Probabilistic Graphical Models

Paul Swoboda & Gerard Pons-Moll

Admin Stuff

- Language: English (all the terminology and books are in English)

  You may ask questions in German.
- Lecturer: Paul Swoboda, Gerard Pons-Moll
- Exercises: Jan-Hendrik Lange, N.N.
- Staff Email: pswoboda@mpi-inf.mpg.de, gpons@mpi-inf.mpg.de
- Course page: mpi-inf.mpg.de/gm
- Slides+script - online
- Announcements: online
- Books:

  Sebastian Nowozin, Christoph Lampert: Structured Learning and Prediction in Computer Vision.

  Bogdan Savchynskyy: Discrete Graphical Models: An Optimization Perspective

  References to books, conference and journal articles after each block

Offers in our research group

Opportunities:

- Bachelor & Master theses
- HiWi-positions

Topics include

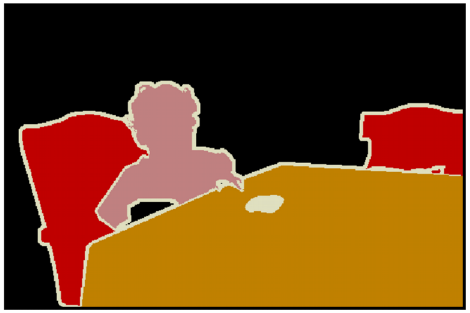
- Machine Learning
- Computer vision
- Optimization for machine learning and computer vision (me)

Come, talk to us!

ML: Main Concepts

Observation, w

Inference



Object state, x

Model $p_w(x)$ Probability theory

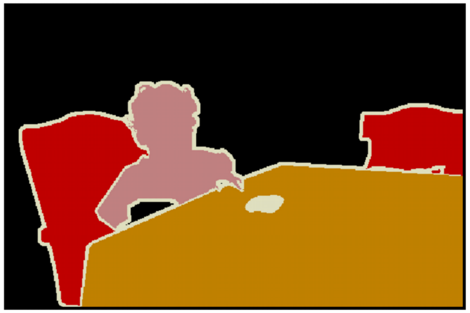
Decision strategy $$x' = \arg\max_x p(x)$$

