Tutorials for “Automated Reasoning WS20/21”
Exercise sheet 1

Exercise 1.1:
Prove by induction that for any propositional formula the number of closing parentheses is equal to the number of opening parentheses (see Definition 2.1.1).

Exercise 1.2:
Determine which of the following formulas are valid/satisfiable/unsatisfiable using propositional semantics, i.e., the definition of $\models$:

1. $\neg(P \vee \neg(P \wedge Q))$
2. $(P \vee Q) \rightarrow (P \wedge Q)$
3. $\neg(P \rightarrow \neg P)$
4. $(P \vee \neg Q) \wedge \neg(\neg P \rightarrow \neg Q)$
5. $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$

Exercise 1.3:
Prove the validity of the following formulas using $\Rightarrow_T$.

1. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
2. $(P \rightarrow Q) \rightarrow ((R \vee P) \rightarrow (R \vee Q))$

Exercise* 1.4:
Consider a satisfiable formula $\phi$ with $A \models \phi$.

1. Prove $\Rightarrow_T$ to be strongly complete with respect to models: if $\{(\phi)\} \Rightarrow^*_T N$ and $N$ is a normal form then there is a sequence $(\phi, \phi_1, \ldots, \phi_n) \in N$ such that $A \models \phi \wedge \phi_1 \wedge \ldots \wedge \phi_n$. 
2. Is $\mathcal{A}$ the only model of $\phi \land \phi_1 \land \ldots \land \phi_n$?

Is is not encouraged to prepare joint solutions, because we do not support joint exams.