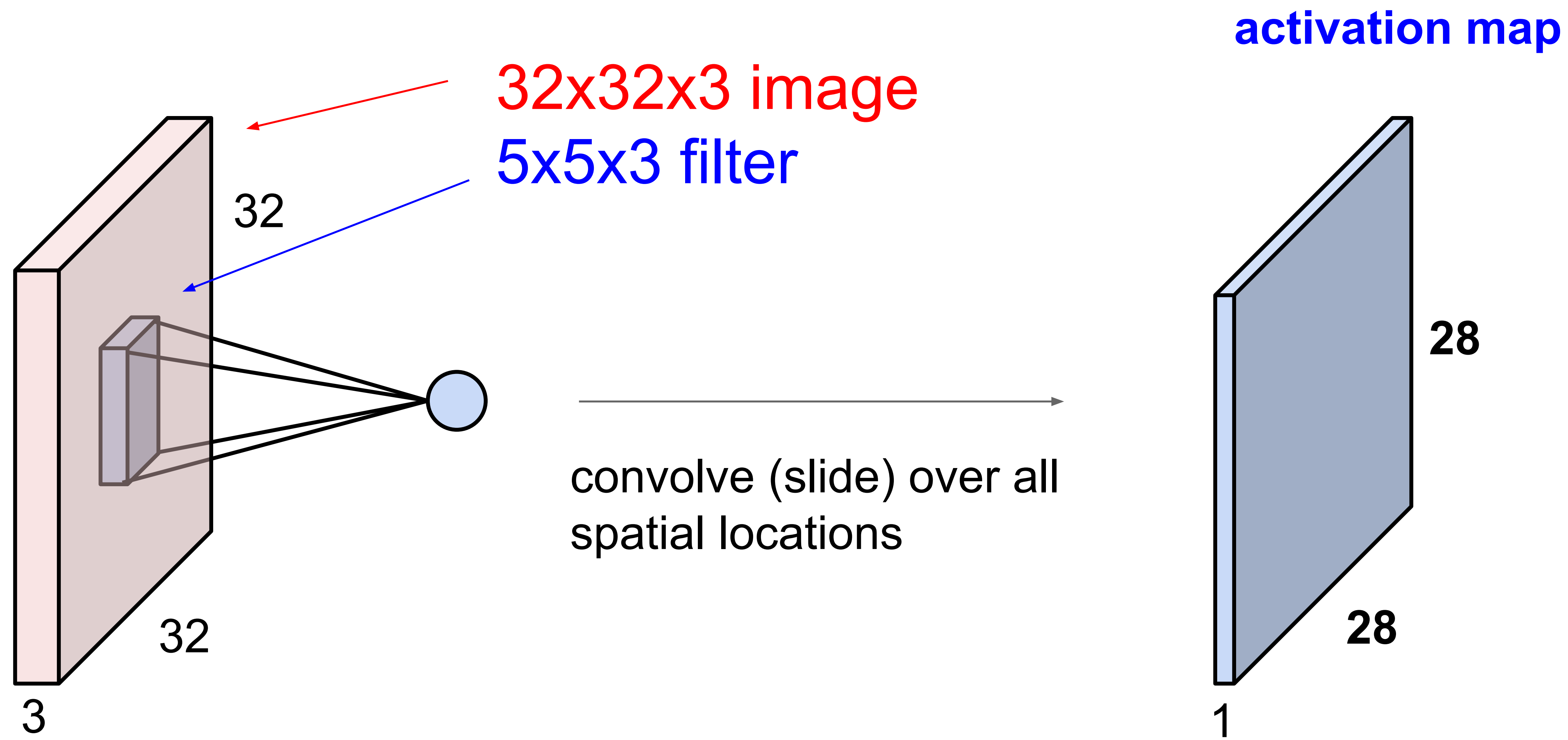
$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1,y-n_2]$$

elementwise multiplication and sum of a filter and the signal (image) preview:

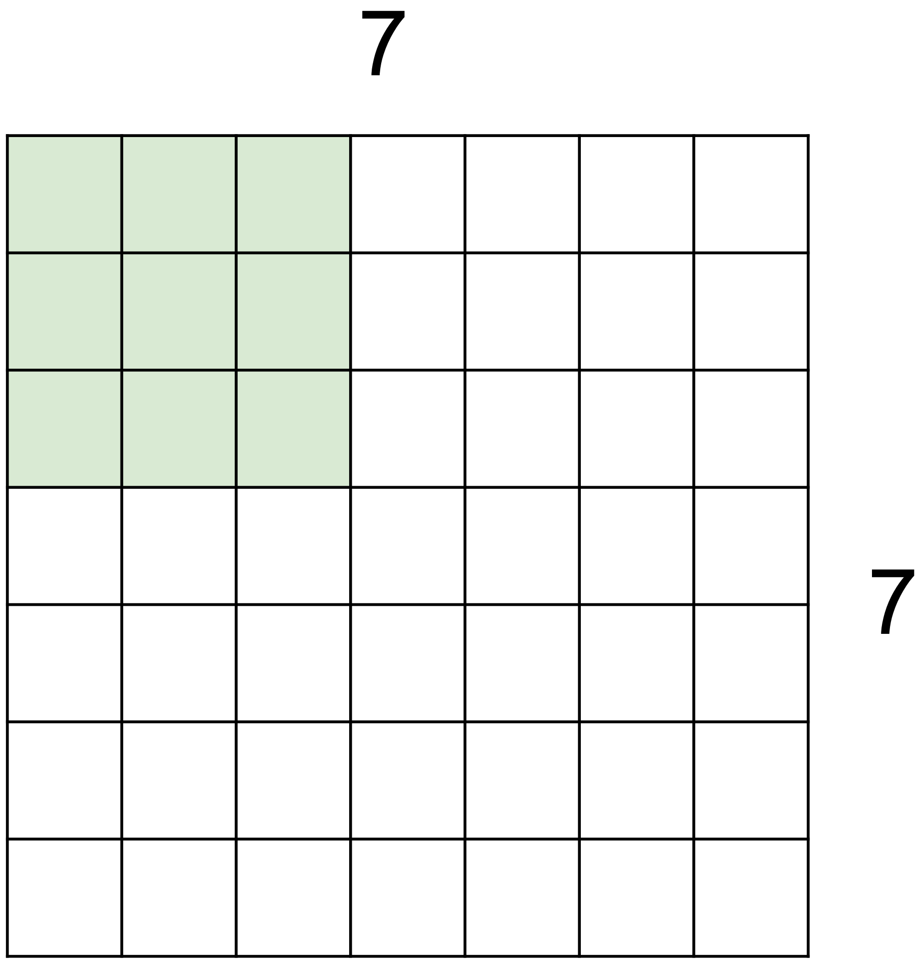


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

A closer look at spatial dimensions:

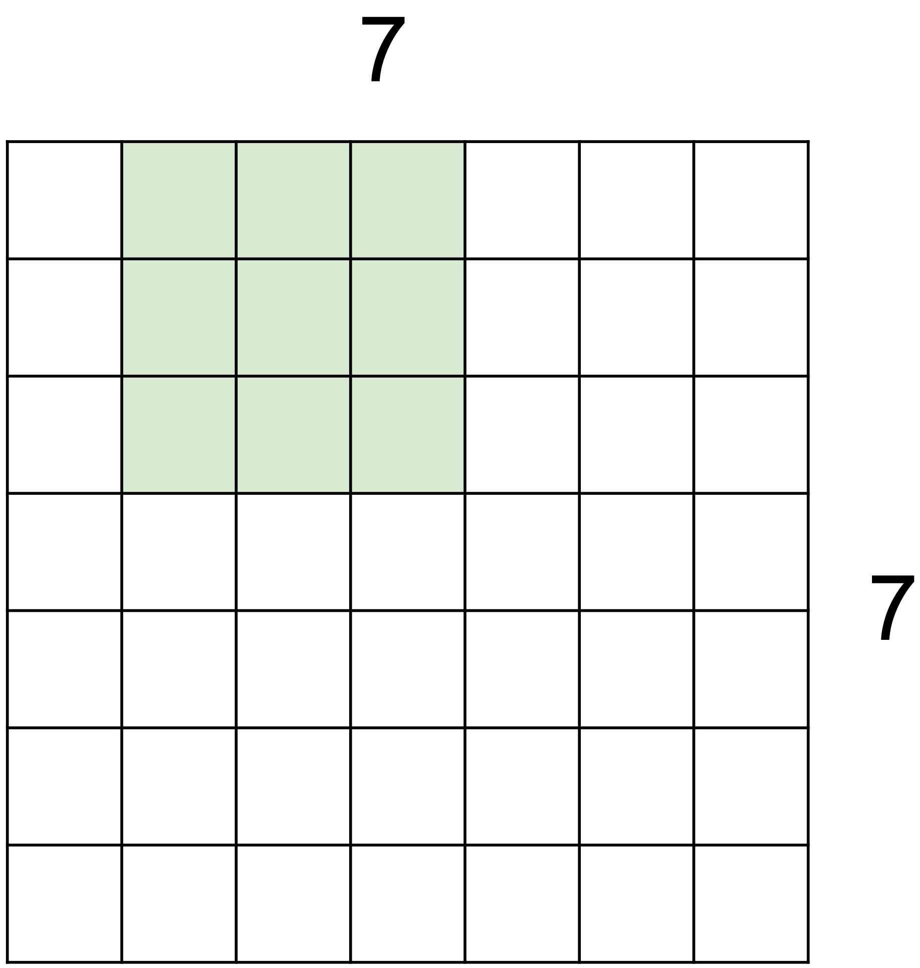


A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



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assume 3x3 filter

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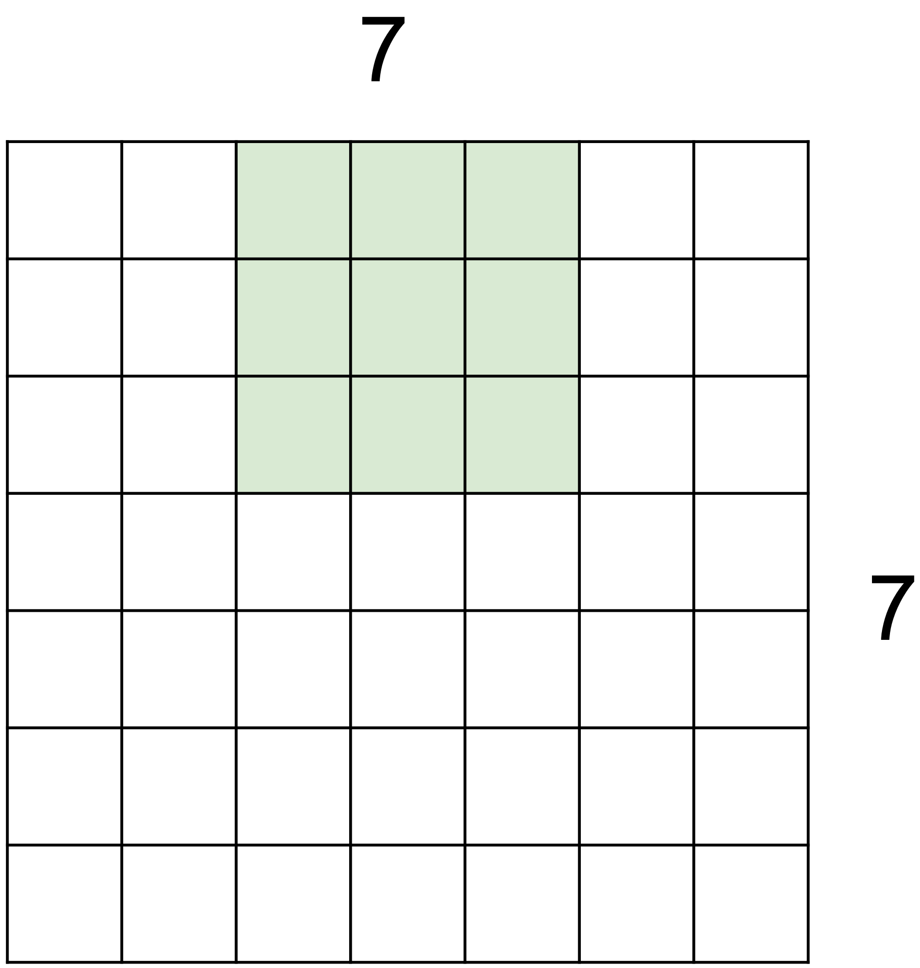