Theory of statistical decisions

Optimization

ML: Main Concepts

Observation, o



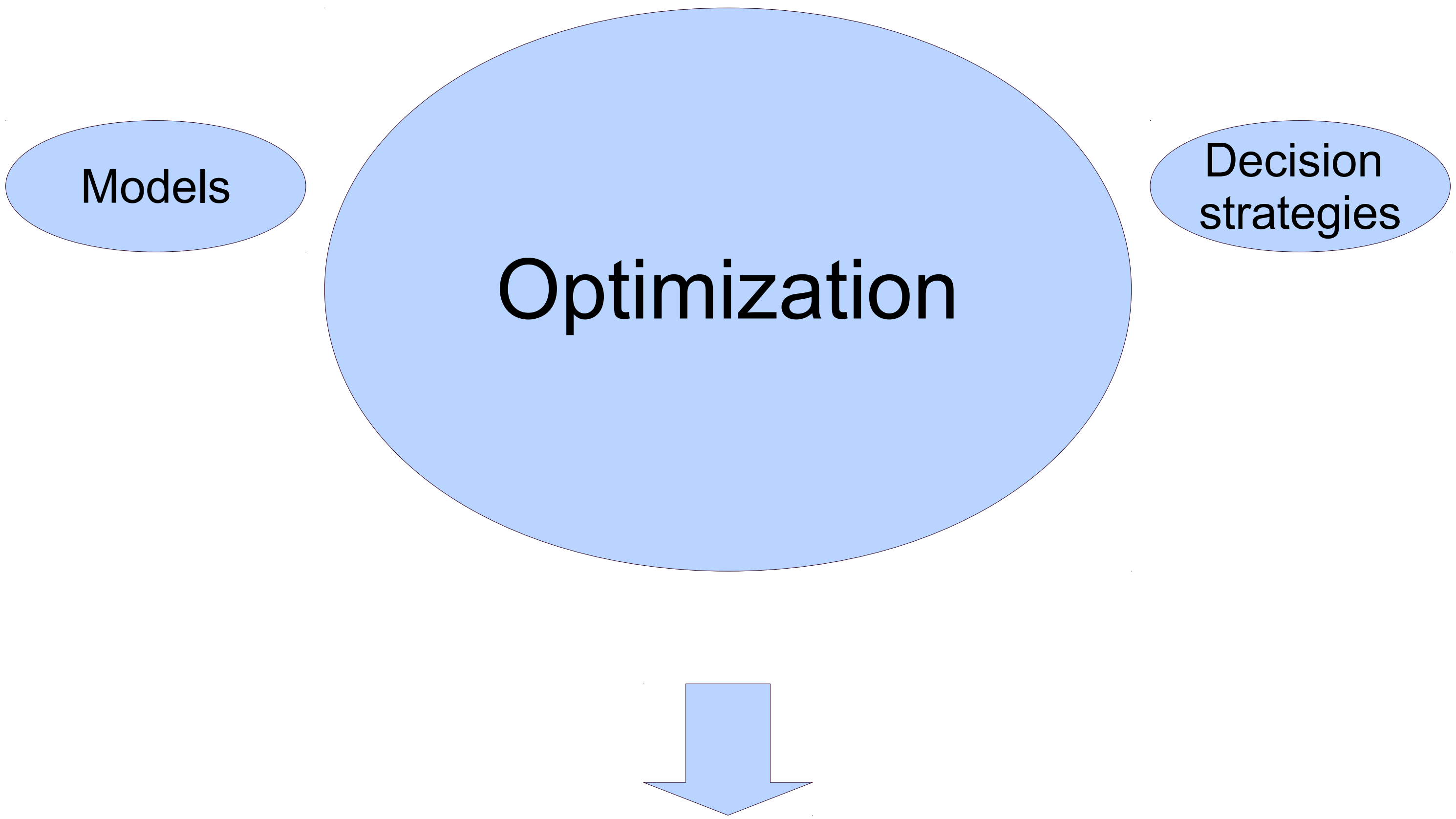
Object state, x

Learning → Mapping o -> w

$$p_w(x)$$

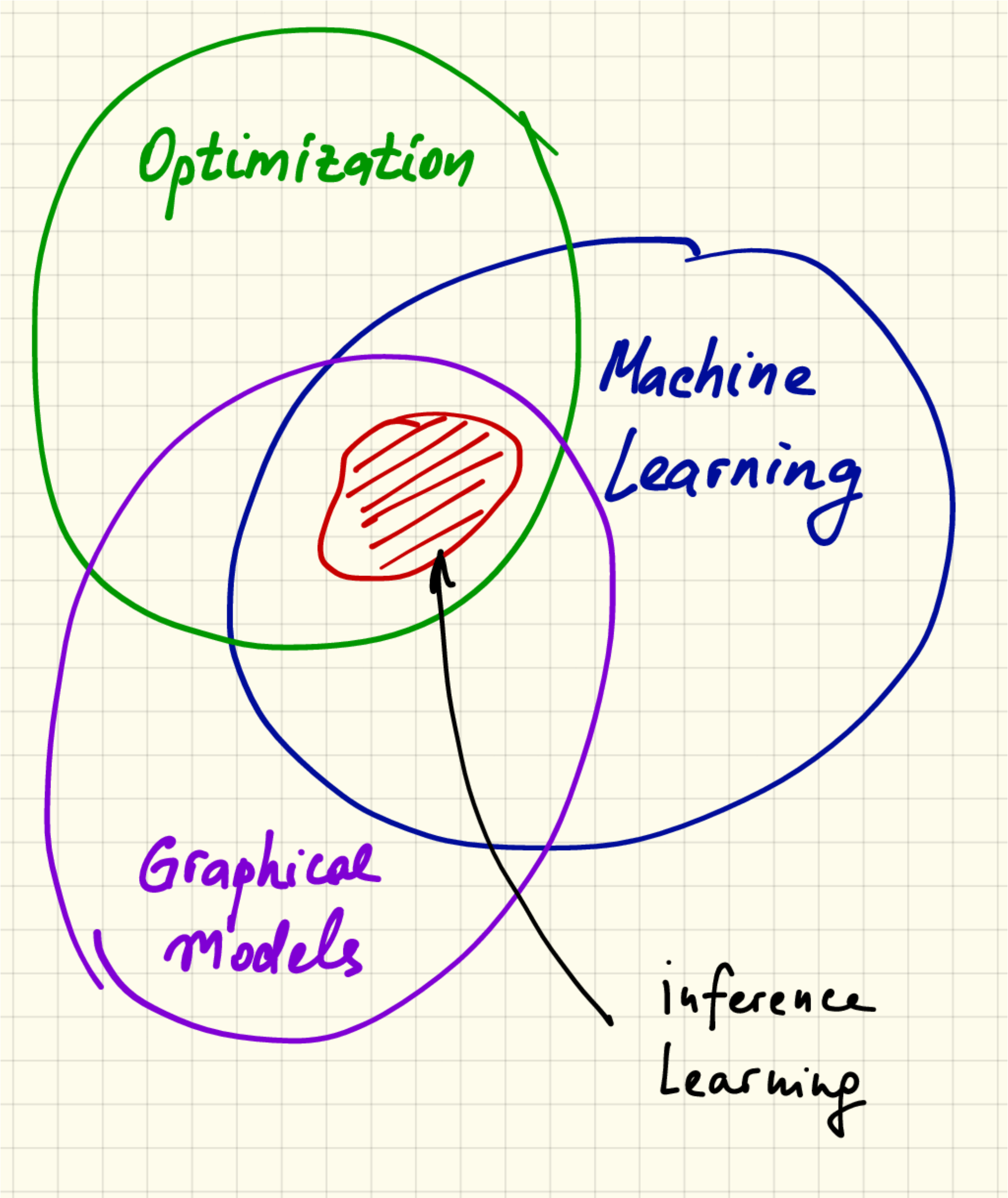
Model	$p_w(x)$	Probability theory
Learning strategy	$\min_w L\left(x, \arg\max_{x'} p_w(x')\right)$	Theory of statistical decisions
		Optimization

Accents of this course



Trade-off between model expressiveness and inference/learning efficiency

