max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Christoph Weidenbach **November27, 2018**

Tutorials for "Automated Reasoning WS18/19"
Exercise sheet 6

Exercise 6.1 (3.7):
Prove or refute the following statements:

1. If ϕ is a first-order formula and x a variable, then ϕ is unsatisfiable if and only if $\exists x.\phi$ is unsatisfiable.
2. If ϕ and ψ are first-order formulas and x is a variable, then $\forall x.(\phi \wedge \psi) \models (\forall x.\phi) \wedge (\forall x.\psi)$ and $(\forall x.\phi) \wedge (\forall x.\psi) \models \forall x.(\phi \wedge \psi)$.
3. If ϕ and ψ are first-order formulas and x is a variable, then $\exists x.(\phi \wedge \psi) \models (\exists x.\phi) \wedge (\exists x.\psi)$ and $(\exists x.\phi) \wedge (\exists x.\psi) \models \exists x.(\phi \wedge \psi)$.

Exercise 6.2 (3.60):
Let the terms r, s, t be defined by

$$\begin{aligned} r &= g(f(x, h(c))) \\ s &= f(h(x), g(f(b, g(x)))) \\ t &= f(h(h(c)), f(c, x)) \end{aligned}$$

Check for each pair of terms $(r, s), (r, t), (s, t)$, whether the terms are comparable using an LPO with precedence $f \succ g \succ b \succ h \succ c$. If they are comparable, say which term is larger.

Exercise 6.3 (3.63):
Consider a signature with constants a, b, unary function g, and unary predicates P, Q. As usual one sort S serves all.

1. Find some Knuth-Bendix ordering (i.e., define weight function and precedence) in such a way that the following will hold:
$P(a) \succ_{kbo} Q(g(b)) \succ_{kbo} P(g(b)) \succ_{kbo} P(b)$

2. Do the same for LPO:
   $P(a) \succ_{lpo} Q(g(b)) \succ_{lpo} P(g(b)) \succ_{lpo} P(b)$

Justify your definitions.

Exercise* 6.4 (3.67):
Prove or provide a counter example for the following statements.

1. If two terms are comparable with respect to an LPO instance, then they are comparable with respect to a KBO instance.
2. If two terms are comparable with respect to a KBO instance, then they are comparable with respect to an LPO instance.

It is not encouraged to prepare joint solutions, because we do not support joint exams.