Löwenheim–Skolem (1926): If a first-order sentence is valid over an infinite domain, then it is valid over any infinite domain.

\[ U = N \]

\[ R \]

It is possible with TOL (LRA)

\[ \text{Nat}(0') \]

\[ y = x + 1 \land \text{Nat}(x) \rightarrow \text{Nat}(y) \]

\[ 0 < x < 1 \rightarrow \neg \text{Nat}(x) \]

\[ 0 < x \rightarrow \neg \text{Nat}(x) \]

\[ y + 1 = x \land x > 1 \land \neg \text{Nat}(y) \rightarrow \neg \text{Nat}(x) \]

\[ \exists N \in \text{Nat} \land N = \text{Nat} \]
\[ P(f(x + g(y))) \lor \neg P(g(y - 3z)) \]  
\[ \not\exists x \lor g(y_1) \not\subseteq y_2 \lor y_3 = x + y_2 \lor P(f(y_3)) \lor y_4 \not\subseteq y - 3z \lor \neg P(g(y_4)) \]  
\[ y \not\subseteq x \land y_3 = x + y_2 \land y_4 \subseteq y - 3z \]  
[\text{Not abstraction}]  
\[ [\text{abstraction}] \]  
\[ g(y_1) \not\subseteq y_2 \lor P(f(y_3)) \lor \neg P(g(y_4)) \]  
[\text{constraint clause form}]
\[ N' = \exists \Delta_1, \ldots, \Delta_{n+3} \subseteq N \]

If \( N \) is unsat, then there exists a subset \( N' \) of clauses that can be resolved together to get
\[ \Delta_1 \vdash \] where \( \Delta \) has a satisfiable solution

iff
\[ N^* = \exists \Delta_1, \ldots, \Delta_{n+3} \in \text{gndL}(N, Q) \]
there exists a subset \( N^* \) of \( \text{gndL}(N, Q) \)
that can be resolved together to get
\[ \Delta_1 \vdash \] where \( \Delta \) simplifies to true

\[ M = \text{Pla}(b), a \leq b, \text{Pla}(a), a \leq 0, b \geq 2, \]
\[ \exists \text{Pl}(b), \text{P}(b/a) \]
\[ M = P(a), \ a < 5, \ b \mapsto P(b) \]
\[ M_1 = \exists a \ b \mapsto \emptyset, \ b \mapsto 15 \]
\[ M_2 = \exists a \ b \mapsto 1, \ b \mapsto 2 \ 3 \]

\[ \forall \ x = 0.1 \ P(x) \]
\[ \exists x = 0 \ 1 \ 7 \ P(x) \]
\[ \exists x = 2 \ 11 \ P(x) \]

\[ D = \exists a, b \ 1 \ 3 \]

\[ M_1 = P(a), \ a < 2 \]
\[ M_2 = P(a), \ a = 0 \]

\[ M_2 = P(a), \ a = 0 \]

\[ \exists a \ 3 \ a \mapsto 3 \]