

**(3.1)** Let  $N = \{(1)\neg P_1 \vee \neg P_2, (2)P_3 \vee P_2 \vee P_4, (3)P_2 \vee \neg P_4, (4)\neg P_3 \vee P_2, (5)P_1 \vee P_2 \vee P_4\}$

$$\begin{aligned}
& (\epsilon; N; \emptyset; 0; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Decide}(P_4)} (P_4^1; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(P_2)} (P_4^1 P_2^{(3)}; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(\neg P_1)} (P_4^1 P_2^{(3)} \neg P_1^{(1)}; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Decide}(P_3)} (P_4^1 P_2^{(3)} \neg P_1^{(1)} P_3^2; N; \emptyset; 2; \top)
\end{aligned}$$

All literals from  $N$  are assigned and  $M \models N$ , hence  $N$  is satisfiable ( $M$  is already a partial model of  $N$  in the penultimate step).

**(3.2)** Let  $N = \{(1)P_1 \vee P_2 \vee P_3, (2)P_4 \vee P_5 \vee P_6, (3)\neg P_3 \vee P_5, (4)\neg P_1 \vee \neg P_4, (5)\neg P_2 \vee \neg P_5, (6)\neg P_3 \vee \neg P_6, (7)\neg P_6, (8)\neg P_5 \vee P_2\}$

$$\begin{aligned}
& (\epsilon; N; \emptyset; 0; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(\neg P_6)} (\neg P_6^{(7)}; N; \emptyset; 0; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Decide}(\neg P_5)} (\neg P_6^{(7)} \neg P_5^1; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(P_4)} (\neg P_6^{(7)} \neg P_5^1 P_4^{(2)}; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(\neg P_1)} (\neg P_6^{(7)} \neg P_5^1 P_4^{(2)} \neg P_1^{(4)}; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(\neg P_3)} (\neg P_6^{(7)} \neg P_5^1 P_4^{(2)} \neg P_1^{(4)} \neg P_3^{(3)}; N; \emptyset; 1; \top) \\
& \Rightarrow_{\text{CDCL}}^{\text{Propagate}(P_2)} (\neg P_6^{(7)} \neg P_5^1 P_4^{(2)} \neg P_1^{(4)} \neg P_3^{(3)} P_2^{(1)}; N; \emptyset; 1; \top)
\end{aligned}$$

All literals from  $N$  are assigned and  $M \models N$ , hence  $N$  is satisfiable.

**(3.3)** I only show the first example. Consider the clauses  $\neg Q_0 \vee \neg P_2 \vee Q_1, \neg Q_1 \vee Q_2, P_0 \vee Q_0, \neg Q_0 \vee P_1, Q_0 \vee P_1\}$  with ordering  $Q_2 \succ P_2 \succ Q_1 \succ P_1 \succ Q_0 \succ P_0$  where maximal literals are marked with \*.

$D$	$N_D$	$\delta_D$	Comment
$P_0 \vee Q_0^*$	$\emptyset$	$\{Q_0\}$	
$Q_0 \vee P_1^*$	$\{Q_0\}$	$\emptyset$	clause is true
$\neg Q_0 \vee P_1^*$	$\{Q_0\}$	$\{P_1\}$	
$\neg Q_0 \vee \neg P_2^* \vee Q_1$	$\{Q_0, P_1\}$	$\emptyset$	negative literal maximal
$\neg Q_1 \vee Q_2^*$	$\{Q_0, P_1\}$	$\emptyset$	clause is true

**(3.4)** Show unsatisfiability of the below clause set  $N$  via the propositional superposition calculus based on the atom ordering  $P_1 \succ P_4 \succ P_5 \succ P_2 \succ P_3$

where now maximal literals are marked with \*.