Accents of this course



$$x^* = \min_{\overline{x} \in X_V} E_\theta(\overline{x}) := \min_{\overline{x} \in X_V} \sum_{v \in V} \theta_v(x_v) + \sum_{uv \in \mathcal{E}} \theta_{uv}(x_u, x_v)$$

Gibbs distribution (factorized):

$$p_\theta(\overline{x}) = \frac{1}{Z(\theta)} \exp(-E(\overline{x}))$$

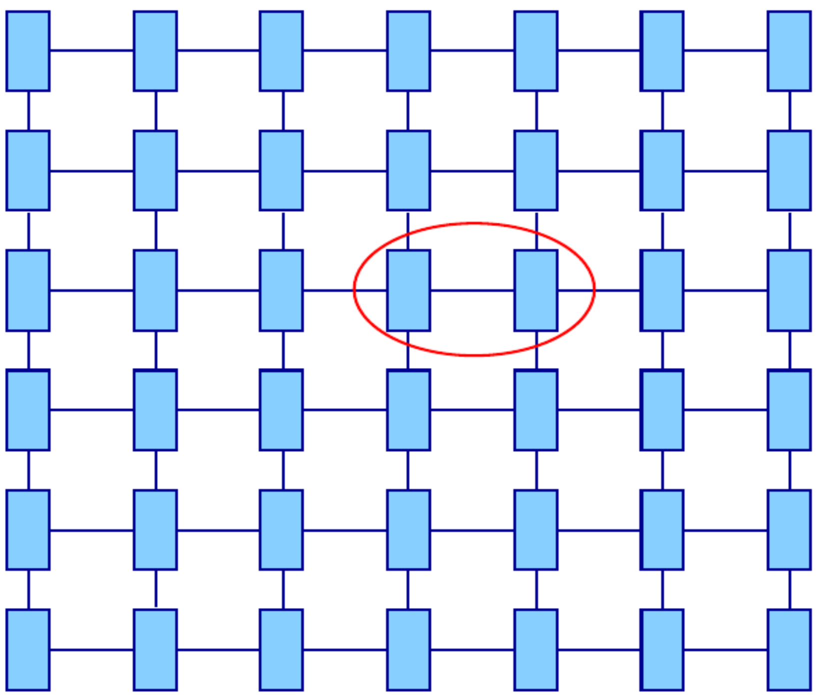
$$= \frac{1}{Z(\theta)} \prod_{v \in V} \Theta_v(x_v) \prod_{uv \in \mathcal{E}} \Theta_{uv}(x_u, x_v)$$

