Exercise 10.1:
Apply the Knuth-Bendix procedure to the set of equations

\[ E = \{ \ f(f(x)) \approx g(x), \ f(a) \approx b \} \]

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence \( g > f > a > b \).

Exercise 10.2:
Apply \( \Rightarrow_{KBC} \) to the following set of equations. Choose an appropriate ordering. As usual one sort for everything.

\[ E = \{ f(g(x), x) \approx h(x), \ f(g(x), h(y)) \approx f(x, y), \ h(a) \approx a \} \]

Exercise 10.3:
Use the congruence closure algorithm to check whether the equational clause

\[ \forall x, \forall y. f(f(x)) \not\approx x \lor f(x) \not\approx y \lor f(f(y)) \not\approx g(y) \lor x \approx y \lor h(x, y) \approx h(x, g(y)) \]

is valid.

Exercise* 10.4:
Show that for any equation \( (l \approx r) \in E \) and \( \text{vars}(l) \not\supseteq \text{vars}(r) \) the rewrite relation \( \rightarrow_{E} \) is not terminating.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.