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FR Informatik

Bromberger/Möhle/Schwarz/Weidenbach **November 17, 2022**

Tutorials for "Automated Reasoning WS22/23"
Exercise sheet 4

Exercise 4.1:
Let $a :\to S$ and $R \subseteq S \times T$. Complete the sort information for g, f, P and variables x, y such that the following formula is well-sorted: $\forall x, y.(R(x, g(x)) \to (f(g(x), a) \approx y \vee P(y) \vee R(x, y)))$

Exercise 4.2:
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Justify, i.e. either prove that the formula is valid or unsatisfiable, or give examples to show that they are satisfiable but not valid.

1. $\forall x.P(x) \to \exists x.P(x)$
2. $\forall x.(P(x) \to P(f(x))) \wedge P(b) \wedge \neg P(f(f(b)))$
3. $[\forall x.(P(x) \to P(f(x))) \wedge P(b)] \to \forall x.P(x)$
4. $[\forall x.(P(x) \to P(f(x)))] \to \exists x.P(x)$
5. $\forall x.(P(x) \vee Q(x)) \to [\forall x.P(x) \vee \forall x.Q(x)]$
6. $\forall x.\exists y.P(x, y) \to \exists x.\forall y.P(x, y)$
7. $\forall x. (P(x) \to P(f(x))) \to \forall x.P(x)$

Exercise* 4.3:
Prove or refute the following statements:

1. If ϕ is a first-order formula and x a variable, then ϕ is unsatisfiable if and only if $\exists x.\phi$ is unsatisfiable.
2. If ϕ and ψ are first-order formulas and x is a variable, then $\forall x.(\phi \wedge \psi) \models (\forall x.\phi) \wedge (\forall x.\psi)$ and $(\forall x.\phi) \wedge (\forall x.\psi) \models \forall x.(\phi \wedge \psi)$.

3. If ϕ and ψ are first-order formulas and x is a variable, then $\exists x.(\phi \wedge \psi) \models (\exists x.\phi) \wedge (\exists x.\psi)$ and $(\exists x.\phi) \wedge (\exists x.\psi) \models \exists x.(\phi \wedge \psi)$.

It is not encouraged to prepare joint solutions, because we do not support joint exams.