$$\Theta_v(x_v) = \exp(-\theta_v(x_v)) \quad \Theta_{uv}(x_u, x_v) = \exp(-\theta_{uv}(x_u, x_v))$$

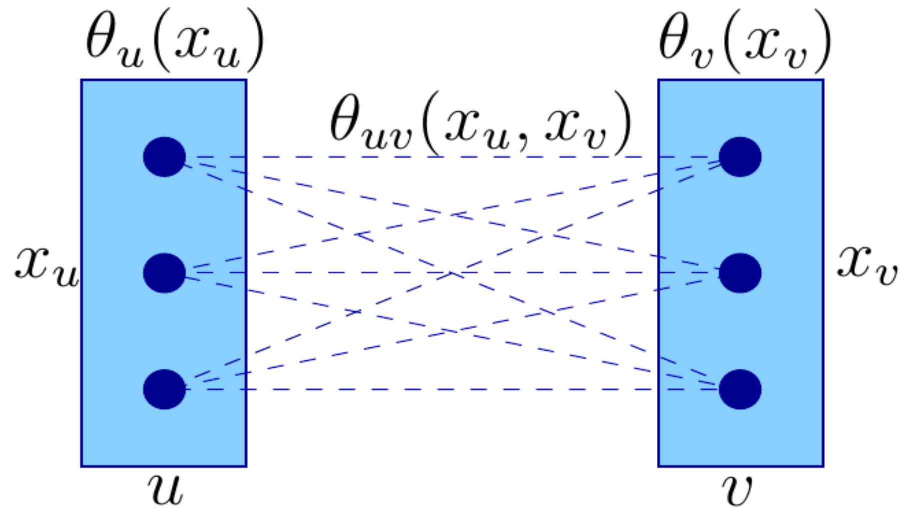
$$Z(\theta, T) = \sum_{\overline{x} \in X_V} \exp(-E(\overline{x})) - \text{partition function}$$

Graphical Models, Inference

$$x^* = \min_{\overline{x} \in X_V} E_\theta(\overline{x}) := \min_{\overline{x} \in X_V} \sum_{v \in V} \theta_v(x_v) + \sum_{uv \in \mathcal{E}} \theta_{uv}(x_u, x_v)$$

