Exercise 7.1:
Refute the following set of clauses using resolution.

\[ N = \{ P(a) \lor P(b), \neg P(x) \lor \neg P(f(x)) \lor Q(f(a)), \neg P(x) \lor P(f(x)), Q(a), \neg Q(f(x)) \lor \neg Q(x), Q(f(x)) \lor \neg P(x) \} \]

Exercise 7.2:
Compute all possible resolution inferences out of the below clauses:

1. \( P(x, x) \lor P(h(x', b), h(c, x'')) \)
2. \( \neg P(y, f(y)) \lor Q(g(y)) \)
3. \( \neg Q(z) \lor P(d, z) \).

Do not compute recursive inferences, i.e., consider only inferences with parents (1), (2), (3).

Exercise 7.3:
Let \( \Omega = \{ f, g, h, b, c \} \) with \( g \) arity 2, \( f \) and \( h \) arity 1 and \( b \) and \( c \) constants. and let

\[
\begin{align*}
t_1 &= g(h(x), h(c)), \\
t_2 &= g(x, x), \\
t_3 &= g(b, f(x)), \\
t_4 &= f(g(x, y)), \\
t_5 &= h(g(x, c)).
\end{align*}
\]

Determine for each \( 1 \leq i < j \leq 5 \) whether \( t_i \) and \( t_j \) are incomparable or comparable (and if so, which term is larger) with respect to

1. a lexicographic path ordering with precedence \( f > g > h > b > c \),
2. a Knuth-Bendix-ordering with precedence $h > f > g > b > c$, where $h$ has weight 0 and all other symbols have weight 1.

Exercise 7.4:
Prove or provide a counter example for the following statements.

1. If two terms are comparable with respect to an LPO instance, then they are comparable with respect to a KBO instance.

2. If two terms are comparable with respect to a KBO instance, then they are comparable with respect to an LPO instance.

Exercise* 7.5:
Prove that LPO is well-defined, i.e., the overlaps between the cases 2.x lead to unique results.

Is is not encouraged to prepare joint solutions, because we do not support joint exams.