This exercise sheet is prototypical for the final exam!

Exercise 14.1:
Use CDCL to decide satisfiability of the following clause set.

1. $P_1 \lor P_2 \lor P_3$
2. $P_4 \lor P_5 \lor P_6$
3. $\neg P_3 \lor P_5$
4. $\neg P_1 \lor \neg P_4$
5. $\neg P_2 \lor \neg P_3$
6. $\neg P_3 \lor \neg P_6$
7. $\neg P_6$
8. $\neg P_5 \lor P_2$

Exercise 14.2:
Consider the following clause set $N$ with respect to an LPO where $g \succ f \succ b \succ a$.

$N = \{f(a, b) \approx b, b \approx a \lor b \approx g(a), b \not\approx g(b), f(a, g(a)) \approx g(b), b \not\approx a\}$

1. Compute $N_I$.
2. Determine the minimal false clause.
3. Compute the superposition inference out of 2., add it to the clause set $N$ compute the new respective $N_I$.

Exercise 14.3:
Refute the following clause set by superposition. Choose an appropriate ordering and selection strategy.

$N = \{f(x, y) \approx g(y, x) \lor R(x, y), \neg R(g(x, y), a), R(f(g(x, y), y), y), \neg R(g(a, x), b) \lor f(x, a) \not\approx g(x, a)\}$
Exercise 14.4:
Use the congruence closure algorithm to check whether the following conjunction of equations
\[ f(f(a)) \approx a \land f(a) \approx b \land f(f(b)) \approx g(b) \land a \not\approx b \land h(a, b) \not\approx h(a, g(b)) \]
is false.

Exercise 14.5:
Consider the below set of inequations and apply the simplex algorithm to it:
\[
\begin{align*}
2x + 5y & \leq -17 \\
3x + 7y & \leq -24 \\
2x + 5y & \geq -17 \\
3x + 7y & \geq -24
\end{align*}
\]

Exercise 14.6:
Prove or refute by counterexample the following statements:

1. If \( \phi \) is a first-order formula and \( x \) a variable, then \( \phi \) is unsatisfiable if and only if \( \exists x.\phi \)
   is unsatisfiable.

2. If \( \phi \) and \( \psi \) are first-order formulas and \( x \) is a variable, then \( \forall x. (\phi \land \psi) \models (\forall x.\phi) \land (\forall x.\psi) \)
   and \( (\forall x.\phi) \land (\forall x.\psi) \models \forall x. (\phi \land \psi) \).

3. If \( \phi \) and \( \psi \) are first-order formulas and \( x \) is a variable, then \( \exists x. (\phi \land \psi) \models (\exists x.\phi) \land (\exists x.\psi) \)
   and \( (\exists x.\phi) \land (\exists x.\psi) \models \exists x. (\phi \land \psi) \).

Exercise 14.7:
Let \( \phi = P(f(c)) \land \forall x. (P(x) \to P(f(x))) \) and \( \psi = \forall x. P(f(f(x))) \) be first-order formulas.
Prove that every Herbrand model of \( \phi \) considering the signature of \( \phi \) only is also a model of \( \psi \).

It is not encouraged to prepare joint solutions, because we do not support joint exams.