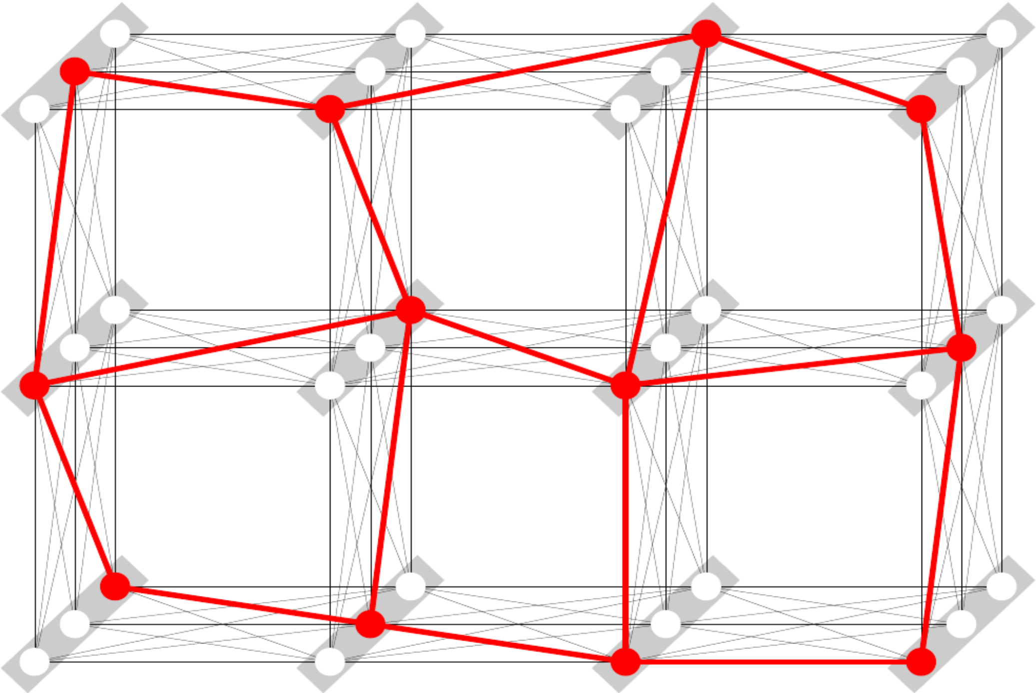


graph $(\mathcal{V}, \mathcal{E})$



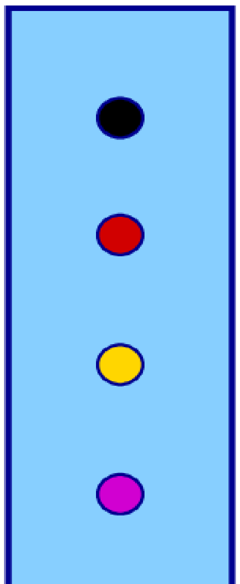
Labeling

$$x^* = \min_{\overline{x} \in X_V} E_\theta(\overline{x}) := \min_{\overline{x} \in X_V} \sum_{v \in V} \theta_v(x_v) + \sum_{uv \in \mathcal{E}} \theta_{uv}(x_u, x_v)$$



Picture: T Werner. A Linear Programming Approach to Max-sum Problem: A Review

Example: Image Segmentation



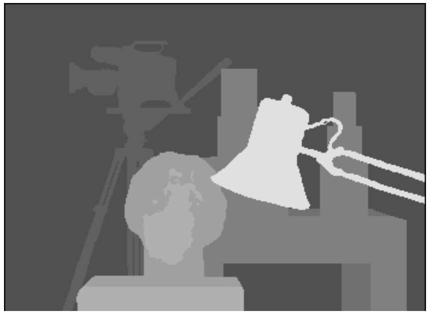
Labels = Segment names, V – set of pixels

$$\theta_v(x_v) \quad \text{- data term}$$

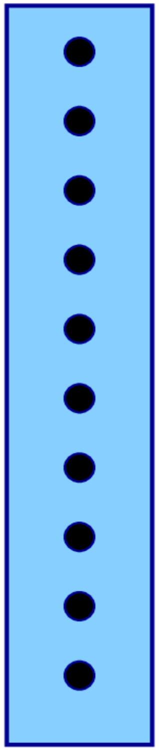
$$\theta_{uv}(x_u, x_v) = \lambda |x_u \neq x_v|$$