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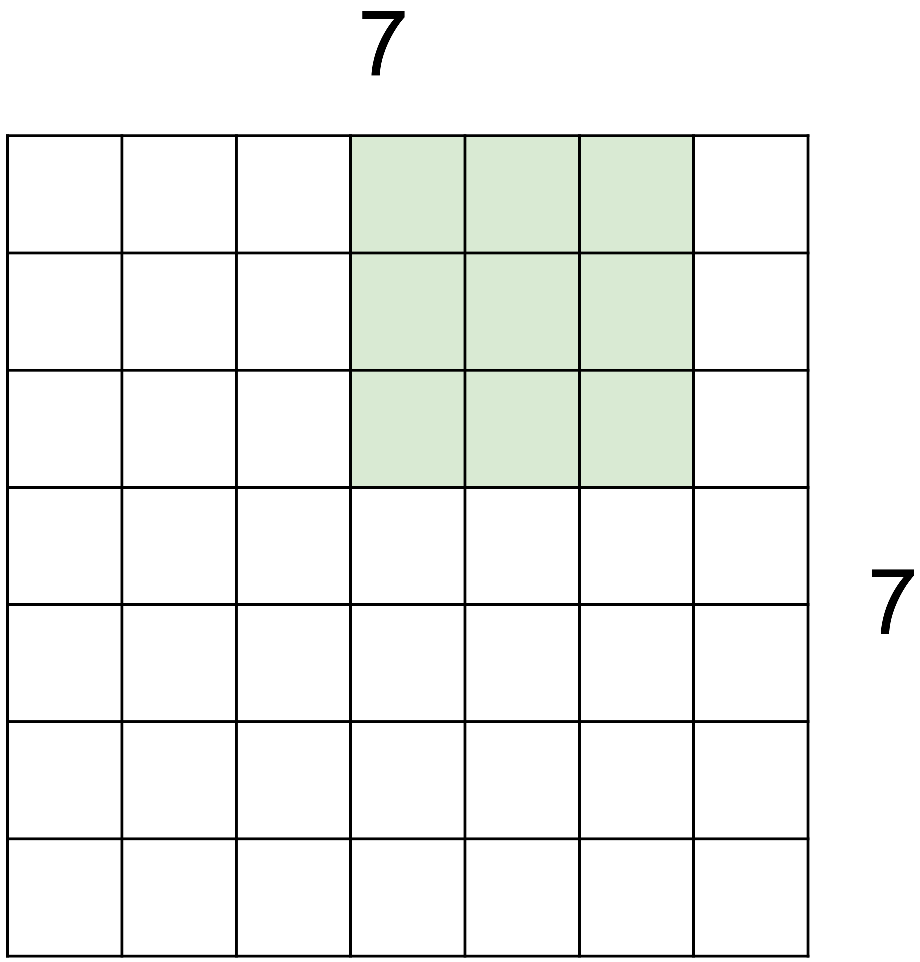
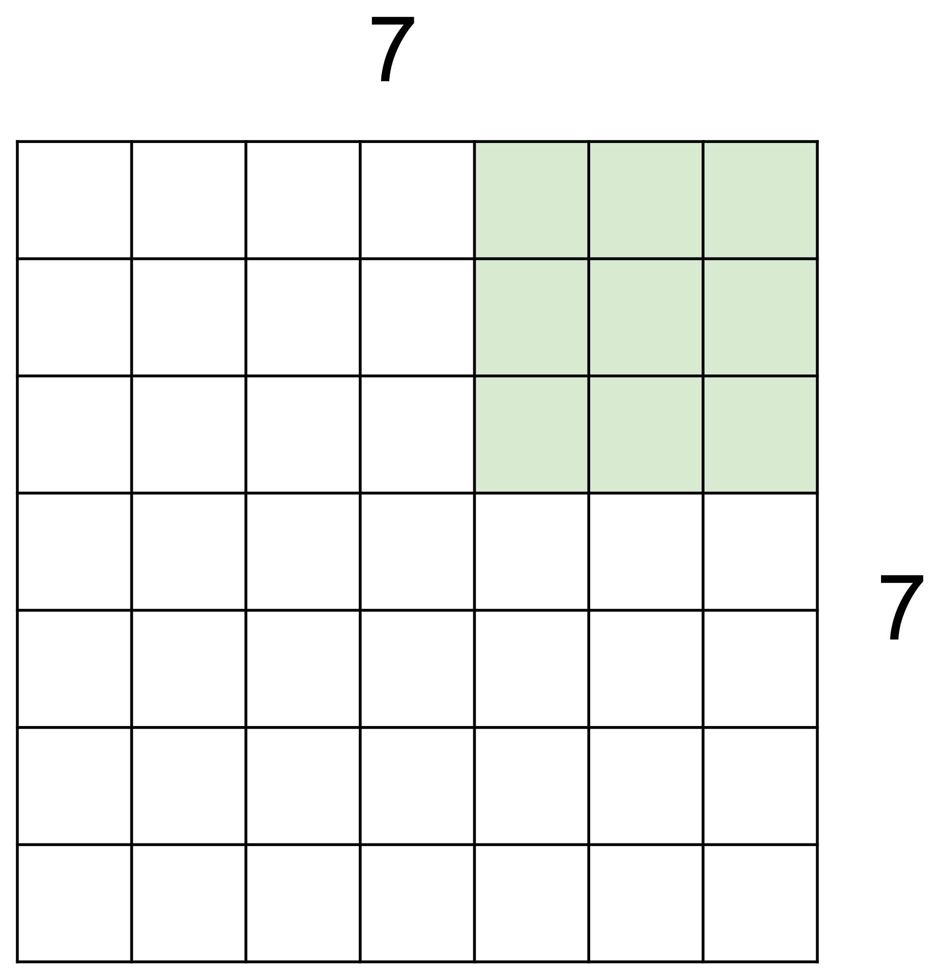


7x7 input (spatially)
assume 3x3 filter

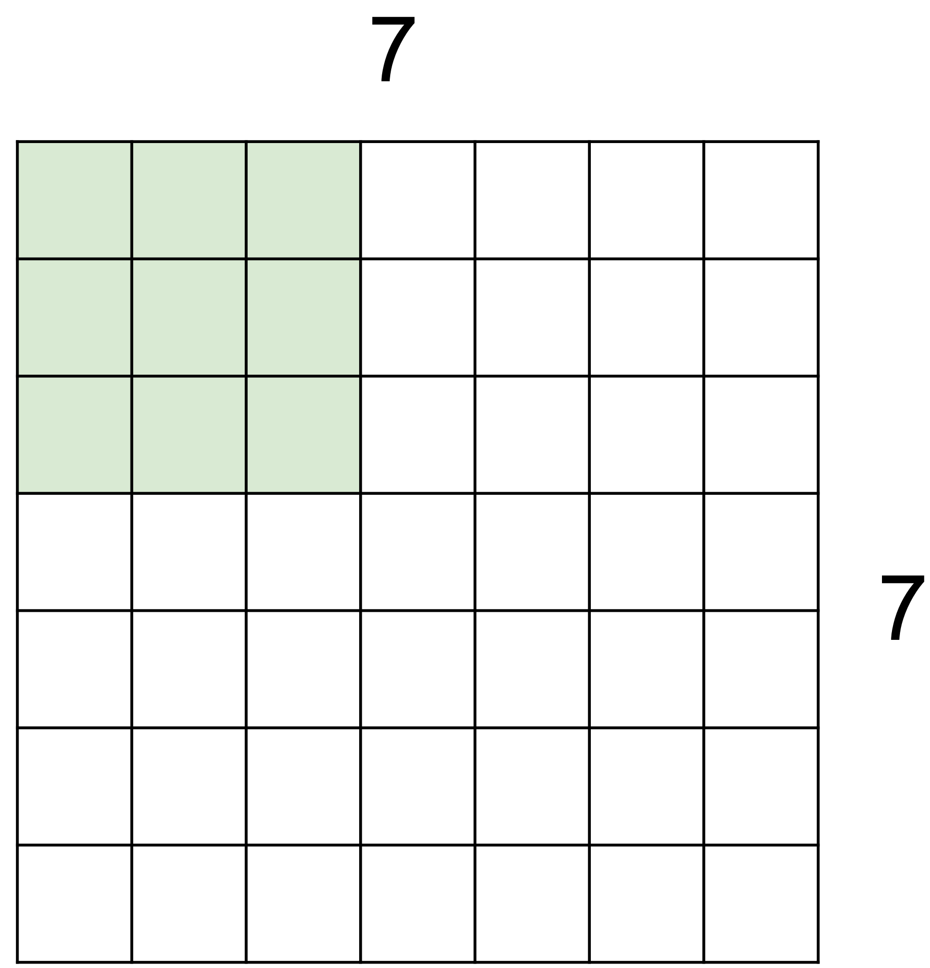
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

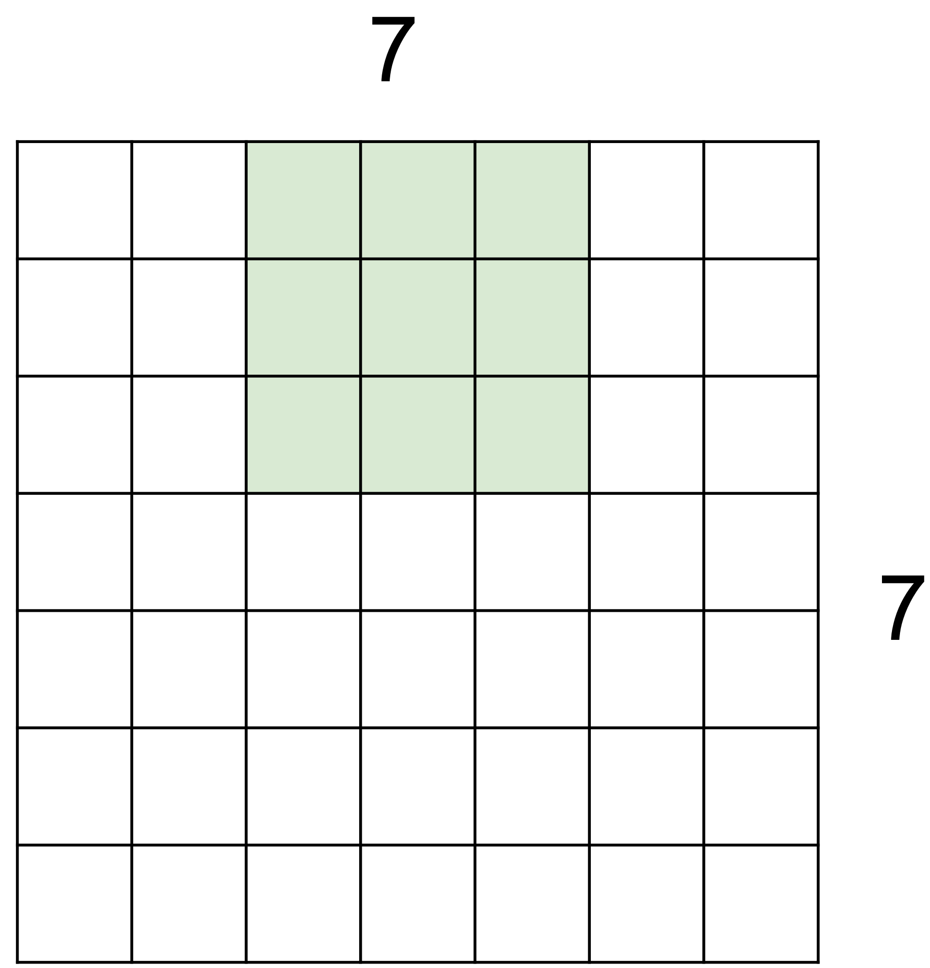
=> 5x5 output

A closer look at spatial dimensions:



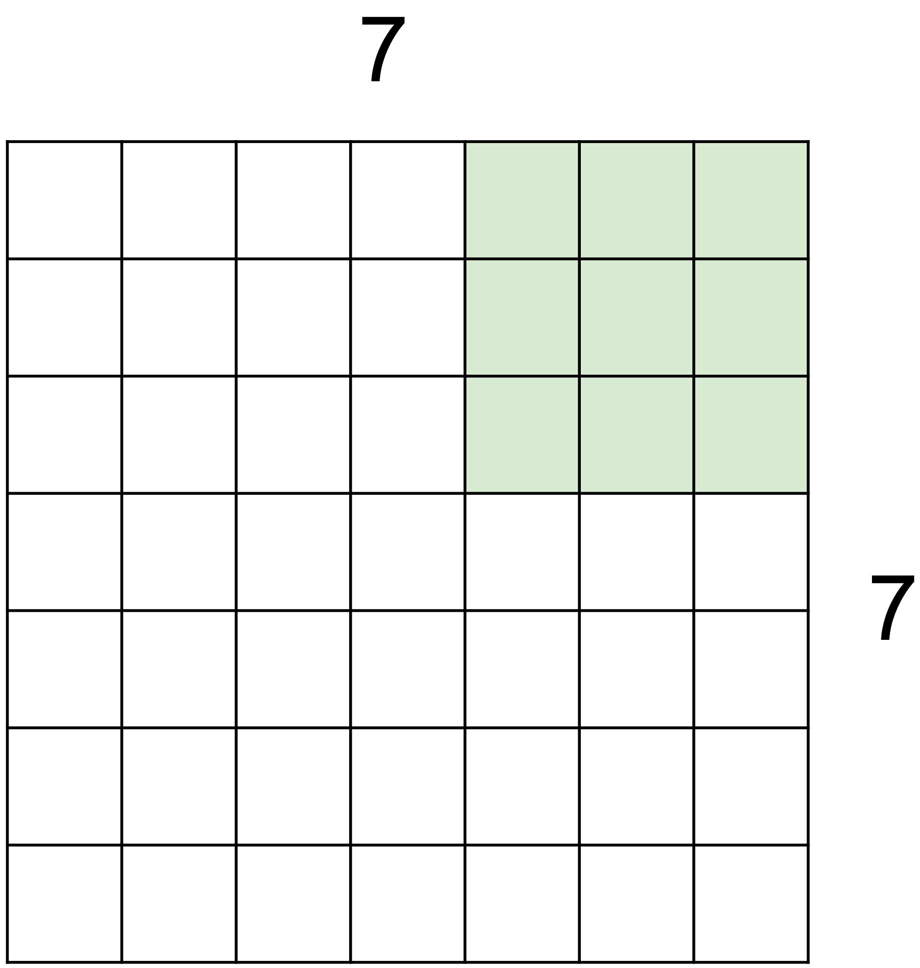
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



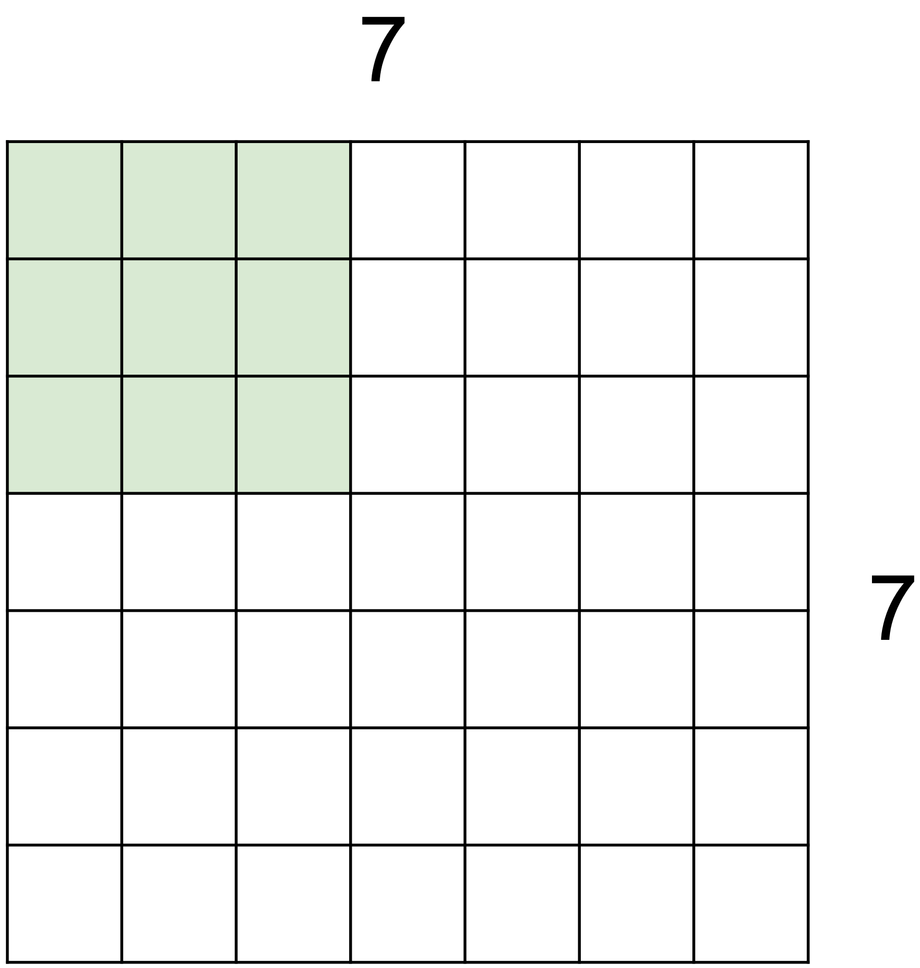
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



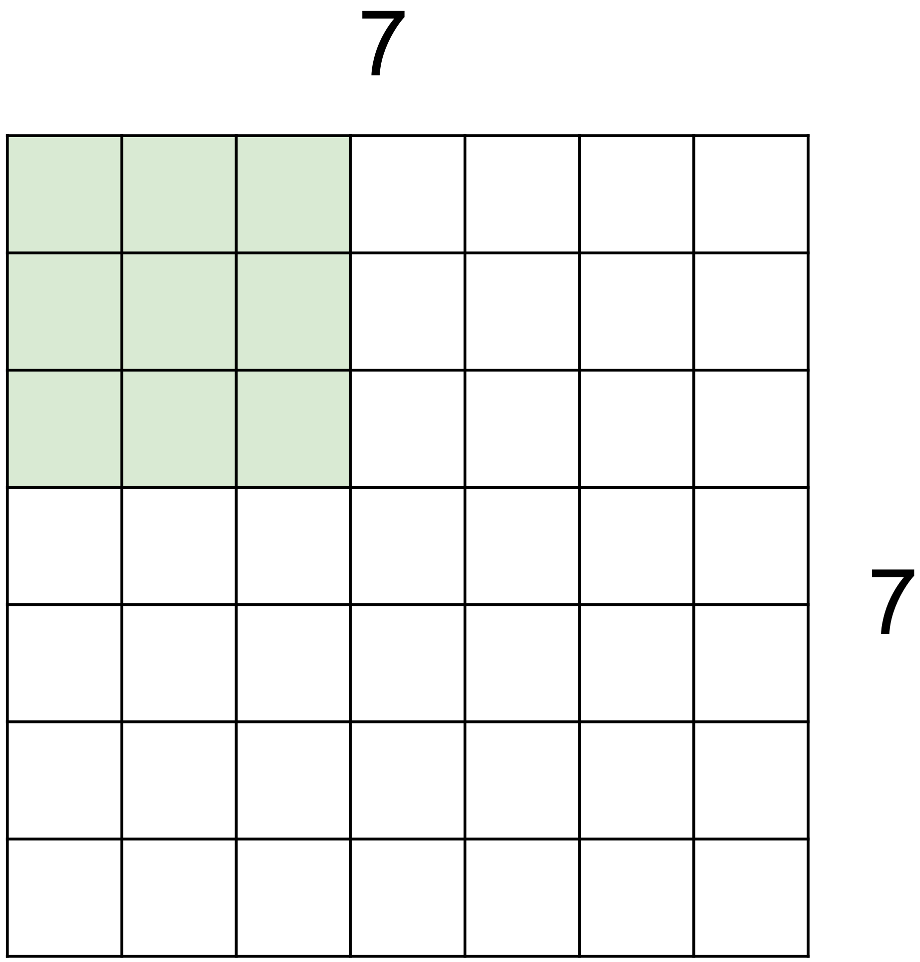
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:



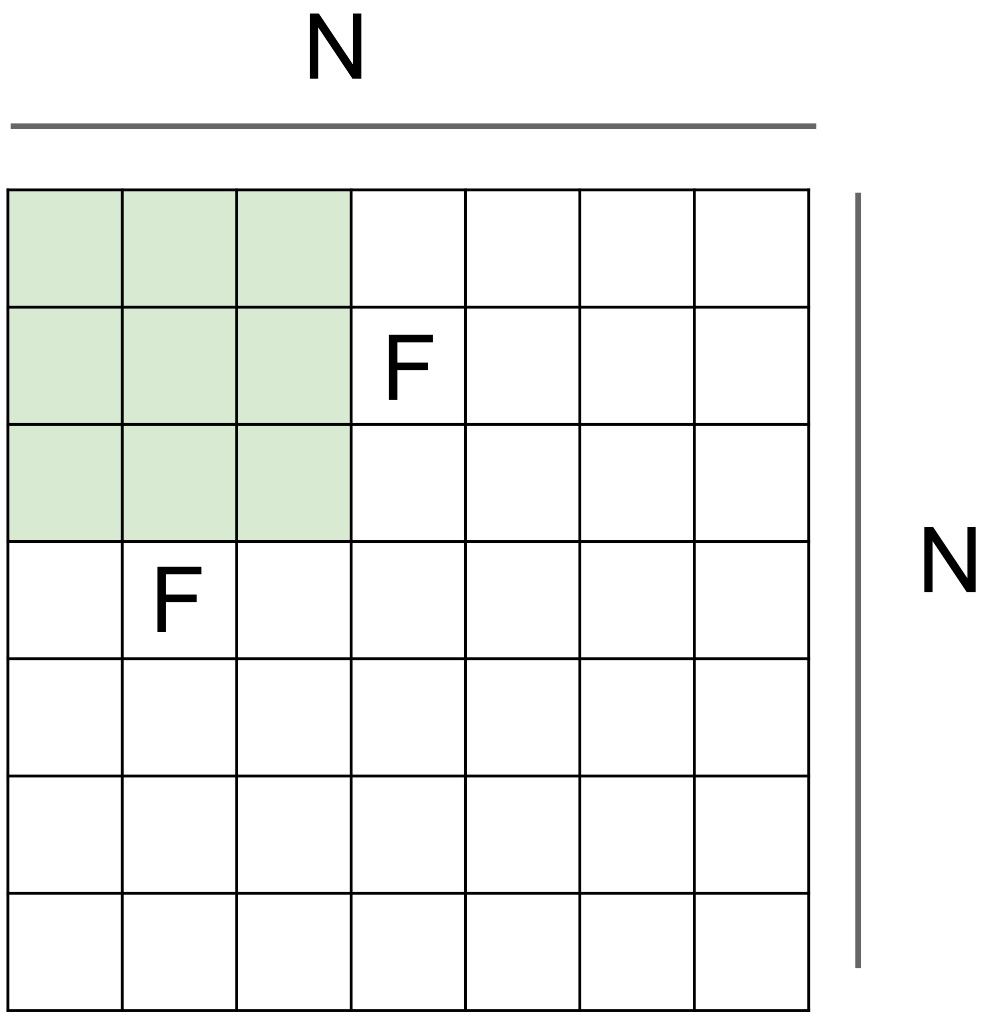
7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

