Exercise 4.1:

Let \( a : \rightarrow S \) and \( R \subseteq S \times T \). Complete the sort information for \( g \), \( f \), \( P \) and variables \( x \), \( y \) such that the following formula is well-sorted: \( \forall x, y. (R(x, g(x)) \rightarrow (f(g(x), a) \approx y \lor P(y) \lor R(x, y))) \)

Exercise 4.2:

Check whether the following first-order formulas are satisfiable, valid or unsatisfiable, where \( a \) and \( b \) are constants and \( g \) is a unary function symbol. Assume a one-sorted universe.

1. \( (\forall x. \exists y. R(x, y)) \rightarrow R(a, b) \)
2. \( (P(a) \land \forall x. (P(x) \rightarrow P(g(x)))) \rightarrow P(g(a)) \)
3. \( (\exists x. P(x)) \rightarrow P(b) \)
4. \( P(b) \rightarrow (\exists x. P(x)) \)

Exercise 4.3:

Use the FM method to decide whether the following conjunction of inequations is satisfiable:

\[
\begin{align*}
    x + y & \geq 16 \\
    4x + 7y & \leq 28 \\
    2x - 7y & \leq 20 \\
    2x - 3y & \geq -9
\end{align*}
\]
Exercise 4.4:
Check via FM whether the following formulas are true/false:

1. $\forall x.\exists y.(2x + y > 7 \land x + y < 6)$
2. $\exists x.\forall y.(2x - y > 7 \land 2x + y > 7)$

Exercise* 4.5:
Provide first-order formulas such that the domain of any interpretation satisfying the formula

1. has exactly 3 elements
2. is infinite

Is is not encouraged to prepare joint solutions, because we do not support joint exams.