Example: Calibrated stereo reconstruction

Left image　　Right image



Disparities



Labels = 1D disparities, V – set of pixels

$$\theta_v(x_v)$$ - data term

$$\theta_{uv}(x_u, x_v) = \lambda |x_u - x_v|$$ - regularizer

Pictures: Middlebury Benchmark

Example: Calibrated stereo reconstruction

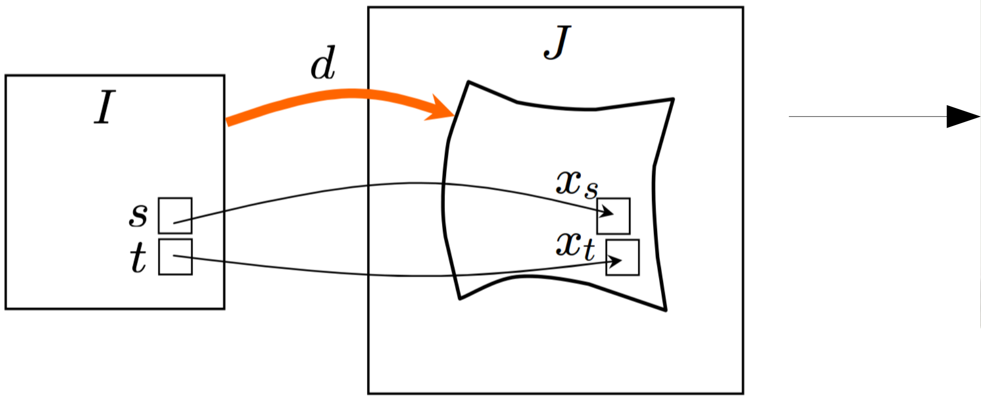
Stereo pair

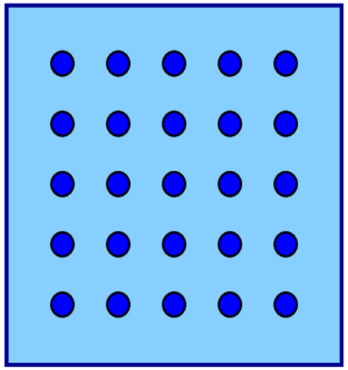
Local reconstruction

Globally consistent reconstruction

Picture: courtesy of D. Schlesinger

Example: Optical flow/Non-rigid matching





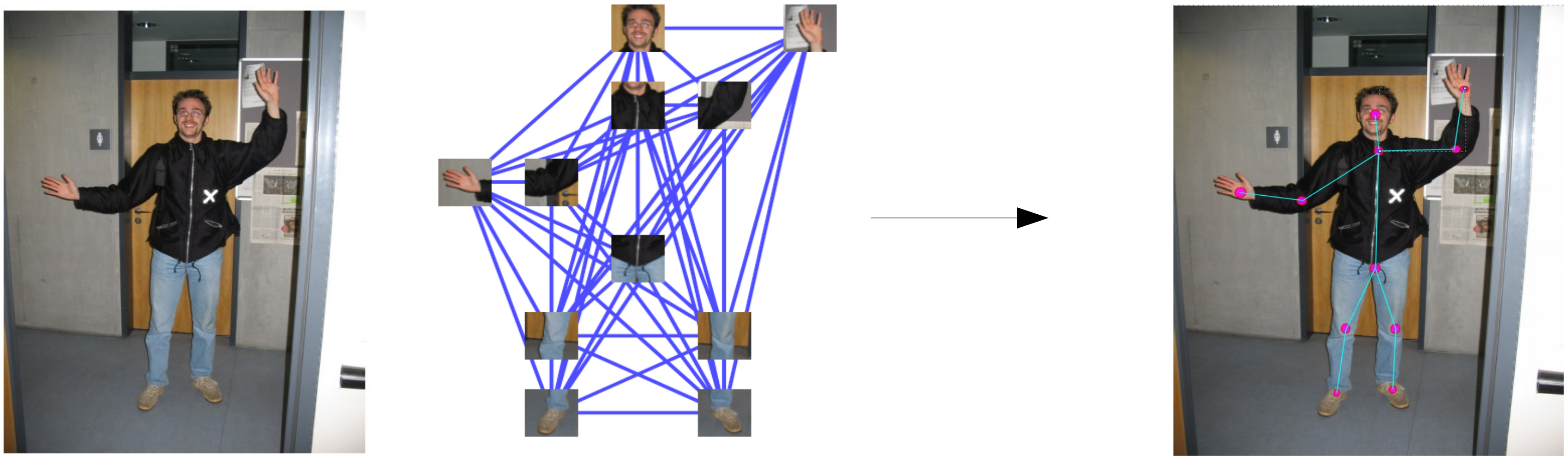
Labels = 2D disparities, V – set of pixels