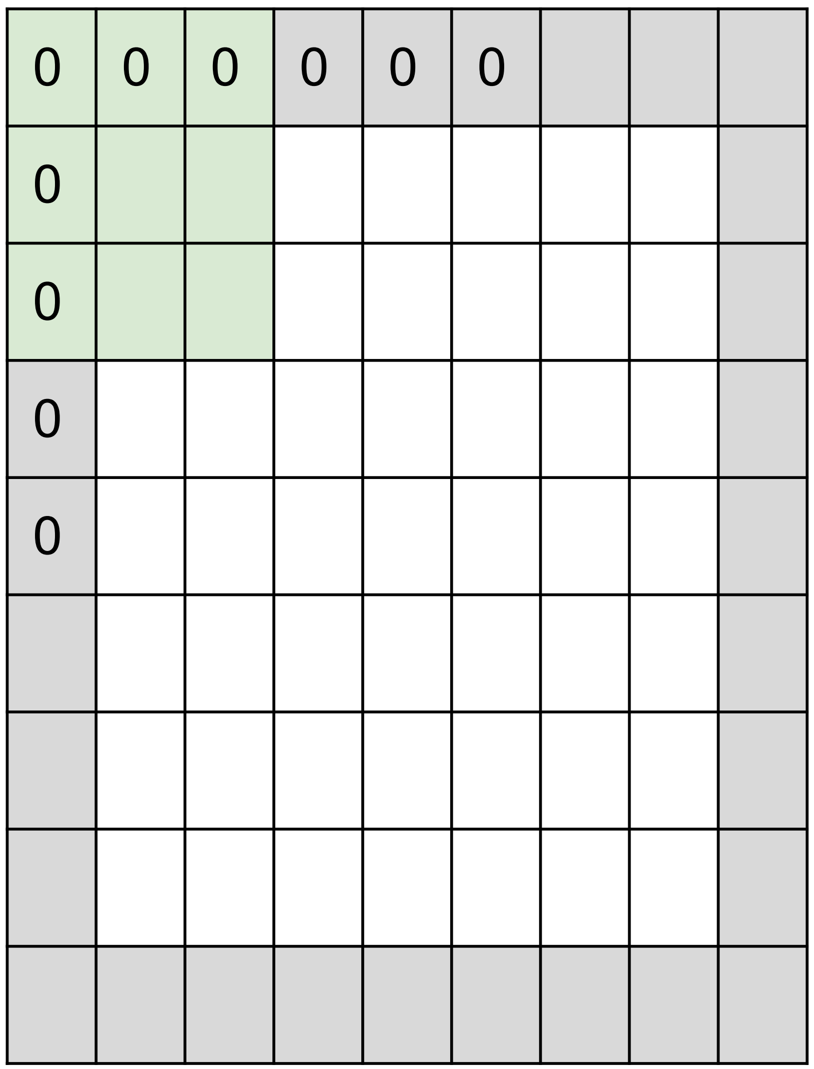
doesn’t fit!
cannot apply 3x3 filter on
7x7 input with stride 3.



Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

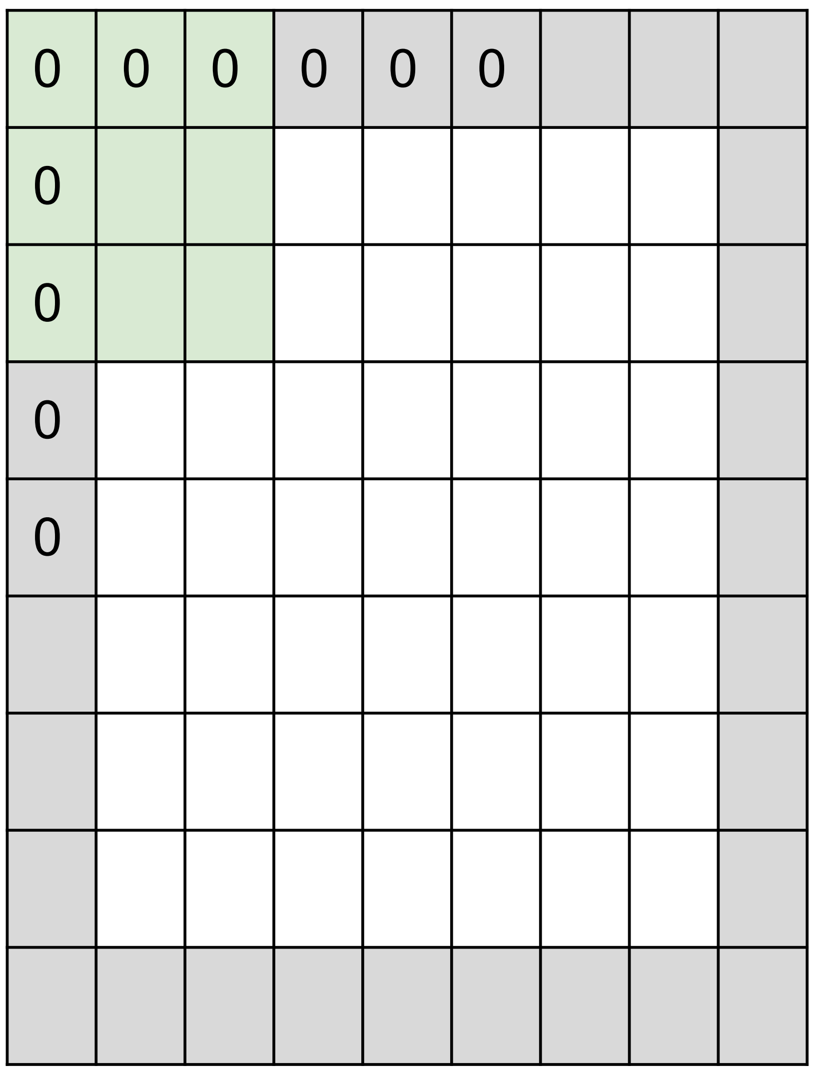
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

(recall:)
(N - F) / stride + 1

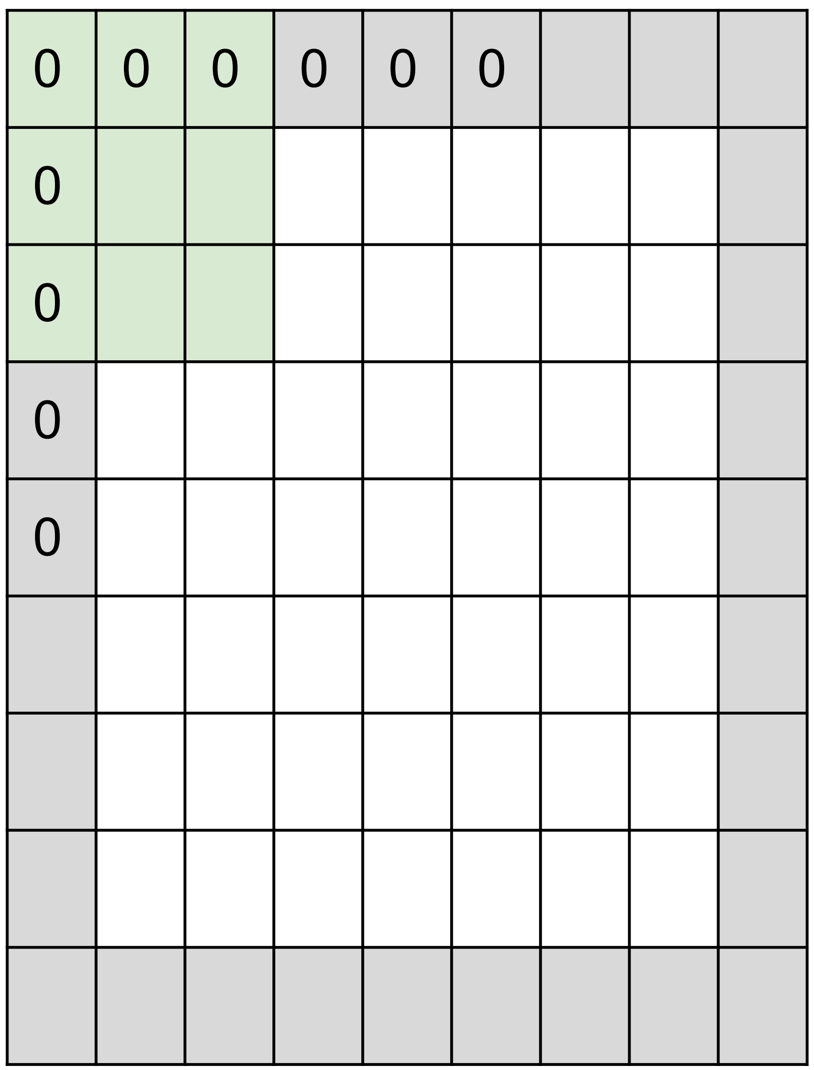
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

