Exercise 11.1:
Refute the following set of clauses via Superposition. You may freely choose an ordering and selection function and apply the well-known simplification rules. As usual variables in different clauses are different, $x, y, z$ denote variables and $f, g$ functions and $a$ is a constant.

\begin{align*}
1 & \neg R(g(x), g(y), g(y), g(x)) \\
2 & \neg R(x, y, z, g(a)) \lor R(x, y, z, f(a)) \\
3 & \neg R(x, g(a), f(a), f(a)) \lor R(x, f(a), g(a), g(a)) \\
4 & \neg R(g(a), f(a), f(a), f(a)) \lor R(f(a), g(a), g(a), g(a)) \\
5 & \neg R(g(a), f(a), f(a), f(a)) \lor R(f(a), g(a), g(a), g(a)) \\
6 & \neg R(f(a), f(a), f(a), f(a))
\end{align*}

Exercise 11.2:
Consider the below clause set $N$ over predicate $R$, function $g$ and constant $a$ with respect to an LPO with precedence $g \succ R \succ a$. As usual one sort for everything and $x, y$ are variables.

\begin{align*}
\alpha, g(a) & \vdash R(g(a), g(a)) \\
1 & \neg R(x, y) \lor R(y, x) \\
2 & \neg R(x, x) \\
3 & R(x, g(x)) \\
4 & \neg R(g(a), a)
\end{align*}

a). Compute $N \prec R(g(a), g(a))$ and determine the minimal false clause.

b). Do the respective superposition inference with the minimal false clause, add it to $N$ giving $N'$ and recompute $(N')_I \prec R(g(a), g(a))$.

Exercise 11.3:
Apply Knuth-Bendix completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$E = \{ f(g(x), y) \approx f(x, y), f(g(a), a) \approx f(b, a), g(g(x)) \approx g(x) \}$$

**Exercise 11.4:**
Convert the following formula in CNF using $\Rightarrow_{\text{ACNF}}$: $P \land \neg[(Q \leftrightarrow R) \lor (S \rightarrow T)]$

**Exercise 11.5:**
Prove by congruence closure that the following ground equations are unsatisfiable: $f(a, g(a)) \approx f(b, g(b)), g(a) \approx h(c), h(d) \approx f(h(d), a), f(b, g(b)) \not\approx f(h(c), a)$.

**Exercise 11.6:**
Check whether the following clause set is satisfiable via CDCL(LRA), where you may use the Fourier-Motzkin procedure for the linear rational arithmetic (LRA) part.

$$N = \{ y < 5 + x \lor y > 5 + x, 2x \approx z + 3, y \leq 3x + 2 - z, y - 11 + 3x \geq 2z \}$$

**Exercise 11.7:**
Which of the following statements are true or false? Provide a proof or a counter example.

a). Let $N$ be a first-order clause set without equality containing a clause $C \lor P(t)$. If all resolvents on $P(t)$ result in tautologies, then the clause $C \lor P(t)$ can be removed from $N$ preserving satisfiability of $N$.

b). Let $E$ be a set of equations. If all equations in $E$ are orientable by an instance of the KBO, then $\Rightarrow_{\text{KBC}}$ terminates on $E$.

c). Let $t = f(t_1, \ldots, t_n)$ and $s = f(s_1, \ldots, s_n)$ be two terms such that the $t_i, s_i$ are either variables or constants. Then the unification problem $E = \{ s = t \}$ has a solution if there exists a substitution $\sigma$ such that $t\sigma = s$ or $s\sigma = t$.

**Exercise 11.8:**
A term $t$ is called *linear* if any variable occurs at most once in $t$. Consider a unification problem $E = \{ s = t \}$ where $s, t$ are both linear and don’t share variables. Prove the following: if $E'$ is a normal form of $E$, i.e., $E \Rightarrow_{\text{SU}} E'$, then the size of $E'$, i.e., the number of function, constant and variable symbols, can be linearly bound in the size of $E$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.