$$\begin{array}{lll}
(1) & P_1^* \vee P_2 \vee P_3 & (2) \quad \neg P_1^* \vee \neg P_2 & (3) \quad \neg P_2^* \vee \neg P_3 \\
(4) & \neg P_1^* \vee \neg P_3 & (5) \quad P_4 \vee P_5 \vee P_1^* & (6) \quad \neg P_4 \vee P_1^* \\
(7) & \neg P_4^* \vee P_2 & (8) \quad \neg P_5^* \vee P_2 & (9) \quad \neg P_5^* \vee P_3 \\
(10) & \neg P_1^* \vee P_4 & & 
\end{array}$$

Initial clause set  $N_0 = N$  then incremented index refers to clause set from previous step.

$$\begin{array}{ll}
\Rightarrow_{\text{SUP}}^{(10)\text{SubRes}(5)} & N_0 \cup \{(12) P_4^* \vee P_5\} \\
\Rightarrow_{\text{SUP}}^{(6)\text{SubRes}(5)} & N_1 \cup \{(13) P_1^* \vee P_5\} \\
\Rightarrow_{\text{SUP}}^{(13)\text{Sup}(2)} & N_2 \cup \{(14) \neg P_2^* \vee P_5\} \\
\Rightarrow_{\text{SUP}}^{(12)\text{Sup}(7)} & N_3 \cup \{(15) P_2^* \vee P_5\} \\
\Rightarrow_{\text{SUP}}^{(14)\text{SubRes}(15)} & N_4 \cup \{(16) P_5\} \\
\Rightarrow_{\text{SUP}}^{(16)\text{SubRes}(8)} & N_5 \cup \{(17) P_2\} \\
\Rightarrow_{\text{SUP}}^{(16)\text{SubRes}(9)} & N_6 \cup \{(18) P_3\} \\
\Rightarrow_{\text{SUP}}^{(17)\text{SubRes}(3)} & N_7 \cup \{(19) \neg P_3\} \\
\Rightarrow_{\text{SUP}}^{(19)\text{SubRes}(18)} & N_8 \cup \{(20) \perp\}
\end{array}$$

**(3.5)** Consider the following CDCL derivation:

$$\begin{array}{ll}
(\epsilon; N; \emptyset; 0; \top) & \\
\Rightarrow_{\text{CDCL}}^{\text{Decide}(P_2)} & (P_2^1; N; \emptyset; 1; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Propagate}(P_1)} & (P_2^1 P_1^{(2)}; N; \emptyset; 1; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Conflict}(3)} & (P_2^1 P_1^{(2)}; N; \emptyset; 1; \neg P_1 \vee \neg P_2) \\
\Rightarrow_{\text{CDCL}}^{\text{Resolve}(P_1)} & (P_2^1; N; \emptyset; 1; \neg P_2) \\
\Rightarrow_{\text{CDCL}}^{\text{Backtrack}} & (\neg P_2^{(9)}; N; \{(9) \neg P_2\}; 0; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Restart}} & (\epsilon; N; U_1; 0; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Decide}^*(\neg P_1, \neg P_3)} & (\neg P_1^1 \neg P_3^2; N; U_1; 2; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Propagate}(\neg P_2)} & (\neg P_1^1 \neg P_3^2 \neg P_2^{(9)}; N; U_1; 2; \top) \\
\Rightarrow_{\text{CDCL}}^{\text{Conflict}(1)} & (\neg P_1^1 \neg P_3^2 \neg P_2^{(9)}; N; U_1; 2; P_1 \vee P_2 \vee P_3) \\
\Rightarrow_{\text{CDCL}}^{\text{Resolve}(P_2)} & (\neg P_1^1 \neg P_3^2; N; U_1; 2; P_1 \vee P_3) \\
\Rightarrow_{\text{CDCL}}^{\text{Backtrack}} & (\neg P_1^1 P_3^{(10)}; N; U_1 \cup \{(10) P_1 \vee P_3\}; 1; \top) \\
U_1 = \{(9) \neg P_2\} & 
\end{array}$$