In practice: Common to zero pad the border

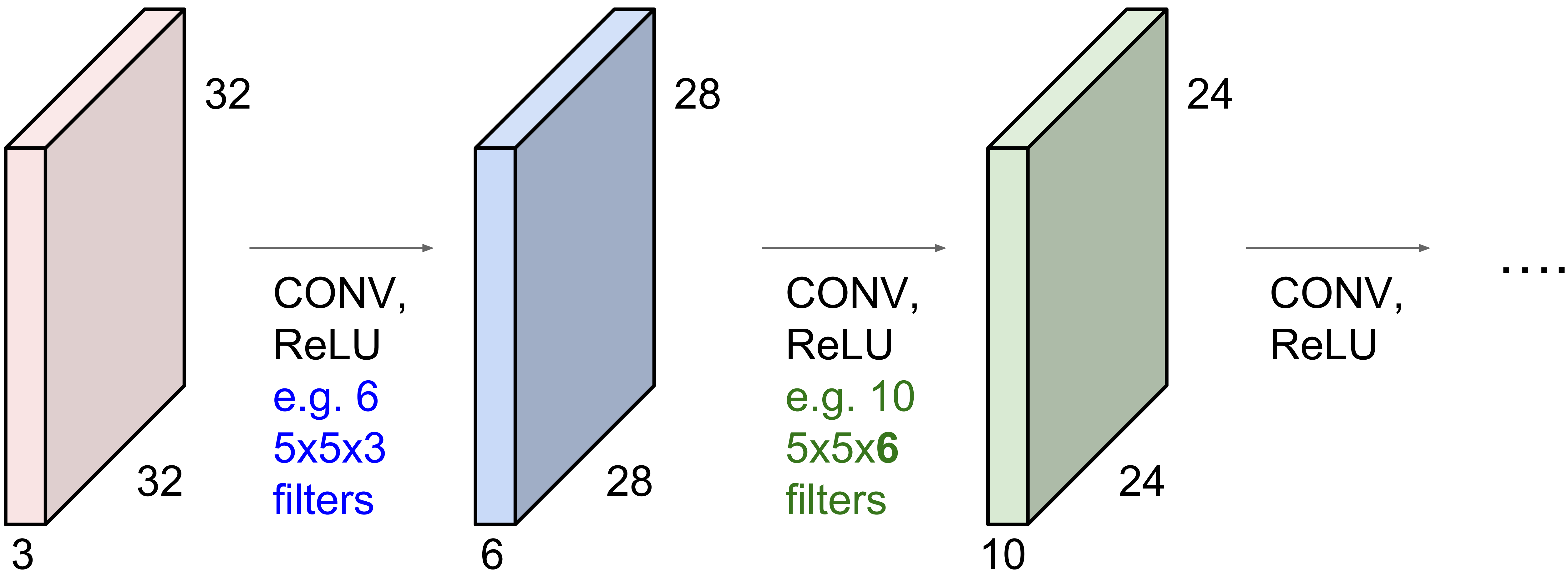


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Remember back to…

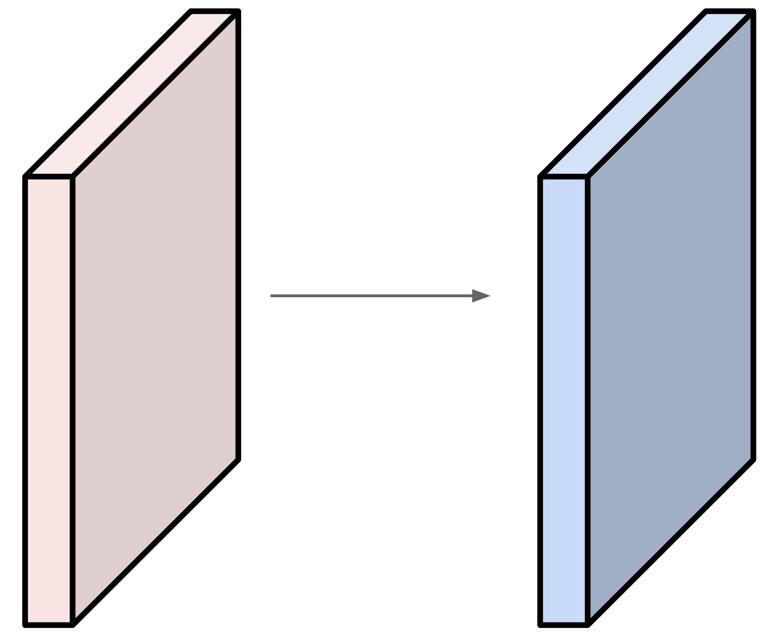
E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.



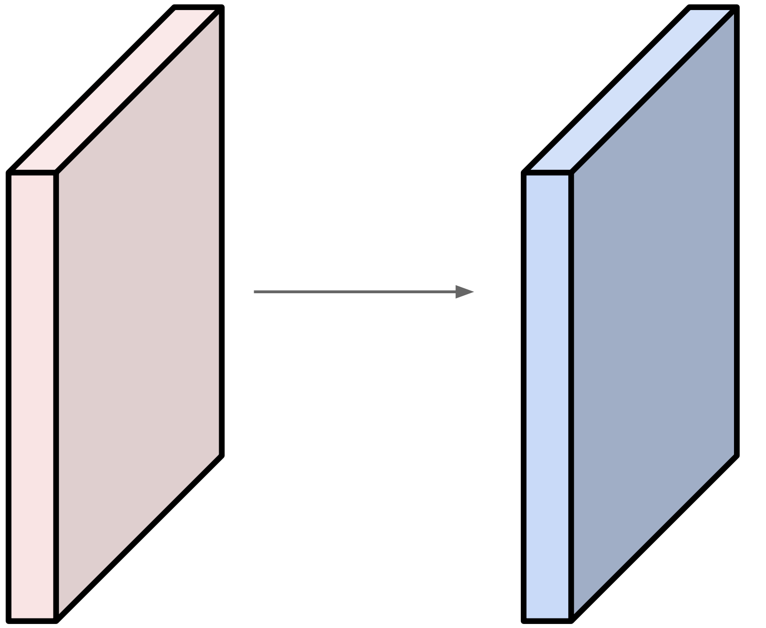
Examples time:

Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Output volume size: ?



Examples time:



Input volume: **32x32x3**

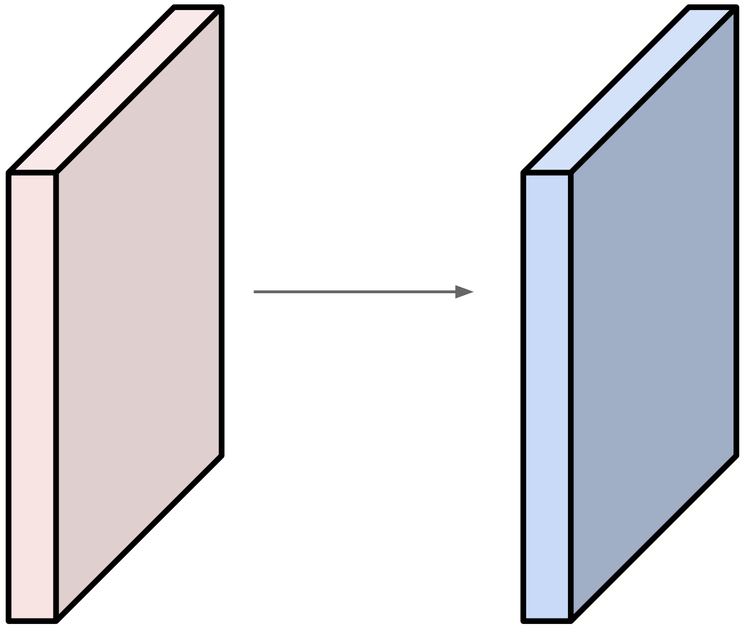
10 5x5 filters with stride 1, pad 2

Output volume size:

(32+2*2-5)/1+1 = 32 spatially, so

32x32x10

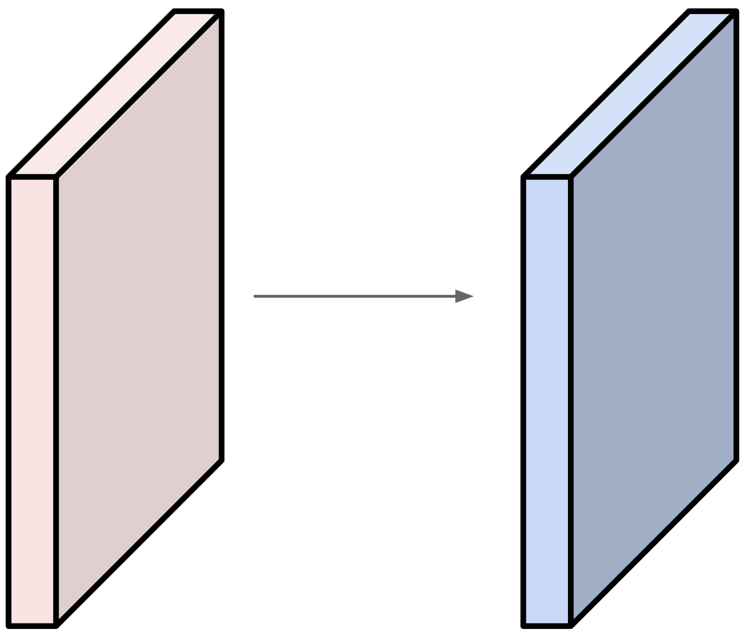
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

Common settings:

K = (powers of 2, e.g. 32, 64, 128, 512)

- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 5, S = 2, P = ? (whatever fits)
- F = 1, S = 1, P = 0

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
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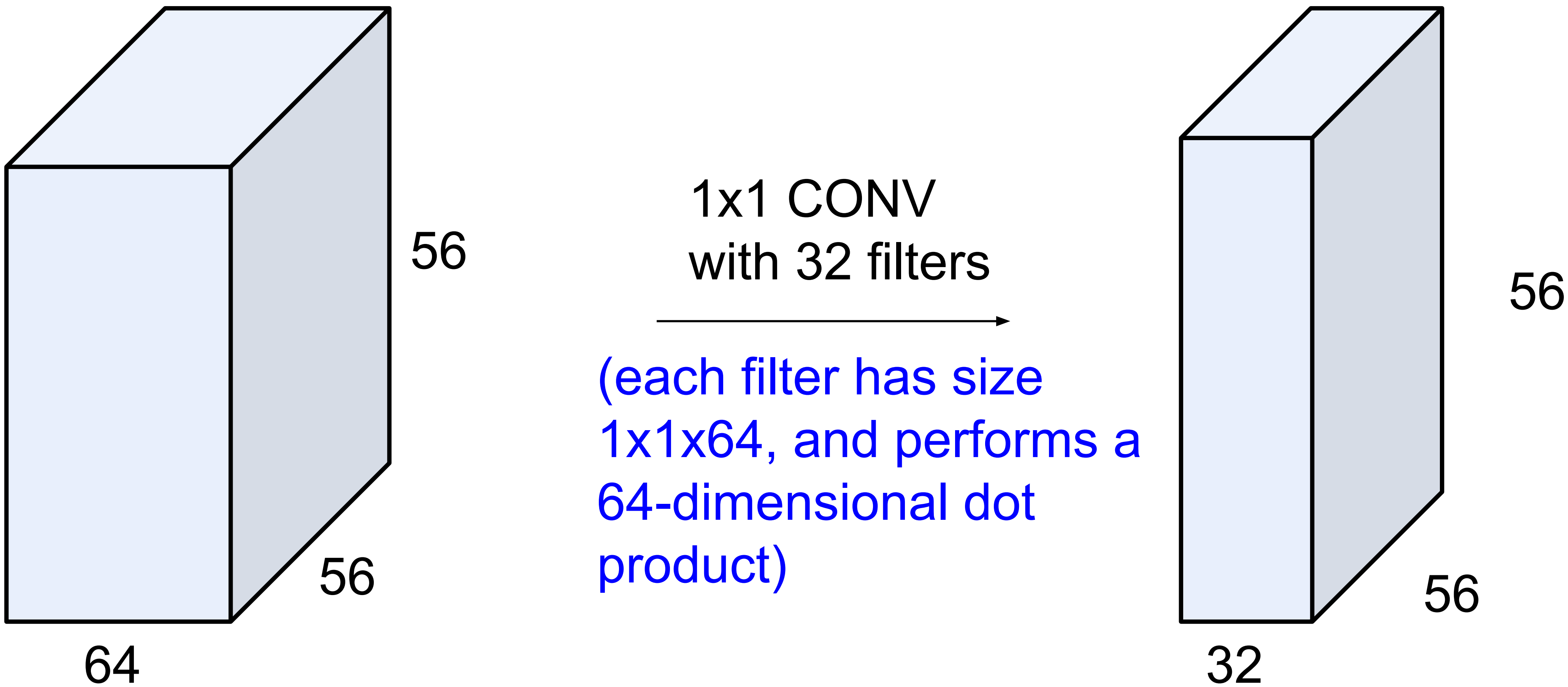
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

(btw, 1x1 convolution layers make perfect sense)



(each filter has size 1x1x64, and performs a 64-dimensional dot product)

Example: CONV layer in Torch

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.

SpatialConvolution

```
module = nn.SpatialConvolution(nInputPlane, nOutputPlane, kW, kH, [dW], [dH], [padW], [padH])
```

Applies a 2D convolution over an input image composed of several input planes. The `input` tensor in `forward(input)` is expected to be a 3D tensor (`nInputPlane x height x width`).

The parameters are the following:

- `nInputPlane` : The number of expected input planes in the image given into `forward()` .
- `nOutputPlane` : The number of output planes the convolution layer will produce.
- `kW` : The kernel width of the convolution
- `kH` : The kernel height of the convolution
- `dW` : The step of the convolution in the width dimension. Default is `1` .
- `dH` : The step of the convolution in the height dimension. Default is `1` .
- `padW` : The additional zeros added per width to the input planes. Default is `0` , a good number is `(kW-1)/2` .
- `padH` : The additional zeros added per height to the input planes. Default is `padW` , a good number is `(kH-1)/2` .

Note that depending of the size of your kernel, several (of the last) columns or rows of the input image might be lost. It is up to the user to add proper padding in images.