$$\theta_v(x_v)$$ - data term

$$\theta_{uv}(x_u, x_v) = \lambda \|x_u - x_v\|^2$$ - regularizer

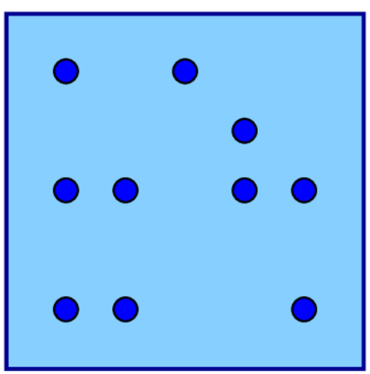
Pictures: http://www.irtc.org.ua/image/

Example: Part-Based Object Detection



Labels = 2D coordinates (sparse),

V – set of parts, Tree-structure

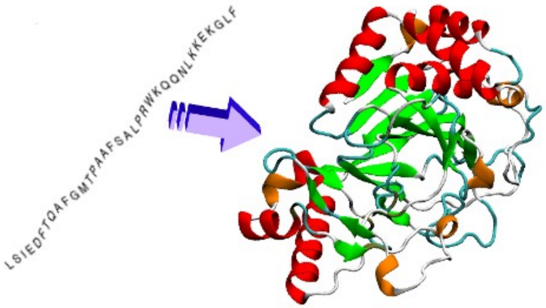


$$\theta_v(x_v)$$ - data term

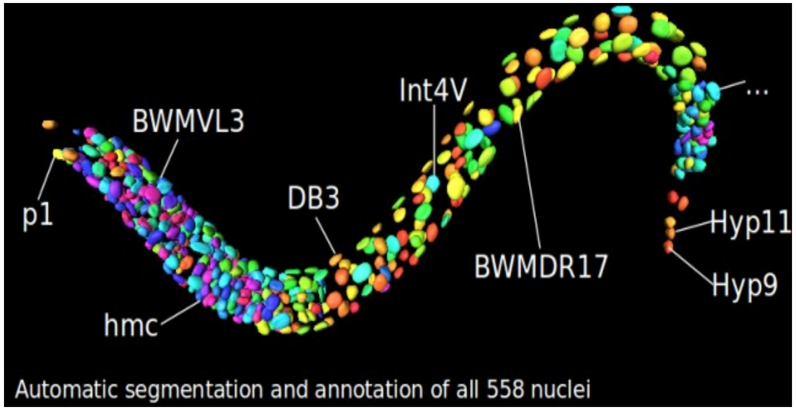
$$\theta_{uv}(x_u, x_v)$$ - geometric prior

Pictures: Bergtholdt, M. and Kappes, J. H. and Schmidt, S. and Schnörr, C.: A Study of Parts-Based Object Class Detection Using Complete Graphs

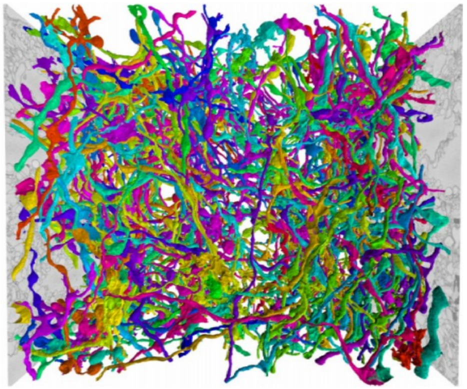
Other examples…



Protein folding



Reconstruction of structure of sim
ple organisms



Segmentation of
neurons

Panorama stitching

Relation to other formalisms

Related areas:

- Hidden Markov Models
- Finite Automata and Formal languages
- Constraint Satisfaction Problems
- Markov Random Fields

Other names in other communities:

Maximum likelihood estimation (MLE) inference
Weighted constraint satisfaction problem
Constraint optimization problem
Energy minimization for graphical models

References

Segmentation: Pascal VOC 2010

Stereo, Panorama: Middlebury Benchmark

Non-rigid matching:

– Shekhovtsov, I. Kovtun, V. Hlavac:
  Efficient MRF Deformation Model for Non-Rigid Image Matching

Part-based object detection:

Bergtholdt, M. and Kappes, J. H. and Schmidt, S. and Schnörr, C.:
A Study of Parts-Based Object Class Detection Using Complete Graphs

Other datasets: OpenGM2 Benchmark, Worm, Neurons, Protein folding