Proof sketch for 7.2.3

3. $N' \models_\tau atr^{-1}(U)$

Ind. on $\Rightarrow^*_{CDCL(\tau)}$

Only interesting cases:

Backtrack D:

$N \models D \Rightarrow N' \models_\tau atr^{-1}(D)$

$N \models T \Rightarrow M \models N \Rightarrow M \models D$

T-Conflict D:

$N' \models T \Rightarrow \beta, atr^{-1}(M) \models_\tau N'$

$\Rightarrow M \models D \Rightarrow L \in M$ s.t. $L \in D$

$\beta \models L \Rightarrow \beta \models D$

$atr^{-1}(L_1), \ldots, atr^{-1}(L_n) \models_\tau \bot$

$\rightsquigarrow \emptyset \models_\tau \neg(atr^{-1}(L_1) \wedge \ldots \wedge atr^{-1}(L_n))$

$\rightsquigarrow \emptyset\ N' \models_\tau \neg(atr^{-1}(L_1) \wedge \ldots \wedge atr^{-1}(L_n))$

4. Remember $N \vee U \models D$ if $(M; N; U; k; D)$

and $D \neq \top$

5. $M \models N$ and $\models \neg \mathrm{atr}^{-1}(M)$

↑ one literal per clause in N is true

↑ all those literals have a common assignment where they evaluate to true

Exactly the condition for N to be satisfiable!

Example: $N' = \underbrace{2x \geq 5}_{P} \wedge (\underbrace{x \leq 1}_{Q} \vee \underbrace{x \geq 6}_{R}) \wedge \underbrace{x \geq 0}_{S}$

$\rightsquigarrow atr(N') = N = P \wedge (Q \vee R) \wedge S$

$(\varepsilon, N, \emptyset, 0, T)$

$\Rightarrow^{*}_{CDCL(T)} ([P^{P}\ S^{S}\ Q^{1}], N, \emptyset, 1, T)$

[Is $\overbrace{2x \geq 5}^{atr^{-1}(P)} \wedge \overbrace{x \geq 0}^{atr^{-1}(S)} \wedge \overbrace{x \leq 1}^{atr^{-1}(Q)}$ is T-satisfiable

contradiction

Simplex says no with Fail Bounds $2x \geq 5 \rightsquigarrow x \geq \frac{5}{2}$

$P \wedge Q$ is already not T-satisfiable]

$\Rightarrow_{CDCL(T)}^{T\text{-}Conflict} (\varepsilon; N; \{\underline{\neg P} \vee \neg Q\}; 0; T)$

$\Rightarrow^{*}_{CDCL(T)} ([P^{P}\ S^{S}\ \neg Q^{\neg P \vee \neg Q}\ R^{Q \vee R}], N, \{\neg P \vee \neg Q\}, 0, T)$

$\Rightarrow^{T\text{-}Success} ([P^{P}\ S^{S}\ \neg Q^{\neg P \vee \neg Q}\ R^{Q \vee R}], N, \{\neg P \vee \neg Q\}, -1, T)$

$$N' = \underbrace{2x \geq 5}_{P} \land (\underbrace{x \leq 1}_{Q} \lor \underbrace{x \geq 6}_{R}) \land \underbrace{x \geq 0}_{S}$$

$$\rightsquigarrow N = P \land (Q \lor R) \land S$$

$$\Rightarrow^{*}_{CDCL(T)} ([P^{P}\, S^{S}], N, \emptyset, 0, T)$$

$$atr^{-1}(P) = x \geq \frac{5}{2} \models \underbrace{x > 1}_{atr^{-1}(\neg Q)}$$

$$\Rightarrow^{T\text{-}Propagate}_{CDCL(T)} ([P^{P}\, S^{S}\, \neg Q^{\neg P \lor \neg Q}], N, \emptyset, 0, T)$$

$$\Rightarrow^{Propagate}_{CDCL(T)} ([P^{P}\, S^{S}\, \neg Q^{\neg P \lor \neg Q}\, R^{Q \lor R}], N, \emptyset, 0, T)$$

$$\Rightarrow^{T\text{-}Success} \ldots$$