If the input image is a 3D tensor `nInputPlane x height x width` , the output image size will be `nOutputPlane x oheight x owidth` where

```
owidth  = floor((width  + 2*padW - kW) / dW + 1)
oheight = floor((height + 2*padH - kH) / dH + 1)
```

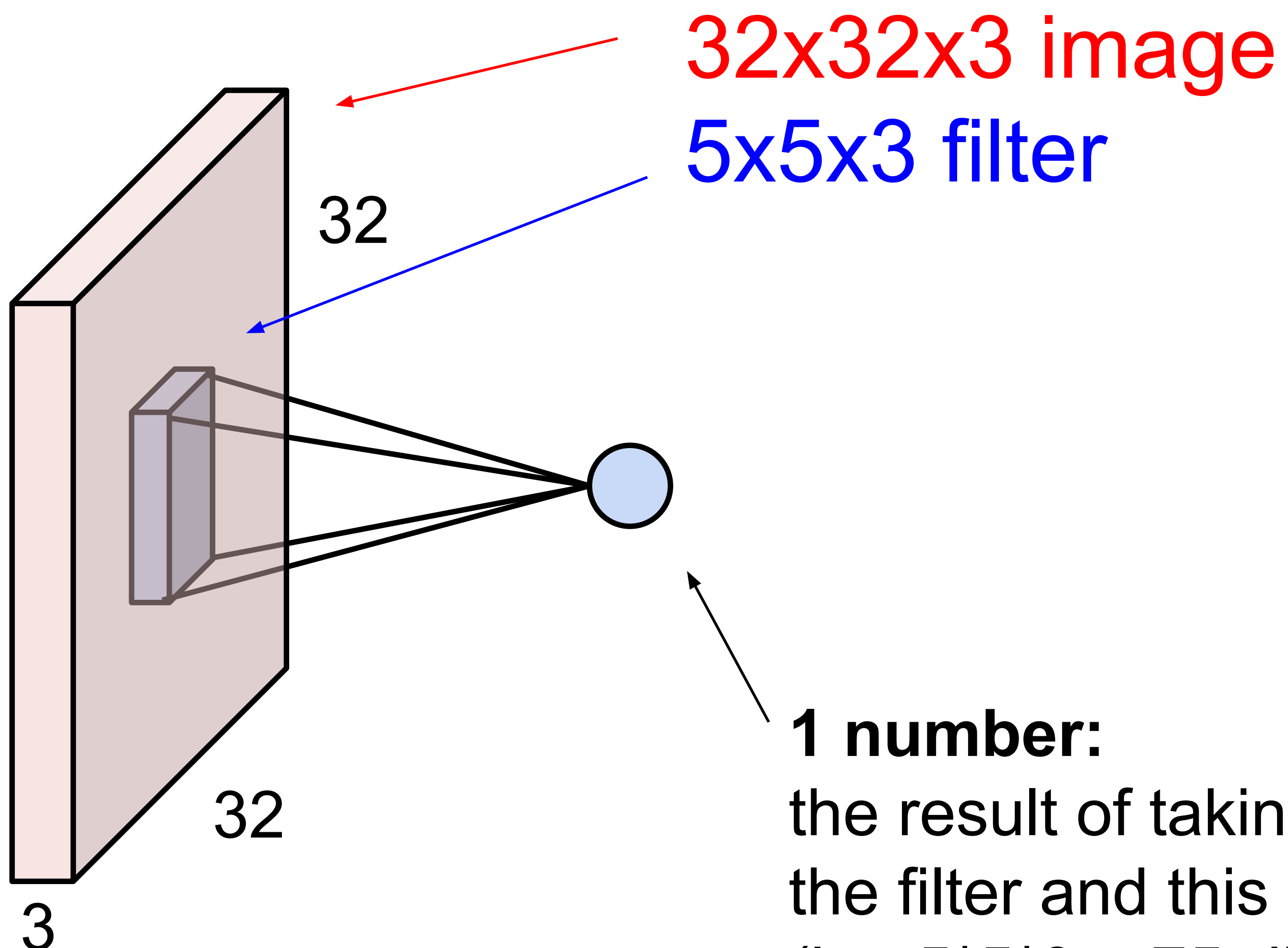
Example: CONV layer in Caffe

```
layer {
  name: "conv1"
  type: "Convolution"
  bottom: "data"
  top: "conv1"
  # learning rate and decay multipliers for the filters
  param { lr_mult: 1 decay_mult: 1 }
  # learning rate and decay multipliers for the biases
  param { lr_mult: 2 decay_mult: 0 }
  convolution_param {
    num_output: 96     # learn 96 filters
    kernel_size: 11    # each filter is 11x11
    stride: 4          # step 4 pixels between each filter application
    weight_filler {
      type: "gaussian" # initialize the filters from a Gaussian
      std: 0.01        # distribution with stdev 0.01 (default mean: 0)
    }
    bias_filler {
      type: "constant" # initialize the biases to zero (0)
      value: 0
    }
  }
}
```

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.

The brain/neuron view of CONV Layer

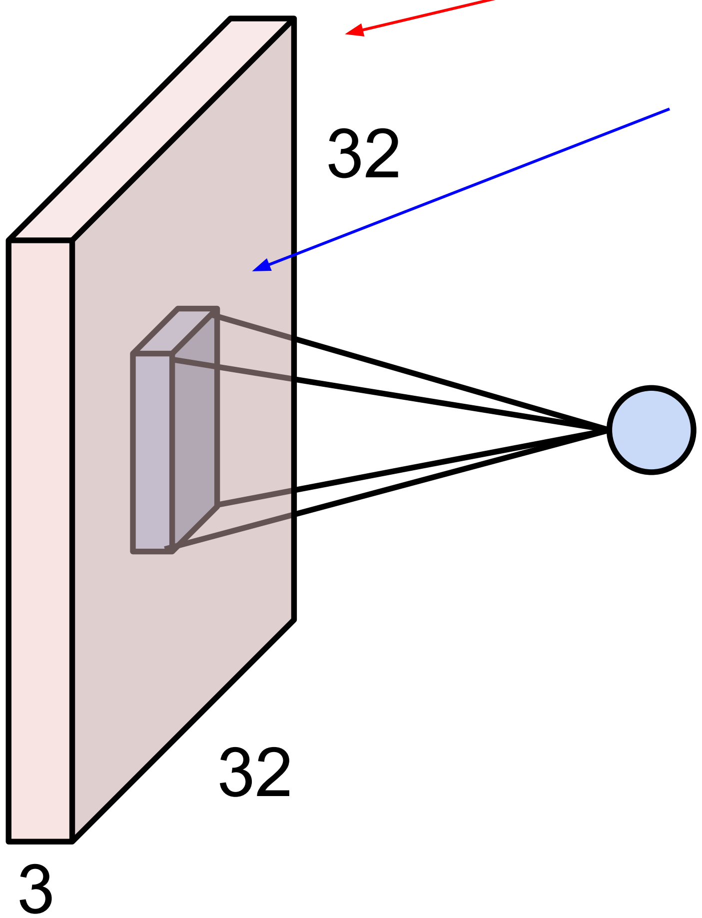


1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

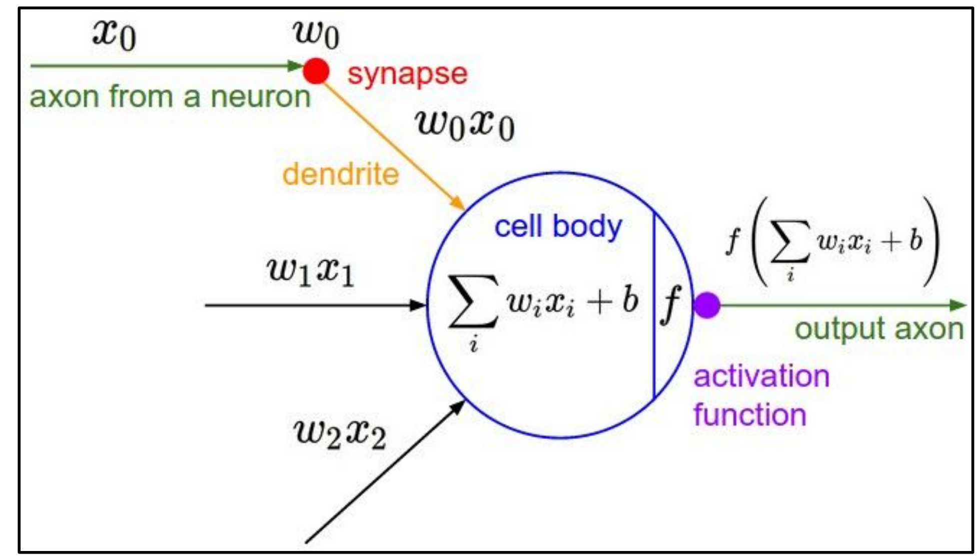
The brain/neuron view of CONV Layer

32x32x3 image

5x5x3 filter

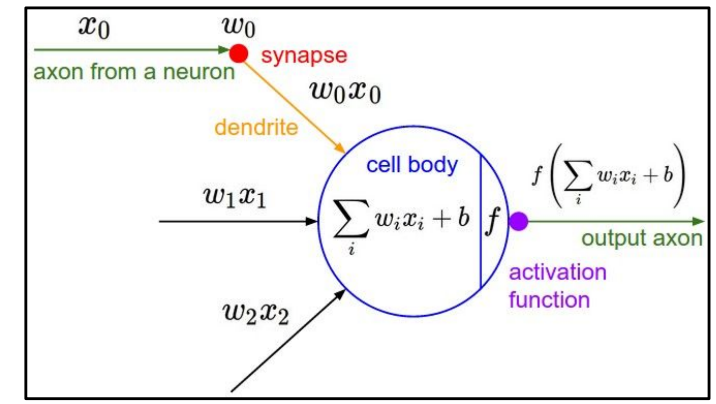


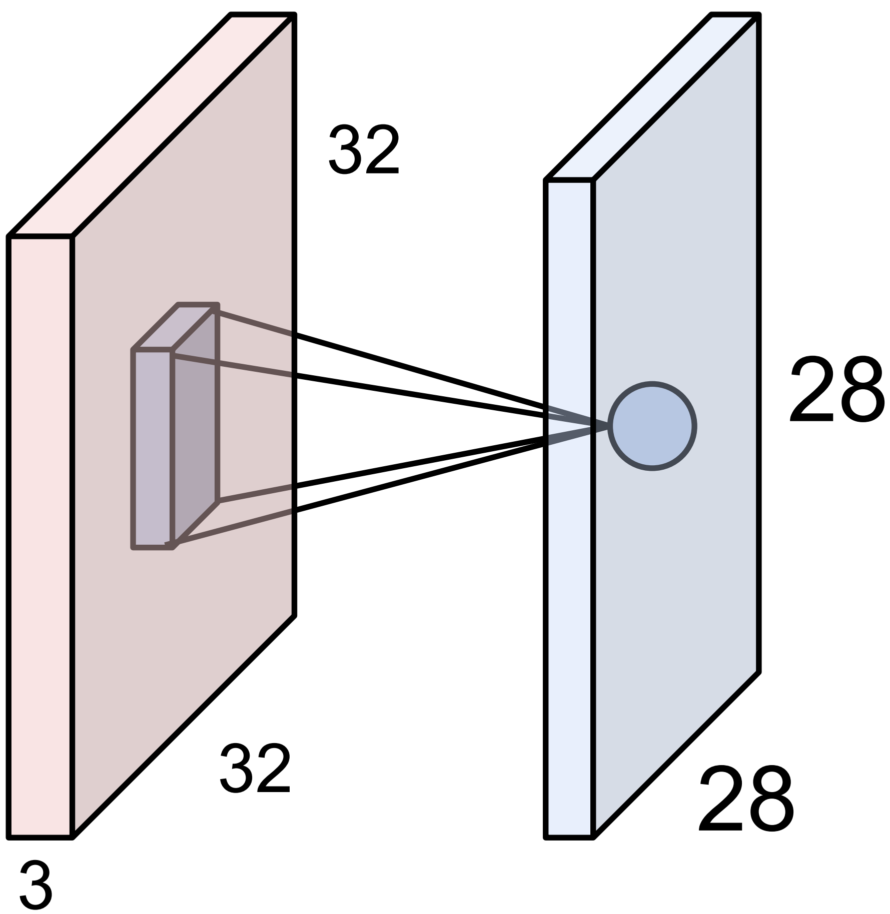
1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)



It's just a neuron with local connectivity...

