



Christoph Weidenbach

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Tutorials for “Automated Reasoning WS18/19”
Exercise sheet 5

Exercise 5.1 (2.56):

Show unsatisfiability of the below clause set N via the superposition calculus based on the atom ordering $P_1 \succ P_4 \succ P_5 \succ P_2 \succ P_3$.

- | | | |
|------------------------------|------------------------------|------------------------------|
| (1) $P_1 \vee P_2 \vee P_3$ | (2) $\neg P_1 \vee \neg P_2$ | (3) $\neg P_2 \vee \neg P_3$ |
| (4) $\neg P_1 \vee \neg P_3$ | (5) $P_4 \vee P_5 \vee P_1$ | (6) $\neg P_4 \vee P_1$ |
| (7) $\neg P_4 \vee P_2$ | (8) $\neg P_5 \vee P_2$ | (9) $\neg P_5 \vee P_3$ |
| (10) $\neg P_1 \vee P_4$ | | |

Exercise 5.2 (New):

Consider the ordering $P_1 \prec P_2 \prec P_3 \prec P_4$. Compute $N_{\mathcal{I}}$ on the below clause set, determine the minimal false clause, compute the respective inference, add the clause to the clause set and then recompute $N_{\mathcal{I}}$ on the updated clause set.

- | | | |
|------------------------------|-----------------------------|-------------------------|
| (1) $\neg P_1 \vee \neg P_2$ | (2) $P_3 \vee P_2 \vee P_4$ | (3) $P_2 \vee \neg P_4$ |
| (4) $\neg P_3 \vee P_2$ | (5) $P_1 \vee P_2 \vee P_3$ | |

Exercise 5.3 (3.1):

Let $a : \rightarrow S$ and $R \subseteq S \times T$. Complete the sort information for g, f, P and variables x, y such that the following formula is well-sorted: $\forall x, y. (R(x, g(x)) \rightarrow (f(g(x), a) \approx y \vee P(y) \vee R(x, y)))$.

Exercise 5.4 (3.3):

Check whether the following first-order formulas are satisfiable, valid or unsatisfiable, where a and b are constants and g is a unary function symbol. Assume a one-sorted universe.

1. $(\forall x. \exists y. R(x, y)) \rightarrow R(a, b)$
2. $(P(a) \wedge \forall x. (P(x) \rightarrow P(g(x)))) \rightarrow P(g(g(a)))$
3. $(\exists x. P(x)) \rightarrow P(b)$

4. $P(b) \rightarrow (\exists x.P(x))$

It is not encouraged to prepare joint solutions, because we do not support joint exams.