The brain/neuron view of CONV Layer



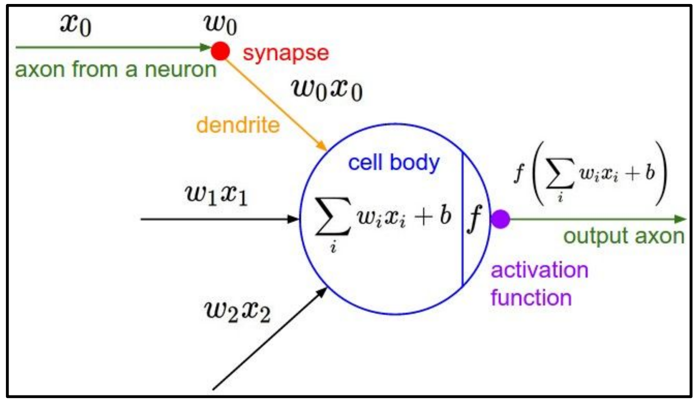


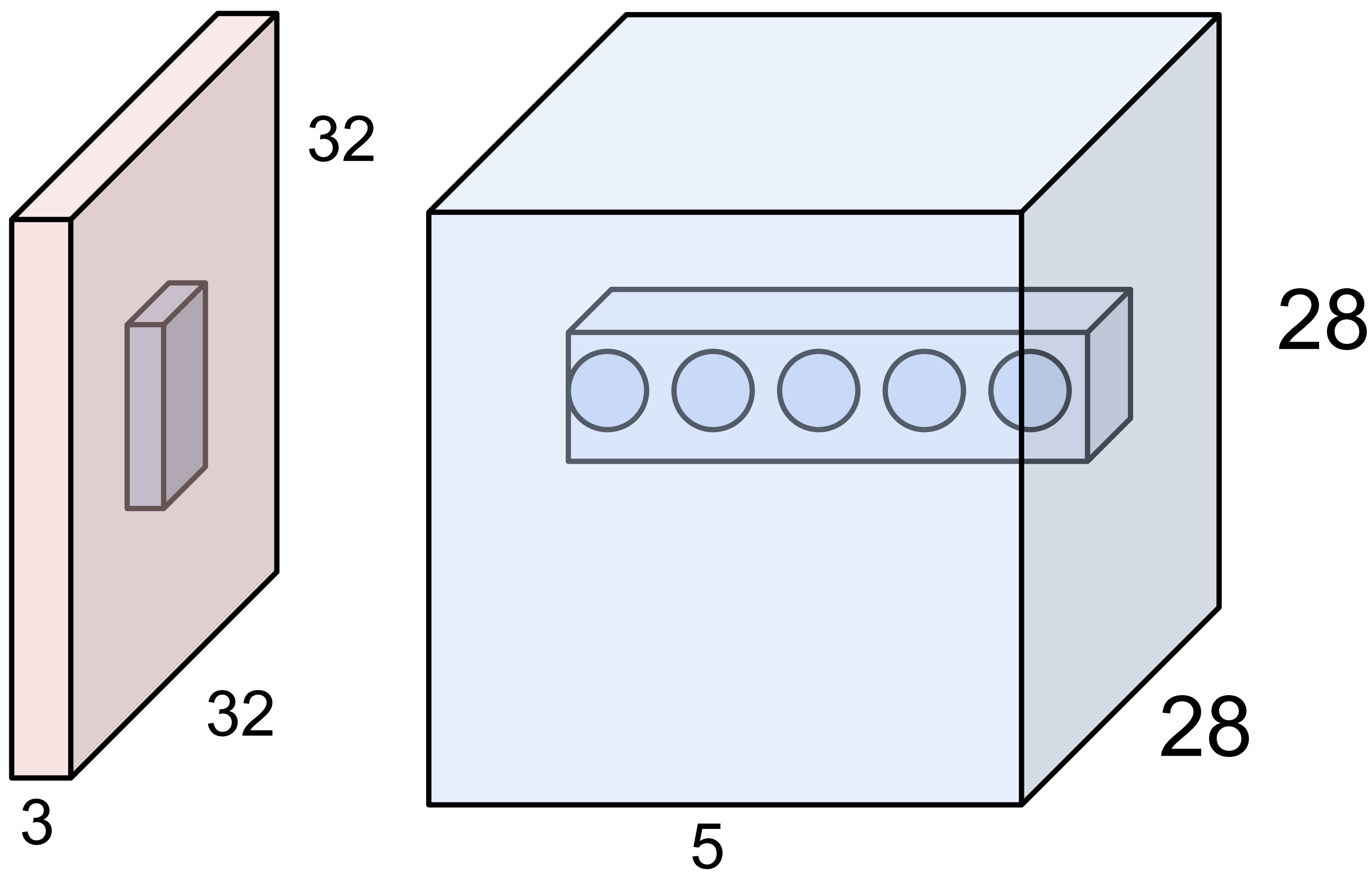
An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

“5x5 filter” -> “5x5 receptive field for each neuron”

The brain/neuron view of CONV Layer





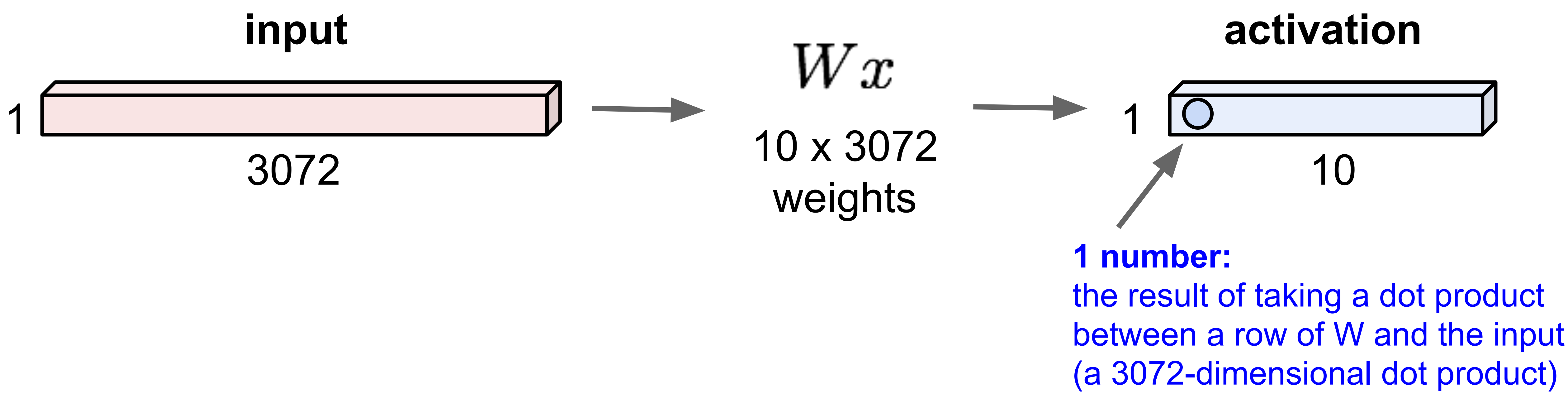
E.g. with 5 filters,
CONV layer consists of
neurons arranged in a 3D grid
(28x28x5)

There will be 5 different
neurons all looking at the same
region in the input volume

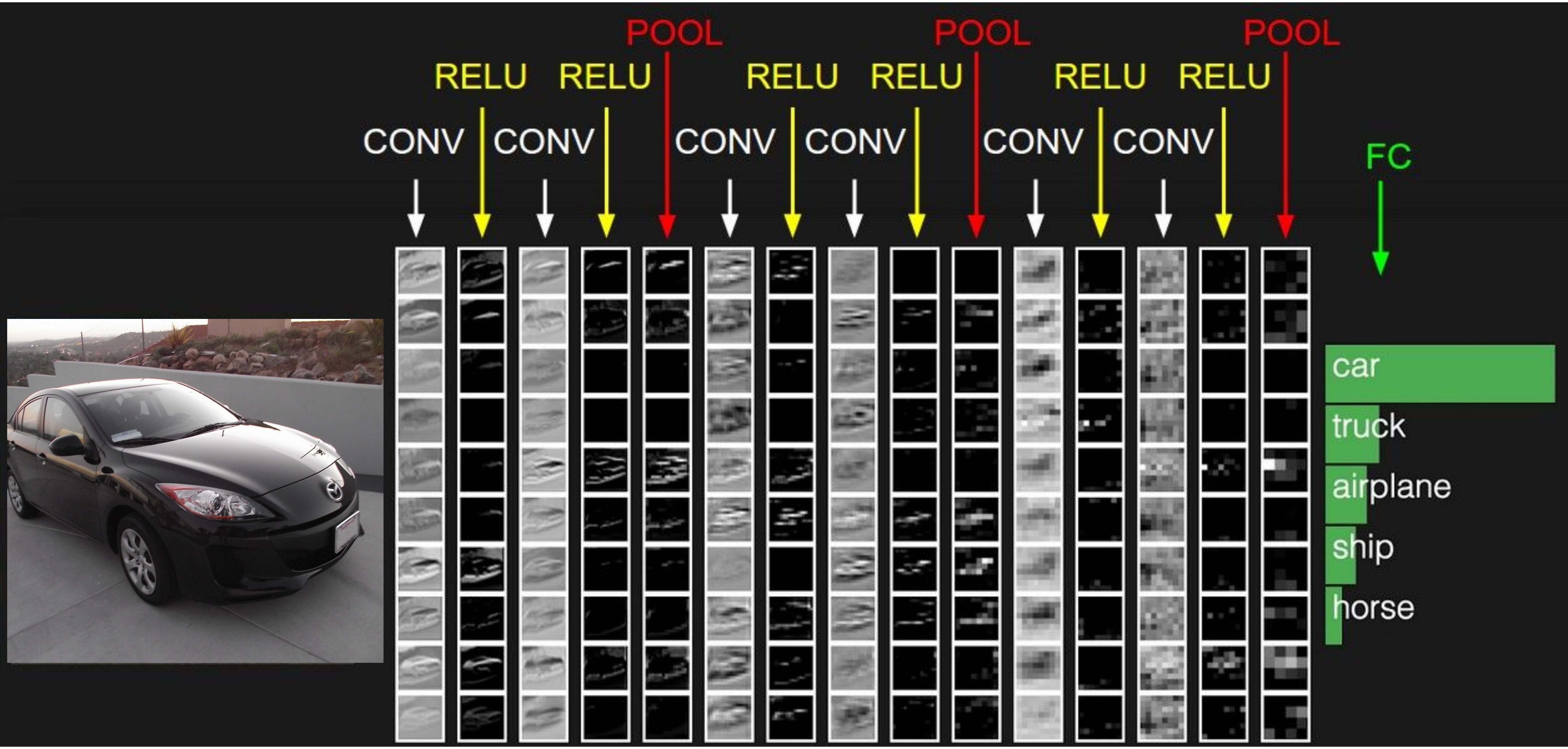
Reminder: Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1

Each neuron looks at the full input volume

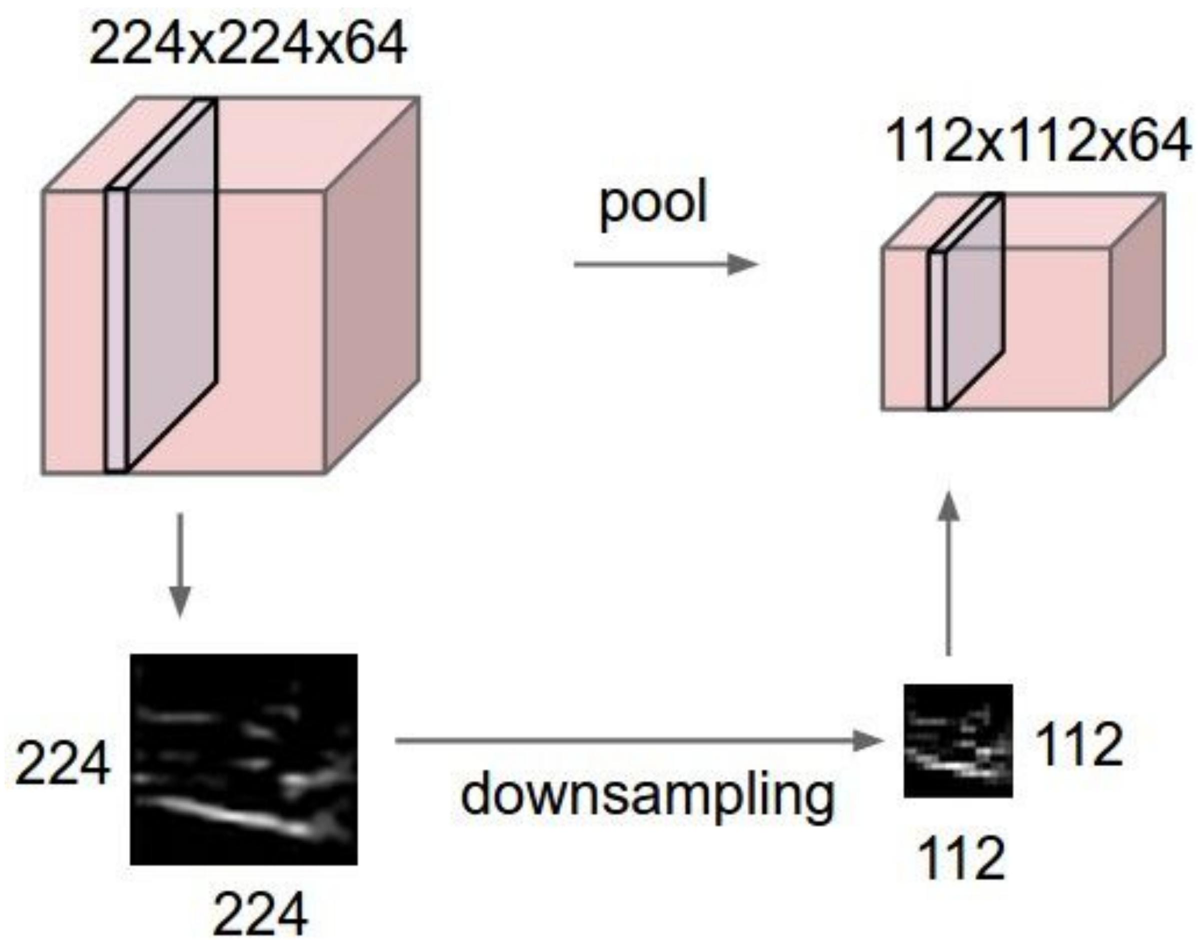


two more layers to go: POOL/FC

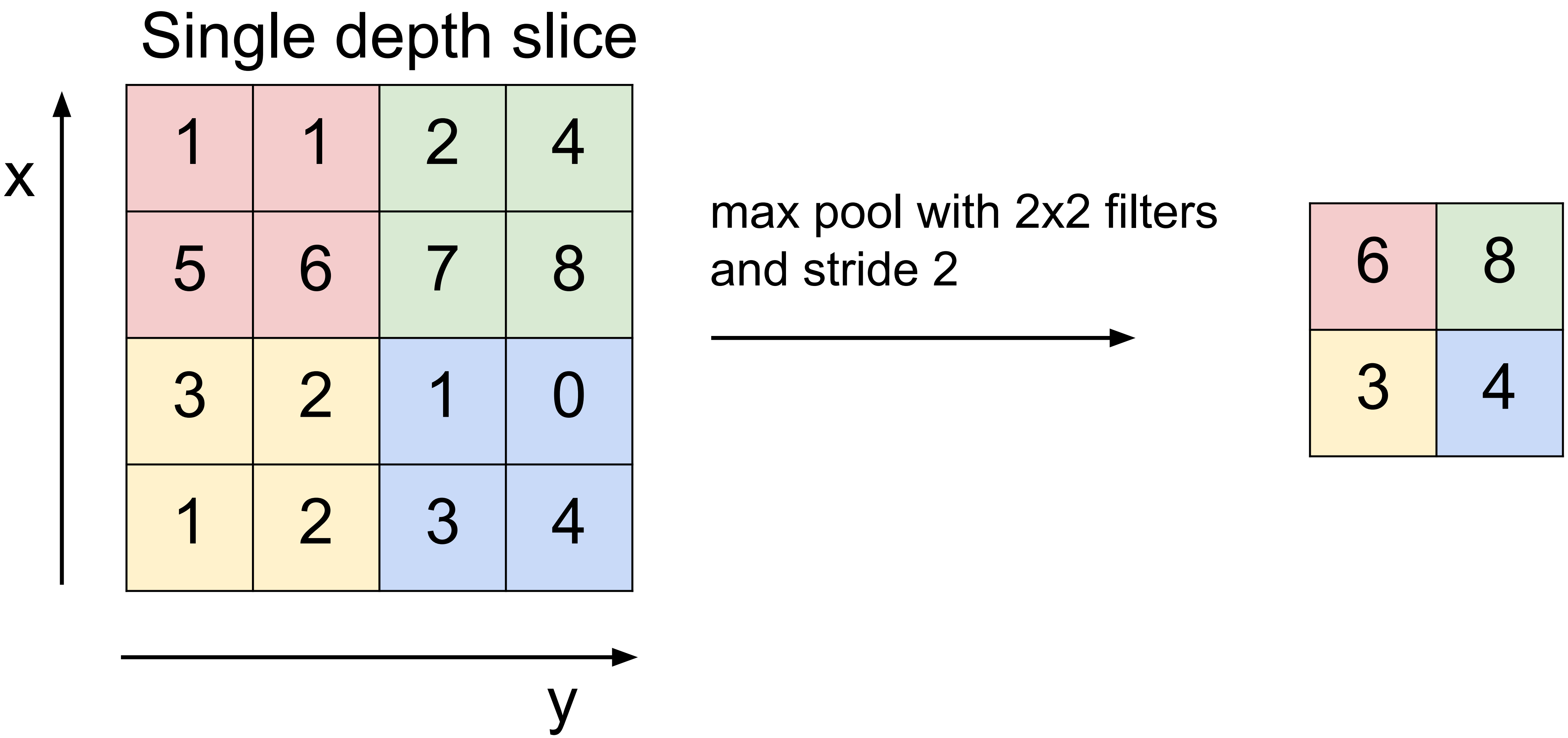


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires three hyperparameters:
  - their spatial extent F,
  - the stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- Note that it is not common to use zero-padding for Pooling layers

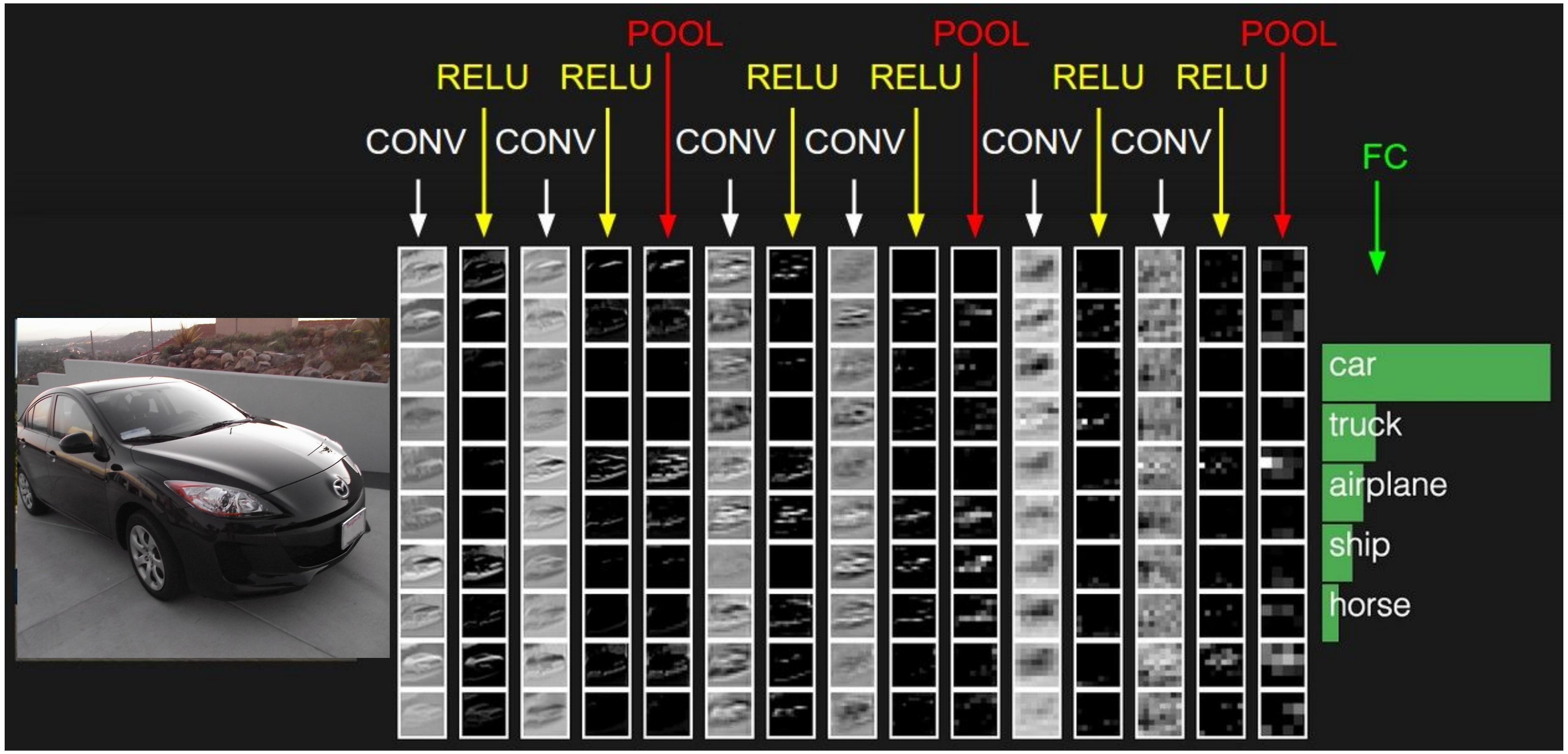
- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires three hyperparameters:
  - their spatial extent F,
  - the stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- Note that it is not common to use zero-padding for Pooling layers

Common settings:

F = 2, S = 2
F = 3, S = 2

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



[ConvNetJS demo: training on CIFAR-10]

ConvNetJS CIFAR-10 demo

Description

This demo trains a Convolutional Neural Network on the [CIFAR-10 dataset](#) in your browser, with nothing but Javascript. The state of the art on this dataset is about 90% accuracy and human performance is at about 94% (not perfect as the dataset can be a bit ambiguous). I used [this python script](#) to parse the [original files](#) (python version) into batches of images that can be easily loaded into page DOM with img tags.

This dataset is more difficult and it takes longer to train a network. Data augmentation includes random flipping and random image shifts by up to 2px horizontally and verically.

By default, in this demo we're using Adadelta which is one of per-parameter adaptive step size methods, so we don't have to worry about changing learning rates or momentum over time. However, I still included the text fields for changing these if you'd like to play around with SGD+Momentum trainer.

Report questions/bugs/suggestions to [@karpathy](#).

http://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html

Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Typical architectures look like

  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX

  where N is usually up to ~5, M is large, 0 <= K <= 2.

  - but recent advances such as ResNet/GoogLeNet challenge this paradigm